Towards Modeling Pellet-Produced Plasmoid Dynamics in Stellarators Using the Nonlinear MHD Code JOREK

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

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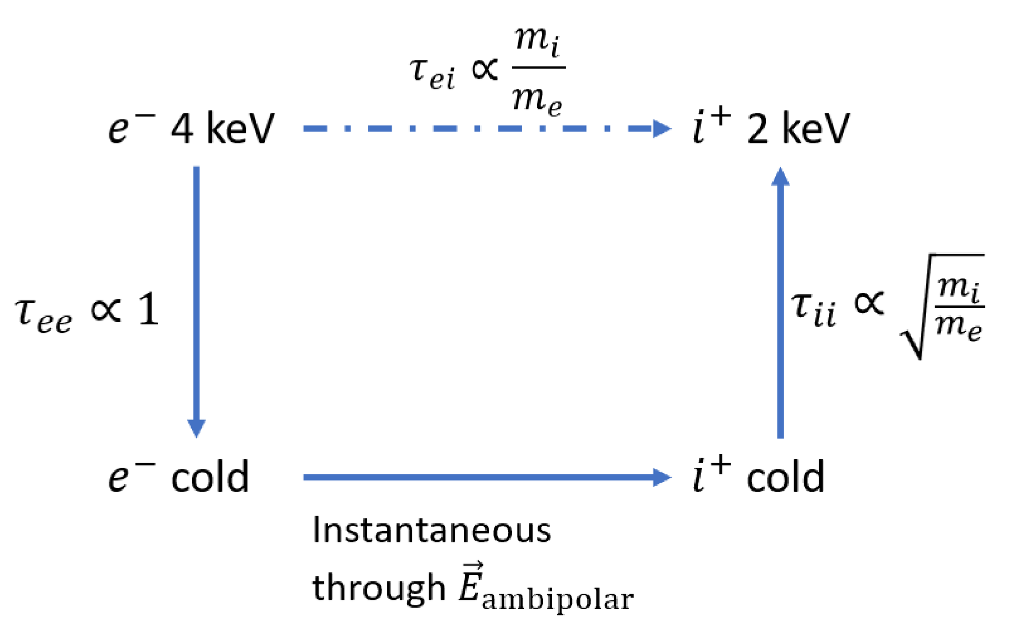
1. MOTIVATION

- Refueling future tokamak and stellarator reactors will likely be based on cryogenic pellet Injection
- Pellet injection is well-studied in tokamaks, however the pellet-supplied density transport is less well understood in stellarators, where core fueling works better than predicted [2, 3]
- Theory suggests that the expansion of the pellet ablation cloud - the "plasmoid" - is accompanied by significant electron-ion energy transfer. [4]
- Previously, available numerical tools could not capture the 3D physics with sufficient spatial and temporal resolution while covering the timescales of the key physical effects in stellarators
- Recently the stellarator extension of the JOREK non-linear 3D MHD code has been developed [5], which can be used to bridge this gap



2. REDUCED MHD

The JOREK fluid model is based on the following coupled MHD equations:

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = \nabla \cdot (\underline{D} \nabla \rho) + S_\rho, & \text{Continuity Eq.} \\ \partial_t p + \vec{v} \cdot \nabla p + \gamma p \nabla \cdot \vec{v} = \nabla \cdot (\underline{\kappa} \nabla T) + S_p, & \text{Energy Eq.} \\ \rho \partial_t \vec{v} + \rho \vec{v} \cdot \nabla \vec{v} + \nabla p = \vec{j} \times \vec{B} + \nabla \cdot (\underline{\nu} \nabla \vec{v}), & \text{Eq. of Motion} \\ \partial_t \vec{B} = -\nabla \times \vec{E}, & \text{Faraday's Law} \\ \nabla \times \vec{B} = \mu_0 \vec{J}, & \text{Ampere's Law} \\ \nabla \cdot \vec{B} = 0 & \text{Gauss's Law} \\ \eta \vec{J} = \vec{E} + \vec{v} \times \vec{B} & \text{Ohm's Law} \end{cases}$$

Due to the numerical complexity of 3D non-linear full MHD simulations, it is useful to reduce the simulated physics to cover longer time scales in realistic stellarator geometries [5].

$$\vec{B} = \underbrace{\nabla \chi}_{\vec{B}_v = \nabla \psi_v \times \nabla \beta_v} + \nabla \psi \times \nabla \chi + \cancel{\nabla \Omega \times \nabla \psi_v}$$

$$\vec{v} = \underbrace{\frac{\nabla \Phi \times \nabla \chi}{B_v^2}}_{\vec{v}_{E \times B}} + v_\parallel \vec{B} + \cancel{\nabla^\perp \zeta}$$

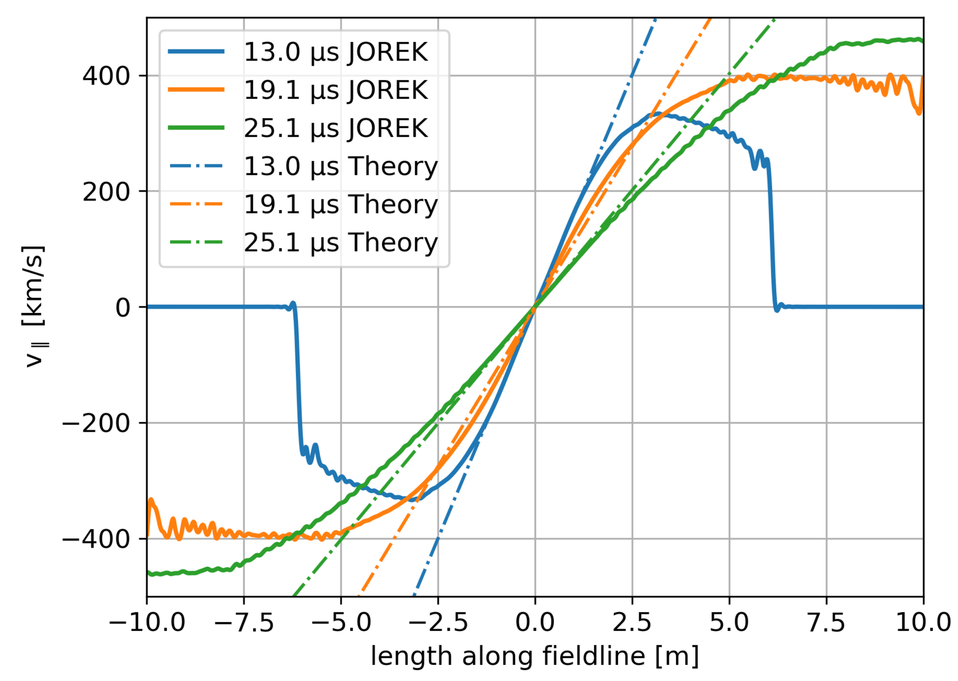
From $\nabla \cdot \vec{v}_{E\times B} \sim \nabla \cdot (\nabla \Phi \times \nabla \chi) = \nabla \chi \cdot (\nabla \times \nabla \Phi) - \nabla \Phi \cdot (\nabla \times \nabla \chi) = 0$ this term eliminates the radial flow compression and thus the propagation of the fast magneto-sonic wave.

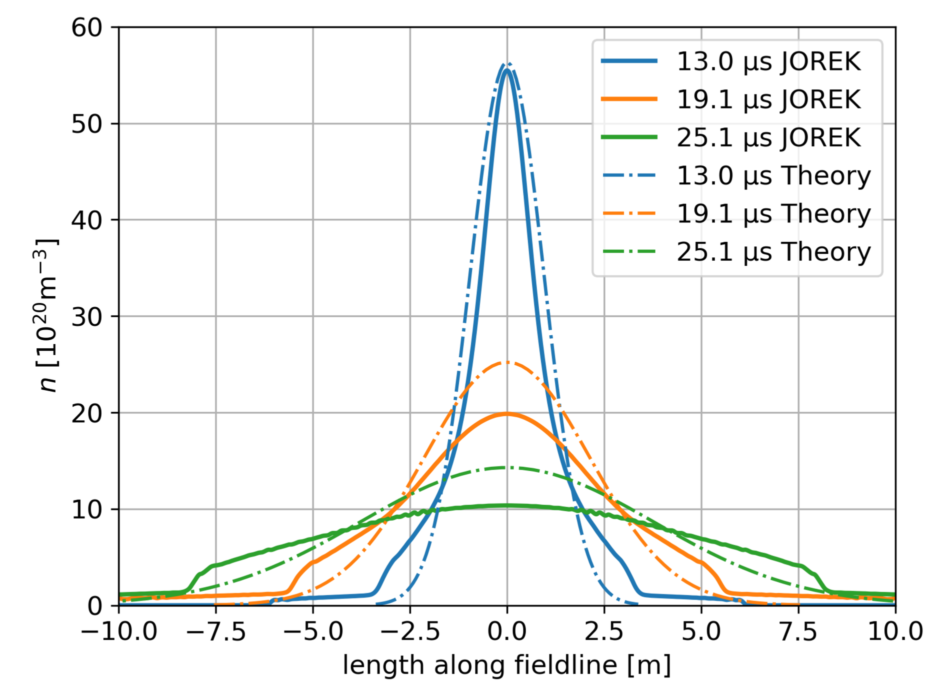
The plasma current and vorticity can then be directly calculated from $j = \triangle^* \psi$ and $\omega = \triangle^\perp \Phi$, where $\triangle^* = B_v^{-2} \nabla \cdot (B_v^2 \nabla^\perp$ and $\triangle^\perp = \nabla \cdot \nabla^\perp$.. For the 2 temperature model, this reduces the time evolved quantities from 9 to 6 $(\psi, \Phi, v_\parallel, \rho, T_i, T_e)$, reducing the memory and compute requirements.

3. VALIDATION OF THE PELLET EXPANSION WITH THEORY

The parallel expansion must first be validated to enable the study of perpendicular pellet dynamics, as the drifts strongly depend on the local temperature and density evolution. In this context, the analytical solution for the 1D expansion of a pellet-produced plasmoid along a field line, as presented by Aleynikov et al. [4, 6], is used for validation.

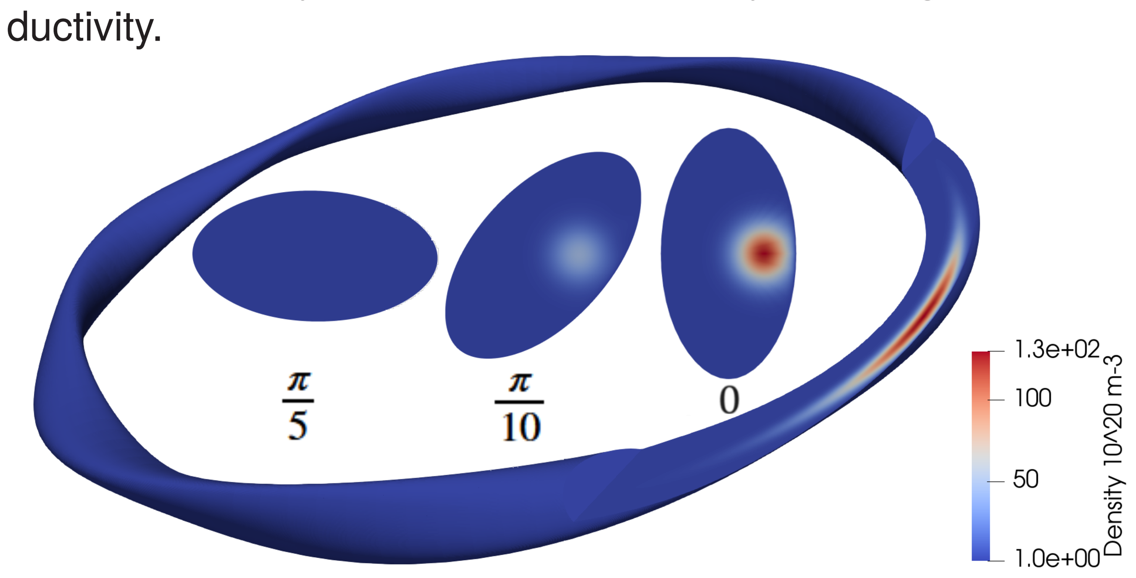
- plasmoid and plasma temperature ramped up linearly
- Heating rate chosen as the Maxwellian eq. time τ_T^{ee}
- Artificially damped drifts
- Tokamak model (for now)
- W7-X-like pellet size [2]





4. INITIAL STELLARATOR PELLET SIMULATIONS

First pellet-produced plasmoid simulations in a Wendelstein 7-A-like stellarator were done using parallel conductive heating. For this, typical values for W7-A were chosen ($n_e = 7.5 \times 10^{19}$m^{-3}) with a relatively low temperature on-axis (500 eV), such that the heat conductivity is still well-described by the Braginskii conductivity.

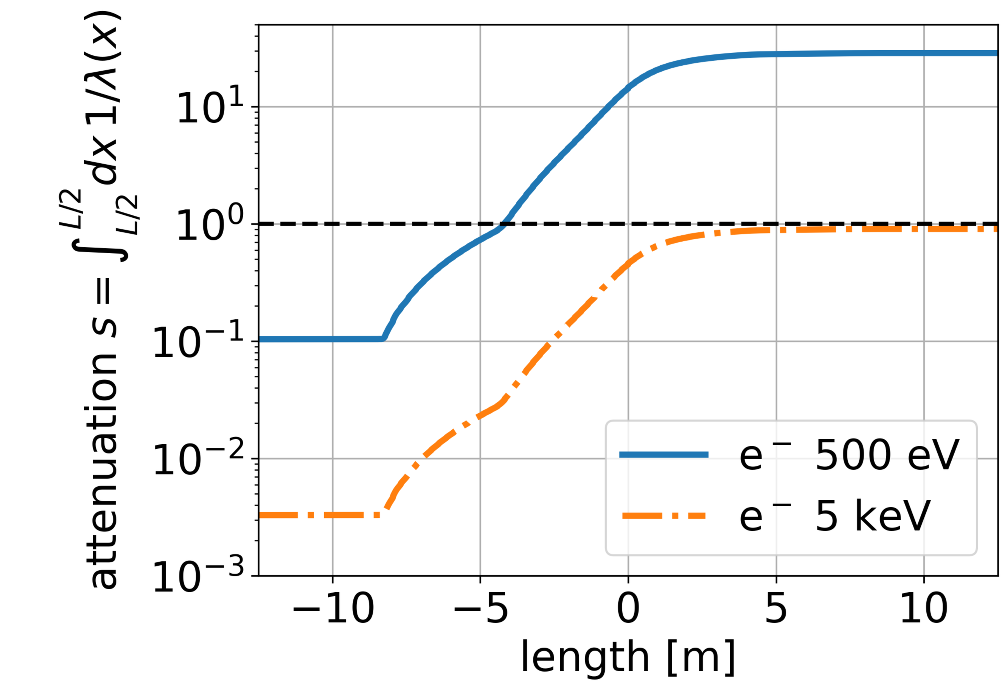


5. NONLOCAL HEATING

For a typical pellet injected into the plasma (e.g. $T_e \approx 500 - 5000$ eV for W7-X) the parallel heating depends on the the mean free path of the ambient electrons inside the pellet-produced plasmoid, which scales as

$$\lambda_e = \frac{T_e^2}{\ln(\Lambda) n_e e^4} \propto \frac{T_e^2}{n_e}.$$

Comparing the attenuation of ambient particles in the plasmoid for $T_e = 500$ eV and 5 keV.



For temperatures near the plasma edge $T_e \approx 500$ eV is still well described by the Spitzer-Härm heat conductivity

$$q_\parallel^e = -\kappa_\parallel^e \nabla_\parallel T_e, \qquad \kappa_\parallel^e = 3.16 \frac{n_e T_e \tau_e}{m_e}.$$

However, the parallel conductive heating model does not capture the nonlocal nature of the transparent plasmoid regime under the considered plasma conditions, where $T_e \approx 5$ keV and $n_e \approx 10^{20}$ m^{-3}.

The key properties for a more accurate heating scheme are:

- Nonlocal
- Global energy conservation
- Low computational cost

Several methods exist for incorporating a nonlocal heating model. Two options are shown here:

1. Rozhanskij [7] **nonlocal heating model**, adapted from Luciani, Mora and Virmont (LMV) [8].

$$q_{\mathrm{LMV}}^e(x) = \int dx' q_\parallel^e(x') w(x, x'),$$

$$w(x, x') = \frac{1}{2\lambda_{\mathrm{eff}}(x')} \exp\left[-\frac{\left| \int_{x'}^{x} dx'' n(x'') \right|}{\lambda_{\mathrm{eff}}(x') n_e(x')} \right],$$

$$\lambda_{\mathrm{eff}}(x') = a\sqrt{Z+1}\lambda_e(x').$$

Here a is a numerical factor and was matched to Fokker-Planck simulations to $a = 32$ by LMV [8] to account for the disproportional influence of the faster particles on the nonlocal heating.

2. Flux surface average heating (Our approach):

$$q_e(x) = \frac{3}{2} \frac{n_e(x)(\langle T_e \rangle_\psi - T_e(x))}{\tau_T^{e,e}(T_e^\infty)}$$

For energy conservation we need

$$\langle T_e \rangle_\psi = \frac{\oiint_\psi dA\, n(\vec{x}) T_e(\vec{x})}{\oiint_\psi dA\, n(\vec{x})}.$$

6. OUTLOOK

- Development of a reliable nonlocal heating model for pellet simulation in stellarators and tokamaks
- Studies of how the plasma drifts affect the evolution of the pellet ablation cloud
- Comparison of pellet-induced plasmoid dynamics in tokamak and stellarator geometries
- The goal is to develop a model capable of explaining pellet fueling in Wendelstein 7-X, including the deposition profile and accurately accounting for energy balance

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Tok Retreat 2025

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.