Study of finite Larmor radius physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

J. Á. Capitán García*[1], P. Ulbl[1], B. J. Frei[1], and F. Jenko[1,2]

[1] Max-Planck-Institut für Plasmaphysik, Boltzmannstraße 2, 85748 Garching b. M. (Germany)

[2] University of Texas at Austin, Austin, Texas 78712 (USA)

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

ABSTRACT

This work discusses an extension of the **gyrokinetic turbulence code GENE-X**, an Eulerian code which solves the Vlasov eq. on a grid using an electromagnetic, collisional, full-f model. The project focuses on **finite (ion) Larmor radius (FLR) effects**, relevant in both edge and particularly core plasma, and their incorporation to the model through gyro-averages. To this end, multiple methods will be implemented, such as the **Padé approximants**. Ultimately, this will enable GENE-X to accurately simulate gyrokinetic turbulence from plasma core to the far SOL in tokamaks and stellarators, crucial for minimizing turbulence-driven transport losses and optimizing energy confinement.

GENE-X GYRO-AVERAGED EQUATIONS

Starting from [1], we **re-derived GENE-X eqs.** to include **full-FLR effects in $\mathcal{H}_0$, $\mathcal{H}_1$** (0th and 1st order of Hamilt.) through **gyro-averages**, defined as

$$\langle \phi(r) \rangle_{R} = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{\rho_\alpha \cdot \nabla} \phi(R)\,, \quad \langle \phi(R) \rangle_{r}^{\dagger} = \int_0^{2\pi} \frac{d\theta}{2\pi} \int dR\, \delta(R + \rho_\alpha - r)\, \phi(R)\,. \quad (1)$$

VLASOV EQ.:

$$\frac{\partial f_\alpha}{\partial t} + \dot{R}\cdot\nabla f_\alpha + \dot{v}_\parallel \frac{\partial f_\alpha}{\partial v_\parallel} = 0\,, \qquad \left(B^* \sim B + \frac{m_\alpha c}{q_\alpha} v_\parallel \nabla \times b + \langle \nabla \times A_1 \rangle \right) \quad (2)$$

$$\text{where} \quad \dot{R} = \frac{B^*}{B_\parallel^*} v_\parallel + \frac{c}{q_\alpha B_\parallel^*} b \times \left(\mu \nabla B + q_\alpha \nabla \langle \phi_1 \rangle_R \right)\,, \quad (3)$$

$$\dot{v}_\parallel = -\frac{B^*}{m_\alpha B_\parallel^*} \left(\mu \nabla B + q_\alpha \nabla \langle \phi_1 \rangle_R \right) - \frac{q_\alpha}{m_\alpha c} \frac{\partial \langle A_{1\parallel} \rangle}{\partial t}\,. \quad (4)$$

FIELD EQS.:

polariz. (from $\mathcal{H}_2$)

$$\text{QN:} \quad \sum_\alpha q_\alpha \langle n_\alpha \rangle_r^\dagger = -\nabla \cdot \left(\sum_\alpha n_\alpha \frac{m_\alpha c^2}{B^2} \nabla_\perp \phi_1 \right)\,, \quad (5)$$

$$\text{Ampère:} \quad \sum_\alpha \frac{q_\alpha}{c} \left\langle \int dW f_\alpha v_\parallel \right\rangle_r^\dagger = -\frac{1}{4\pi} \left| \nabla_\perp^2 A_{1\parallel} \right|\,, \quad (6)$$

$$\text{Ohm:} \quad \sum_\alpha \frac{q_\alpha}{c} \left\langle \int dW \left(\frac{\partial f_\alpha}{\partial t} \right)^* v_\parallel \right\rangle_r^\dagger = -\left(\frac{1}{4\pi} \nabla_\perp^2 \frac{\partial A_{1\parallel}}{\partial t} + \sum_\alpha \frac{q_\alpha^2}{m_\alpha c^2} \left\langle \left\langle \frac{\partial A_{1\parallel}}{\partial t} \right\rangle_R \int dW \frac{\partial f_\alpha}{\partial v_\parallel} v_\parallel \right\rangle_r^\dagger \right)\,. \quad (7)$$

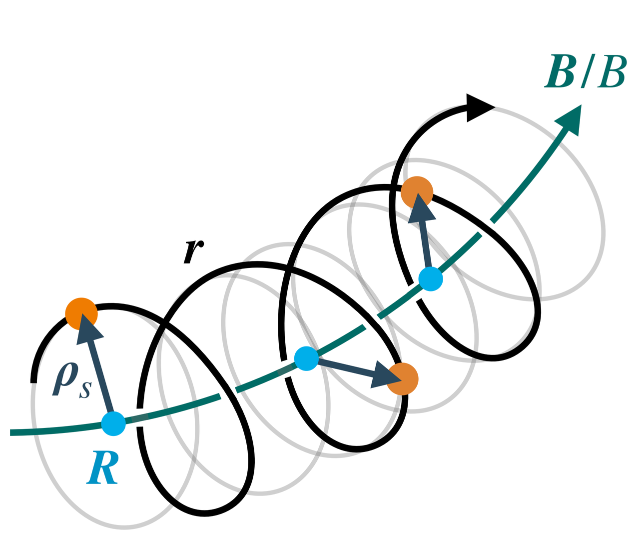
ENERGY CONSERVATION:

$$\sum_\alpha \int dV \int dW\, q_\alpha f_\alpha \left\langle \frac{\partial \phi_1}{\partial t} - \frac{v_\parallel}{c} \frac{\partial A_{1\parallel}}{\partial t} \right\rangle_R = \sum_\alpha \int dV\, q_\alpha \left(\left\langle \int dW f_\alpha \right\rangle_r^\dagger \frac{\partial \phi_1}{\partial t} - \left\langle \int dW f_\alpha \frac{v_\parallel}{c} \right\rangle_r^\dagger \frac{\partial A_{1\parallel}}{\partial t} \right)\,. \quad (8)$$

FLR MODELS

FLR in →	$\mathcal{H}_0$, $\mathcal{H}_1$	$\mathcal{H}_2$
GENE-X current	LWA	LWA
GENE-X target	FULL	LWA
$(\star) \equiv k_\perp^2$	FULL	LWA
$(\star) \equiv \tau_\alpha^{-1}(1-\Gamma_0)$	FULL	FULL

for (⋆) *cf.* eq. (12)



↳ *already being implemented in GENE-X*

PADÉ APPROXIMANT: MOTIVATION

Fourier $\mathcal{F}$ in gyro-average, eq. **(1)**,

$$\langle \phi(r) \rangle_\theta = \frac{1}{(2\pi)^3} \int dk\, J_0\, e^{ik\cdot R}\, \hat{\phi}(k)\,, \quad (9)$$

where $J_0 = J_0(\rho_\alpha k_\perp) \equiv$ Bessel f. of 1st kind, **and FLR operator in $\mathcal{F}$.**

To calculate the gyro-average, Padé approximant on J_0,

$$J_0(\rho_\alpha k_\perp) \overset{P0/2}{\approx} \frac{1}{1 + \rho_\alpha^2 k_\perp^2/4}\,. \quad (10)$$

Applying $\mathcal{F}^{-1}$ on **(9)**,

$$(1 - 1/4\, \rho_\alpha^2 \nabla_\perp^2)\, \langle \phi(r) \rangle_\theta = \phi(R)\,, \quad (11)$$

a **differential eq.** for $\langle \phi(r) \rangle_\theta$.

↳ **elliptic solver**

PADÉ APPROXIMANT: ITG DISPERSION RELATION

FLR models studied in the derived **electrostatic slab ion temperature gradient (ITG) dispersion relation** in Fourier space **with diffusion** in ⊥ and ∥ directions. From GENE-X eqs. and at 1st order,

$$\sum_\alpha \frac{1}{T_\alpha} \Gamma_0 + \sum_\alpha \frac{1}{T_\alpha} \Gamma_0 \xi_{\text{eff}} Z - \sum_\alpha \frac{1}{\sqrt{2 T_\alpha}} \frac{k_\perp}{k_\parallel} \left(\Gamma_0 Z + \eta \left(\Gamma_0 \xi_{\text{eff}} \left(1 + \xi_{\text{eff}} Z\right) - \frac{1}{2} \Gamma_0 Z + a \left(\Gamma_1 - \Gamma_0\right) Z \right) \right)$$

$$+ \sum_\alpha k_\perp^2 = 0\,, \quad (12)$$

(⋆) **term from polarization** ⟵

where $\tau_\alpha = T_\alpha / T_e$, $a = \tau_\alpha k_\perp^2$,

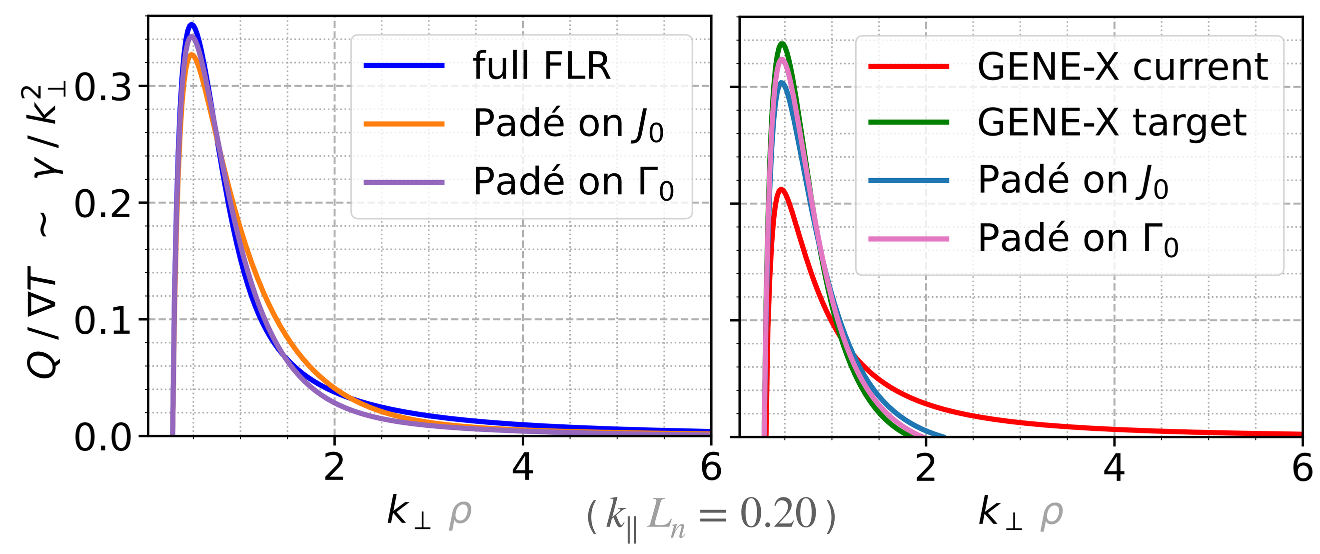
$\Gamma_0(a) = I_0(a)\, e^{-a}$ ($I_0 \equiv$ modif. Bessel f. of 1st kind),

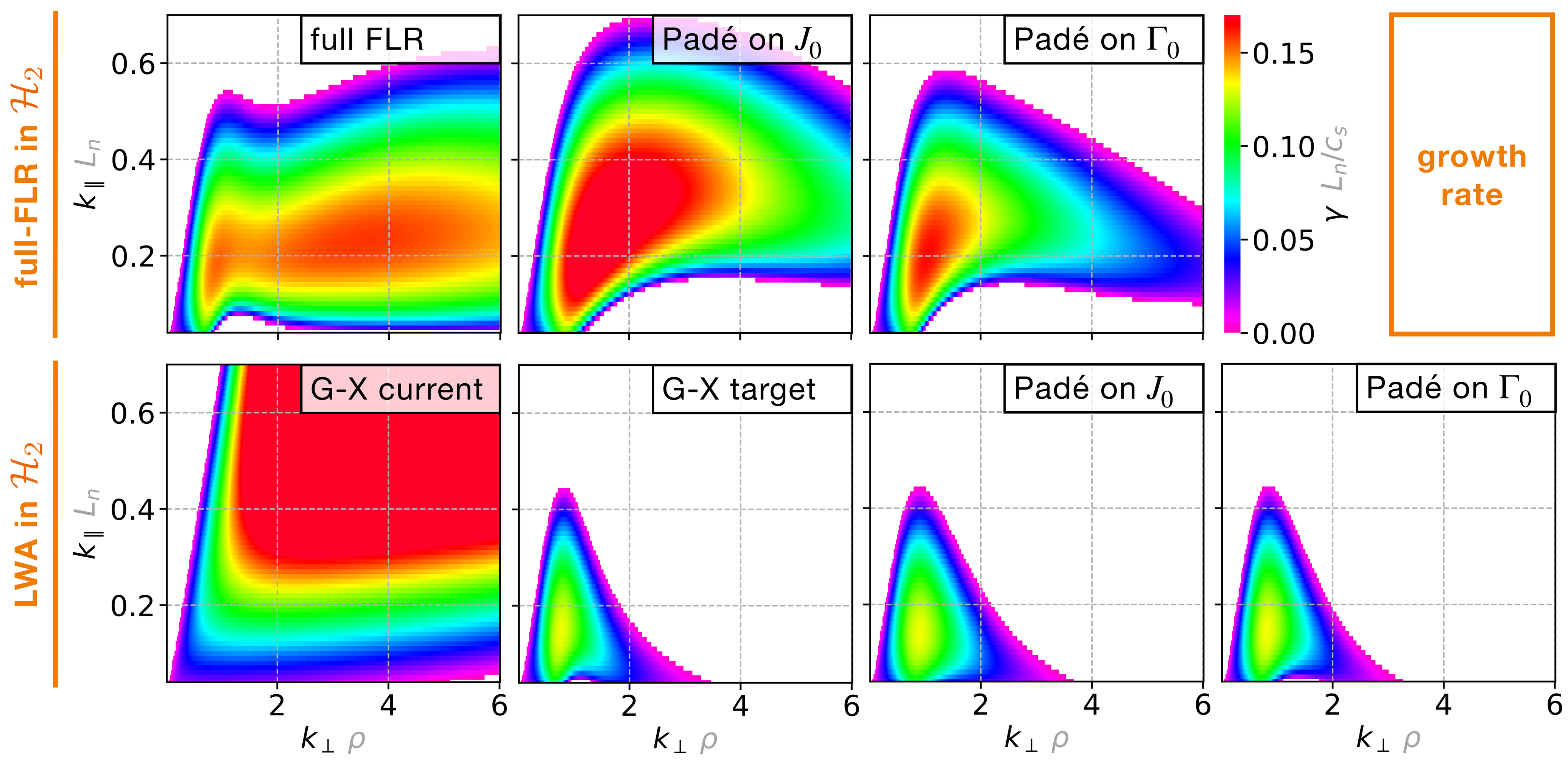
$\xi_{\text{eff}} = \omega_{\text{eff}} / (k_\parallel v_{T\alpha})$, $\omega_{\text{eff}} = \omega_r + i\gamma + i\, q_\alpha^2/m_\alpha D_\perp^2 k_\perp^2 + i\, D_\parallel^2 k_\parallel^2$, $v_{T\alpha} \equiv$ thermal vel., $D \equiv$ diff. coeffs.,

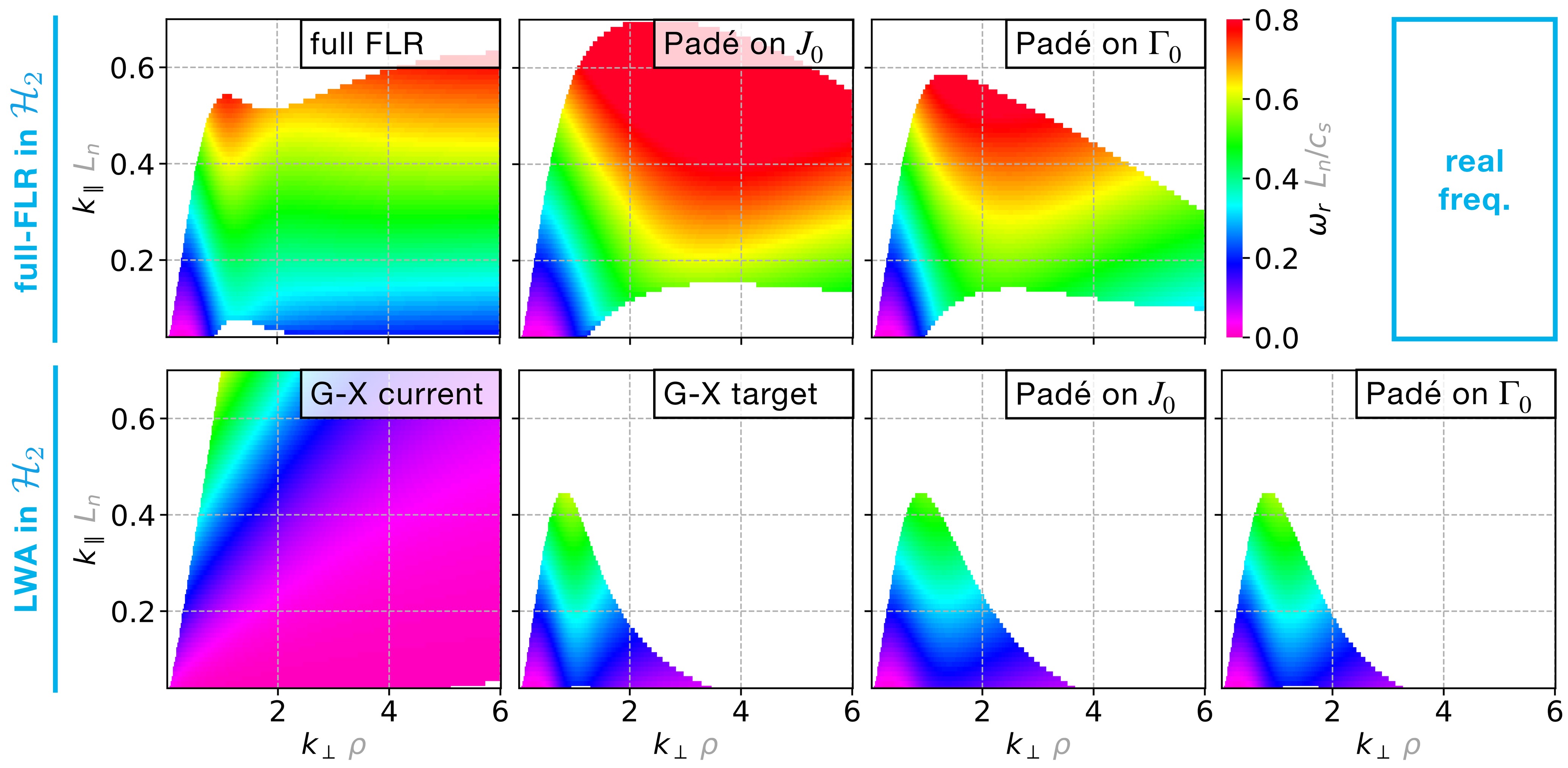
$Z(\xi_{\text{eff}}) = \int_{-\infty}^{\infty} ds\, \frac{1}{\sqrt{\pi}} \frac{1}{s - \xi_{\text{eff}}} e^{-s^2} \equiv$ plasma dispersion function [2].

Eq. **(12)** is solved for cyclone-base-case-like parameters [5].

- When merging Vlasov and QN eqs., respective FLR operators J_0 combine into $\Gamma_0 \sim J_0^2$. **Physics of the system thus in Γ_0.**
- Normalized units used [1, 3].
- Eq. **(12)** exactly matches [3, 4] if adiabatic electrons and full-FLR polariz. are assumed.







CONCLUSION AND OUTLOOK

This discussion outlined a strategy to incorporate **full-FLR effects in $\mathcal{H}_0$ and $\mathcal{H}_1$ in GENE-X code.**

- The FLR operator J_0, eq. **(9)**, motivates **Padé approximants**. Analyzing the case of eq. **(12)**, Padé on Γ_0, from where $\langle J_0 \rangle_W \approx \Gamma_0^{1/2} \overset{P0/2}{\approx} (1 - \rho_\perp^2 \nabla_\perp^2)^{-1/2}$ can be retrieved, is the most physically rigorous approach. Padé on directly J_0 provides approximated physics, but a more straightforward implementation, eq. **(11)**.
- In a following step, a **gyro-matrix method** [6] based on grid interpolations will be included, accounting for FLR to all orders.

[1] Ulbl, P., PhD thesis (Technische Universität M., München, 2023).

[2] B. D. Fried, *The plasma dispersion function* (Elsevier, 1961).

[3] B. J. Frei *et al.*, Journal of Plasma Phys. **88**, 10.1017/s0022377822000344 (2022).

[4] S. Brunner, PhD thesis (EPFL, Lausanne, 1997).

[5] A. M. Dimits *et al.*, Phys. of Plasmas **7**, 969–983 (2000).

[6] T. Görler *et al.*, Journal of Computational Phys. **230**, 7053–7071 (2011).

Poster session I — May 7, 10:00 - 12:00
TOK Retreat (Schloss Hirschberg, 2025)

*Corresponding author:
jose-angel.capitan-garcia@ipp.mpg.de

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

EUROfusion