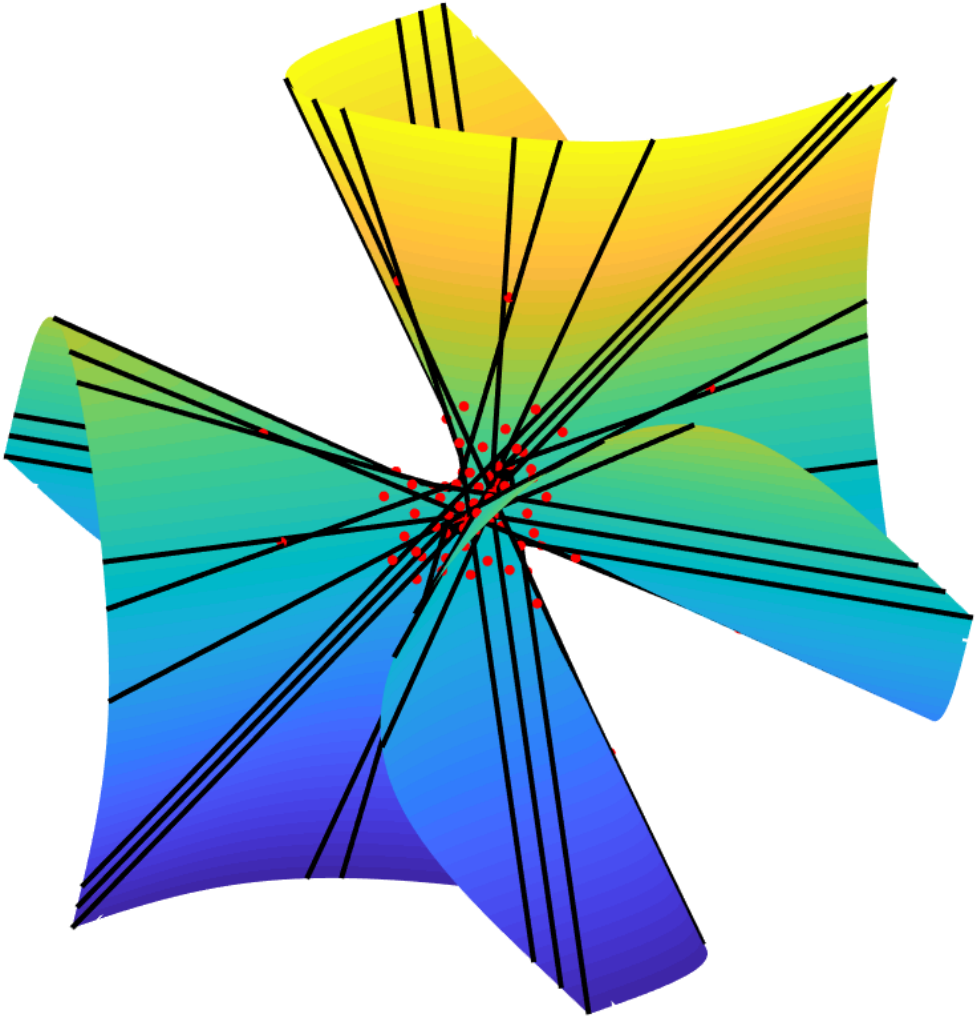
NUMERICAL (NONLINEAR) ALGEBRA IN THE REAL WORLD



3 – 6 FEB 2025 | DRESDEN, GERMANY

WELCOME

Dear workshop participant,

We are so glad you are here with us this week to advance research in real applications of numerical techniques in nonlinear algebra!

Nonlinearity is required to accurately model many scientific phenomena. This workshop brings together scientists from a broad array of topics through biology, chemistry, physics and more, to discuss nonlinear modeling problems and techniques, specifically aligned with those from numerical methods and in algebraic geometry. Our goals include introducing PhD students and early-career researchers to the various techniques, and to facilitate collaboration across different experience levels and scientific fields.

Our schedule provides lots of time and space for collaboration, both between the ex cellent talks and parallel to them so you can choose your focus. A group meal for all participants together on Wednesday here at MPI-CBG will help us build research and personal relationships. On Thursday in the Lightning Talks, we have an opportunity for you to share about things you worked on this week, or something exciting related to the workshop. We hope you will reach out if you have needs or desires, so this can be a productive and collaborative time together.

Silviana Amethyst, Türkü Çelik, Paul Breiding, Sarah Eggleston, Viktoriia Borovik

General information

Organizers

Silviana Amethyst
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Sarah Eggleston
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Paul Breiding
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Locations

The workshop will take place in different rooms of the MPI-CBG. During the course, please wear your badge at all times for security purposes.

The main doors are open
from 8am – 8pm
from Monday to Friday.

Contact Workshop Office

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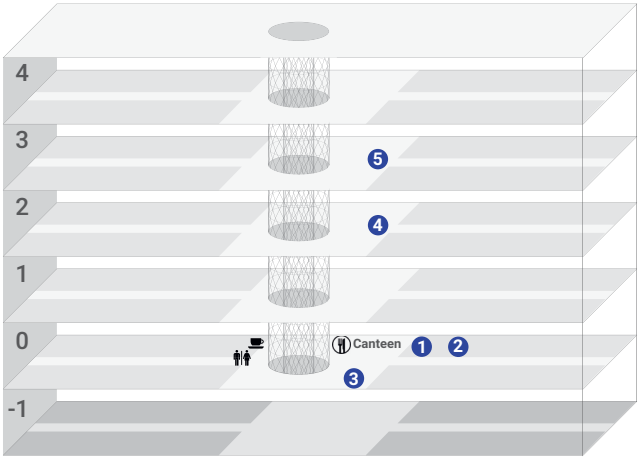
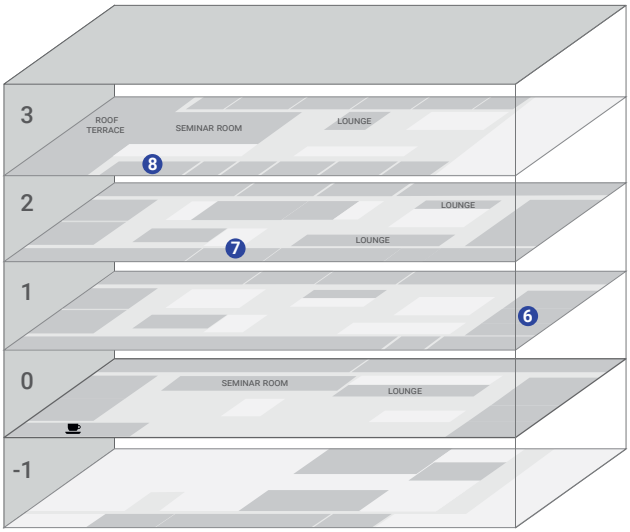
MPI-CBG Building



1 - Large Auditorium
2 - Small Auditorium
3 - Atrium
4 - Seminar Room 2
5 - Seminar Room 3
6 - CSBD 121
7 - CSBD 205
8 - CSBD 306

CSBD Building



Badges

Please help us to be green by returning the reusable plastic holders of your badge at the end of the conference at the reception.

Photography

During the course, an MPI-CBG photographer will be taking photographs. If you would not like to appear in these, please inform the photographer or a member of the Course Office.

Phone Numbers

Ambulance: 112

MPI-CBG Reception:
+49 (0) 351 210 2010

Within the MPI-CBG just dial the last 4 digits from any inhouse phone.

Internet

WiFi is available in the course venues (no password required). Eduroam is also available onsite.

Unencrypted public guest WiFi in common areas:

SSID: MPI-hotspot

Encrypted WiFi with members of participating institutions, available in whole institute:

SSID: eduroam

Course Website

https://plan.events.mpg.de/event/355/

Social Media

Follow us at:
math-mpicbg@bsky

Public transit

The 62 bus takes you quickly to beautiful Altstadt (Pirnaischer Platz or Prager Straße stops), and the number 6 tram takes you across the river to Neustadt (Albertplatz is a good stop). The number 12 tram is also nearby, taking you to Altstadt. The DVB phone app has the most accurate schedule information. Tickets may be purchased on the bus/tram by tap-to-pay, or at a yellow machine at some stops. There's a ticket machine a bit west of MPI-CBG at the intersection of Pfotenhauerstraße and Fetscherstraße. If you ride more than 2 times in a day, a day pass "Tageskarte" is the most economical option.

Journey Planner: www.dvb.de/en

Ticket fares trams and busses: www.dvb.de/en-gb/tickets-en/ticket-overview

Health & Safety Notes

Do not smoke in any area in the MPI-CBG.

Eating and drinking is not allowed in the Auditorium and all laboratories.

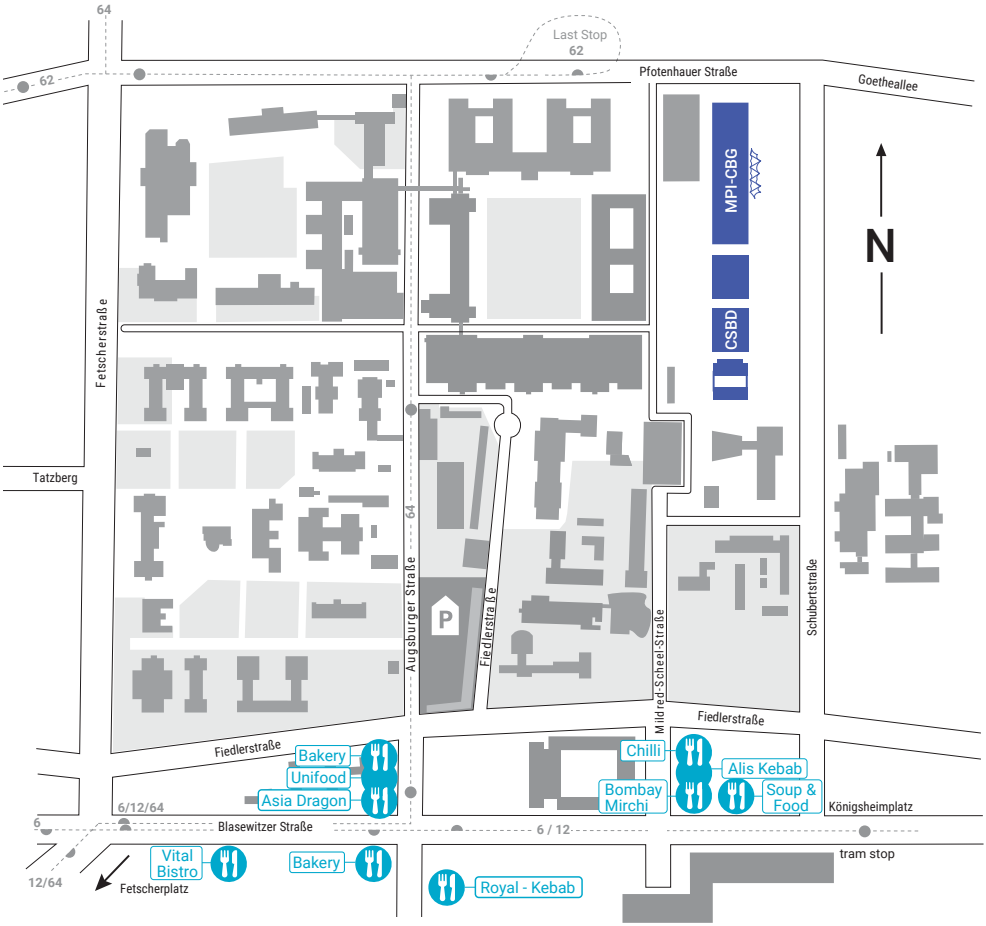
Do not enter any other areas or laboratories than the course locations unless instructed to do so.

If first aid is required:

- A first aid kit is located at the switchboard.
- A quiet room with first aid kit is located in the ground floor (room 009). The key for the room is available at the switchboard.
- In case of an emergency dial 333 from any internal phone.
- An emergency room is located in house 58 of the university hospital.
- Please report all accidents to the Course Office.

In case of fire:

Press the nearest fire alarm button and/or call the emergency number 333 from any internal phone. In case of an evacuation of the building, a loud fire alarm will go off. On hearing the alarm, leave the building immediately following the escape routes. The fire wardens are wearing a bright yellow vest. Do not use the lifts and the center staircase. The assembling point is located opposite of the main entrance.



Catering

Morning and afternoon coffee and snacks are included, as is a catered group dinner on Wednesday in the MPI-CBG canteen. Please wear your badge at all times to help identify yourself as a participant. No food or drinks are allowed in the Auditoriums.

Lunch

Participants are welcome to dine at the on-campus canteen, or go out to a local restaurant. The canteen accepts both cash and card. Lunch is at the participants own expense.

Dinner

A catered conference meal is planned on Wednesday at 6pm at the MPI-CBG canteen, at no cost to participants. The buffet will include vegetarian and vegan options. For the other evenings, participants are encouraged to self-assemble into groups and go to one of Dresden's many delicious restaurants.

Speakers

VIKTORIIA BOROVIK

Osnabrück University

https://www.vborovik.website

Numerical homotopies from SAGBI bases

I will present homotopy continuation methods for solving 0-dimensional polynomial systems, where each polynomial is expressed as a general linear combination of prescribed, fixed polynomials. This approach involves selecting a specific starting system for homotopy continuation, leveraging the theory of SAGBI (Khovanskii bases) and toric geometry. For square systems, SAGBI homotopies can significantly reduce the number of solution paths to track compared to polyhedral homotopies, which are currently the default in HomotopyContinuation.jl.

As a direct application of this theory, I will demonstrate the SAGBI homotopy with two examples: (1) finding approximate stationary states for coupled driven nonlinear resonators — a problem in nonlinear dynamics, and (2) computing approximate solutions to the electronic Schrödinger equation in coupled cluster theory, which arises in quantum chemistry.

TÜRKÜ ÖZLÜM ÇELIK

Max Planck Institute of Molecular Cell Biology and Genetics

https://www.mpi-cbg.de/celik

Wave Parameters from Discrete Data

We study approximating underlying mathematical structures via point cloud data within the framework of integrable systems, with a particular emphasis on the Kadomtsev-Petviashvili (KP) equation. This is a pivotal element in the

theory of integrable systems, and models nonlinear wave interactions. Our objective is to determine the finite-genus KP solution parameters that optimally correspond to a prescribed dataset of discrete point values. The methodology employs foundational principles from Fourier analysis alongside standard optimization techniques. A beautiful mathematical challenge we run into is the Schottky problem, which asks for characterization of Jacobians of algebraic curves among abelian varieties. This is joint with Daniele Agostini, Bernard Deconinck, Charles Wang.

FABIAN FAULSTICH

Rensselaer Polytechnic Institute

https://fabianfaulstich.com

Numerical Algebraic Geometry and Correlated Electrons

We discuss the algebra and combinatorics underpinning coupled cluster (CC) theory for quantum many-body systems. The high-dimensional eigenvalue problems that encode the electronic Schrödinger equation are approximated by polynomial systems at various levels of truncation. The exponential parametrization of the eigenstates gives rise to truncation varieties. These generalize Grassmannians in their Plücker embedding. We offer a detailed study of truncation varieties and their CC degrees, a complexity measure for solving the CC equations. We also discuss the solutions of the CC equations.

PIERRE HAAS

Max Planck Institute of Molecular Cell Biology and Genetics / Max Planck Institute for the Physics of Complex Systems

https://www.mpi-cbg.de/haas

Stories of ecological structures

The dynamics of ecological communities are intrinsically linked to ecological structures such as the network of species interactions or the phenotypic substructure of different species. In this talk, I will give two examples of minimal models that elucidate the interplay of these structures and ecological dynamics. In both cases, I will highlight open questions that (new) (numerical) algebraic methods could answer.

In the first part of my talk, I will analyse the effect of the network of competitive, mutualistic, and predator-prey interactions on stability of coexistence [1]. I will show that the possibility of stable coexistence in ecologies with Lotka-Volterra dynamics is determined completely by "irreducible ecologies", and I will explain how exhaustive analysis of all such interaction networks of N<6 species suggests that, strikingly, these irreducible ecologies form an exponentially small subset of all ecologies, as do the mathematically curious "impossible ecologies" in which stable coexistence is non-trivially impossible.

In the second part of my talk, I will introduce a minimal model of spatial structure in the competition of two species. One of these species switches, both stochastically and in response to the other species, to a phenotype resilient to competition [2]. In particular, I will ask: How does this phenotypic switching affect travelling waves by which one species invades the other? Combining exact and numerical results, I will reveal that, very surprisingly, phenotypic switching does not change the speed of these travelling waves.

[1] Meng, Horvát, Modes, and Haas, arXiv:2309.16261
[2] Gupta and Haas, in preparation

JONATHAN HAUENSTEIN

University of Notre Dame

https://acms.nd.edu/people/jonathan-hauenstein/

Four computations in real numerical algebraic geometry

Many problems in science and engineering can be formulated as computing information about the set of real points satisfying polynomial equations and inequalities. Some examples include synthesizing a linkage in kinematics, computing and analyzing the steady-state solutions to a polynomial dynamical system, and reconstructing a scene in computer vision. Since different problems could require a different amount of information about the real solution set, this talk will summarize four computations in real numerical algebraic geometry: 1) existence of a real solution, 2) smooth points and dimension of the real solution set, 3) decomposition into smoothly connected components, and 4) cell decomposition for a complete description of the real solution set. Each computation will be illustrated with an example along with a discussion about benefits and potential drawbacks. This talk covers computational methods created jointly with a variety of collaborators including Silviana Amethyst, Dan Bates, Gian Mario Besana, Joe Cummings, Sandra Di Rocco, Wenrui Hao, Hoon Hoon, Katherine Harris, Cliff Smyth, Andrew Sommese, Agnes Szanto, and Charles Wampler.

OSKAR HENRIKSSON

University of Copenhagen

https://oskarhenriksson.se

The numerical algebraic geometry of steady state equations

The steady states of a chemical reaction network with power-law kinetics can be described by a polynomial system with fixed support and coefficients that depend on parameters called rate constants and total amounts. This so-called vertical parametrization of the coefficients typically introduces dependencies among them, such that the generic root count over the complex numbers drops below the mixed volume bound predicted by the BKK theorem. In this talk, I will give an overview of recent results on the generic geometry of these systems and their incidence varieties. I will also discuss tropical techniques for constructing homotopies that allow solving the systems by tracing an optimal number of paths.

This is based on joint work, partly in progress, with Elisenda Feliu, Paul Helminck, Beatriz Pascual-Escudero, Yue Ren, Benjamin Schröter, and Máté Telek.

FRANK JÜLICHER

Max-Planck-Institute for the Physics of Complex Systems

https://www.pks.mpg.de/biological-physics/frank-juelicher

Active matter hydrodynamics: from droplets to tissues

TORKEL LOMAN

University of Cambridge

https://elliit.se/torkel-loman/

Catalyst: Fast and flexible modelling of reaction networks

Chemical reaction networks (CRNs) are a type of model commonly used in biology and chemistry. Their applications include the investigation of cellular system functions, designing drugs (pharmacology), forecasting epidemic progression (epidemiology), and optimisation of chemical synthesis pipelines. The Catalyst.jl modelling tool provides an interface for creating such CRN models in the Julia programming language (a recently developed programming language comparable to e.g. Matlab and Python). Next, it enables various forms of model analysis and simulation (e.g. deterministic/stochastic, nonspatial/spatial). Connections to packages in the wider Julia ecosystem also enable workflows such as bifurcation analysis and the finding of steady states through homotopy continuations. This talk will show how CRN models can be implemented and analysed in Julia, and how this can be applied to real-world problems across biology.

JIAYI LI

Max Planck Institute of Molecular Cell Biology and Genetics

https://jl2ml.github.io

Taming Non-convexity in Shallow Neural Networks with Algebraic Activations

One of the key challenges in optimizing neural networks is the inherent high-dimensionality and non-convexity of the objective function. A single neuron with Sigmoid activation is known to have the number of local minima grow exponentially in the dimension based on square loss. Properly tuned gradient-based methods converge to a stationary point, prompting the question: which stationary point do these methods typically find, and how can we bound their convergence rates?

Beyond popular ReLU and sigmoid functions, recent work has explored polynomial and rational activations. Polynomial activations have shown promise in computer vision tasks, while rational activations have been applied to solving PDEs and training Generative Adversarial Networks. Notably, well-known activations such as ReLU and other smooth functions can be approximated by polynomials or rational functions up to a desired accuracy.

In this talk, I will (1) describe the training dynamics of shallow neural networks with these algebraic activations, focusing on rational networks as a representative case, (2) characterize their stationary points and investigate how poles, factorization symmetries, and higher-dimensional parameter spaces complicate gradient-based optimization, and (3) discuss the existence and elimination of "spurious valleys" (connected components of sub-level sets that exclude a global minimum) in different architectures. I will demonstrate the theoretical findings with numerical experiments.

AIDA MARAJ

Max Planck Institute of Molecular Cell Biology and Genetics

https://www.mpi-cbg.de/maraj

Toric Geometry in Brownian motion tree models and their generalizations

Algebraic geometry has recently provided a new approach to advancing problems in multivariate Gaussian models. This is achieved by identifying Gaussian distributions with symmetric matrices and analyzing the polynomials that vanish on these matrices, known as ideals. The talk will focus on Brownian motion tree (BMT) models, a type of Gaussian model used in phylogenetics, and their generalizations to phylogenetic trees with colored and zeroed nodes. The set of concentration matrices on BMT models has hidden toric geometry. We use it to provide formulas on the maximum likelihood degree and its dual. Their generalization is not always toric. We share conditions for toricness under a linear transformation.

ANDREAS MÜNCH

University of Oxford

https://people.maths.ox.ac.uk/muench/

Continuum models of solutions and gels of charge regulating polyelectrolytes such as RNA and DNA and their phase behaviour under varying environmental conditions

We develop continuum models for solutions of polyelectrolytes as a minimal system for biological molecules. We investigate their propensity to undergo phase separation by a combination of bifurcation analysis and time-dependent numerical solutions. In the case of polyelectrolyte gels, we describe the impact of salt concentration in the environment on the pattern forming behaviour under the effect of elasticity and on the structure of the electric double layer. We also explore the often neglected property of charge regulation on the phase behaviour of macro-ion solutions.

DMITRII PAVLOV

Max Planck Institute for Mathematics in the Sciences

https://dmmpavlov.github.io/

Gibbs manifolds

Gibbs manifolds are images of affine spaces of symmetric matrices under the exponential map. They arise in applications such as optimization, statistics and quantum physics, where they extend the ubiquitous role of toric geometry. The Gibbs variety is the zero locus of all polynomials that vanish on the Gibbs manifold. In this talk, I will describe some properties of this variety, and present two implicitization algorithms (one symbolic and one numerical) for it. In particular, we will see on examples that numerical methods allow to compute more complicated Gibbs varieties. This is based on joint work with Bernd Sturmfels and Simon Telen.

IREM PORTAKAL

Max Planck Institute for Mathematics in the Sciences

https://www.irem-portakal.de

Computing equilibra in game theory

In 1950, Nash published a very influential two-page paper proving the existence of Nash equilibria for any finite game. The proof uses an elegant application of the Kakutani fixed-point theorem from the field of topology. It has, however, been noted that in some cases the Nash equilibrium fails to predict the most beneficial outcome for all players. To address this, generalizations of Nash equilibria such as correlated and dependency equilibria were introduced. This talk explores these notions of equilibria and presents methods to compute them from a nonlinear algebraic perspective, offering insights into their computational challenges.

IVO SBALZARINI

Max Planck Institute of Molecular Cell Biology and Genetics / TU Dresden

https://www.mpi-cbg.de/sbalzarini

Particle Methods: Numerical Computation on Point Clouds

Point clouds provide an expressive abstraction for tasks across computational mathematics. They can be used as collocation point sets in numerical analysis, but also as a data structure for machine learning and statistical inference. I provide an introduction to how differential operators can be consistently approximated on point clouds by solving polynomial systems. This includes a meshfree geometric-computing framework that leverages polynomial regression in a unisolvent Newton-Chebyshev basis to represent complex-shaped and dynamic surfaces. Together, these render particle methods a viable choice for problems involving non-parametric or dynamic geometries, as exemplified by our work on active matter models of biological tissue morphogenesis.

GEORGY SCHOLTEN

Max Planck Institute of Molecular Cell Biology and Genetics

https://www.mpi-cbg.de/harrington

Globtim: An attempt at global optimization over compact domain via polynomial approximations

Let f be a measurable function defined over the n-dimensional unit-cube and enjoying a quadratic growth property around all of its local minimizers on that domain. Our objective is to design an algorithm that can compute all local minimizers of f.

We work in a computational framework where the function f is given by an evaluation program Γ. This program takes as input rational points in that domain and returns the value of f in finite precision at these points.

We are considering both the framework where this evaluation program is exact – if the image of f can be represented with a finite amount of bits – or noisy – in that case, we assume that the evaluation function takes an extra parameter and returns an approximation that is η-close to the true value of f".

SIMON TELEN

Max Planck Institute for Mathematics in the Sciences

https://simontelen.webnode.page

Chebyshev varieties

Chebyshev varieties are algebraic varieties parametrized by Chebyshev polynomials or their multivariate generalizations. We determine the dimension, degree, singular locus and defining equations of these varieties. We explain how they play the role of toric varieties in sparse polynomial root finding, when monomials are replaced by Chebyshev polynomials. We present numerical root finding algorithms that exploit our results.

Joint work with Zaïneb Bel-Afia and Chiara Meroni.

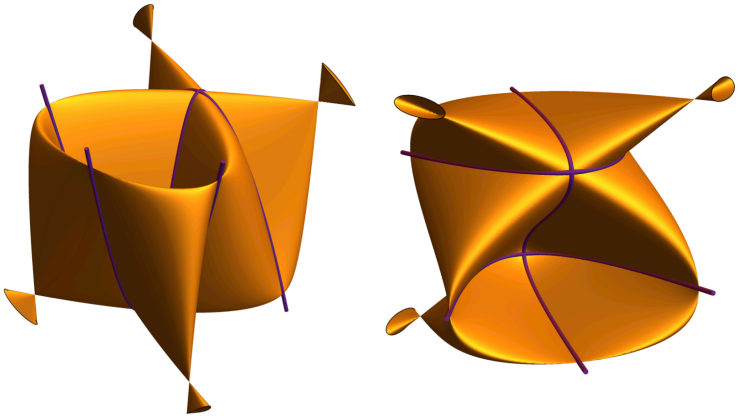
ADA WANG

Harvard University

https://wangk9562.wixsite.com/kexinwang

Computing Arrangements of Hypersurfaces

In this talk, I will present a Julia package, HypersurfaceRegions.jl, for computing all connected components in the complement of an arrangement of real algebraic hypersurfaces in R^n. The package is based on a modified implementation of the algorithm from the paper "Smooth Connectivity in Real Algebraic Varieties" by Cummings et al. I will outline the theory behind the algorithm and our implementation. I will demonstrate the use of the package through various examples.



The picture shows two views of a singular Chebyshev surface. The surface is the image of an affine plane in three-space under the coordinate-wise cosine map. The purple singular curves form the image of an arrangement of affine lines in that plane. (© Simon Telen)

Programme

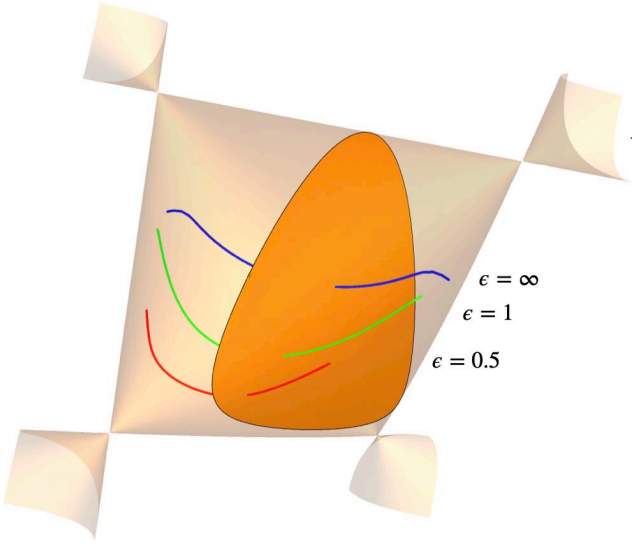
MONDAY 3 FEBRUARY		
13:00 – 14:00	Lunch *(Participants go to lunch, either at our canteen or at a nearby restaurant. Either way, self-pay.)*	Canteen
14:00 – 14:30	Registration and greeting	Atrium
14:30 – 14:45	Welcome	Large Auditorium
14:45 – 15:30	Jon Hauenstein - Four computations in real numerical algebraic geometry	Large Auditorium
15:30 – 16:15	Coffee & collaboration	Atrium
16:15 – 17:00	Andreas Münch - Continuum models of solutions and gels of charge regulating polyelectrolytes such as RNA and DNA and their phase behaviour under varying environmental conditions	Large Auditorium
17:00 – 18:00	Evening collaboration	Large Auditorium
TUESDAY 4 FEBRUARY		
09:00 – 09:45	Frank Jülicher - Active matter hydrodynamics: from droplets to tissues	Large Auditorium
09:45 – 10:30	Pierre Haas - Stories of ecological structures	Large Auditorium
10:30 – 11:15	Coffee & collaboration	Atrium; Seminar Room 2
11:15 – 12:00	Ada Wang - Computing Arrangements of Hypersurfaces	Large Auditorium
12:00 – 12:45	Ivo Sbalzarini - Particle Methods: Numerical Computation on Point Clouds	Large Auditorium
12:45 – 13:00	Announcements	Large Auditorium
13:00 – 14:00	Lunch *(Participants go to lunch, either at our canteen or at a nearby restaurant. Either way, self-pay.)*	Canteen
14:00 – 14:45	Collaboration	Large Auditorium
14:45 – 15:30	Torkel Loman - Catalyst: Fast and flexible modelling of reaction networks	Large Auditorium
15:30 – 16:15	Coffee & collaboration	Atrium; Seminar Room 2
16:15 – 17:00	Aida Maraj - Toric Geometry in Brownian motion tree models and their generalizations	Large Auditorium
17:00 – 18:00	Evening collaboration	Large Auditorium

WEDNESDAY 5 FEBRUARY		
09:00 – 09:45	Oskar Henriksson - The numerical algebraic geometry of steady state equations	Large Auditorium
09:45 – 10:30	Viktoriia Borovik - Numerical homotopies from SAGBI bases	Large Auditorium
10:30 – 10:45	Group photo	Large Auditorium
10:45 – 11:15	Coffee & collaboration	Atrium; Seminar Room 3
11:15 – 12:00	Fabian Faulstich - Numerical Algebraic Geometry and Correlated Electrons	Large Auditorium
12:00 – 12:45	Irem Portakal - Computing equilibra in game theory	Large Auditorium
12:45 – 13:00	Announcements	Large Auditorium
13:00 – 14:00	Lunch *(Participants go to lunch, either at our canteen or at a nearby restaurant. Either way, self-pay.)*	Canteen
14:00 – 14:45	Collaboration	Large Auditorium
14:45 – 15:30	Georgy Scholten - Globtim: An attempt at global optimization over compact domain via polynomial approximations	Large Auditorium
15:30 – 16:15	Coffee & collaboration	Atrium; Seminar Room 2
16:15 – 17:00	Türkü Özlüm Çelik - Wave Parameters from Discrete Data	Large Auditorium
17:00 – 18:00	Evening collaboration	Large Auditorium
18:00 – 20:00	Conference meal at MPI-CBG canteen	Canteen

THURSDAY 6 FEBRUARY		
09:00 – 09:45	Simon Telen - Chebyshev varieties	Small Auditorium
09:45 – 10:30	Dmitrii Pavlov - Gibbs manifolds	Small Auditorium
10:30 – 11:15	Coffee & collaboration	Atrium; Seminar Room 3
11:15 – 12:00	Jiayi Li - Taming Non-convexity in Shallow Neural Networks with Algebraic Activations	Small Auditorium
12:00 – 12:45	Lightning presentations	Small Auditorium
12:45 – 13:00	Announcements	Small Auditorium
13:00 – 14:00	Lunch *(Participants go to lunch, either at our canteen or at a nearby restaurant. Either way, self-pay.)*	Canteen
14:00 – 15:15	Open collaboration time	Small Auditorium
15:15 – 16:00	Coffee & collaboration	Atrium; Seminar Room 2
16:00 – 17:25	Open collaboration time	Small Auditorium

Notes

A sequence of Gibbs manifolds arising in the entropic regularization process for a certain semidefinite program. (© Dmitrii Pavlov)



Notes

Cover: One sample point in each connected component of the Clebsch cubic with the 27 lines lying on it removed.