PIC Simulations 1
Tutorial lecture

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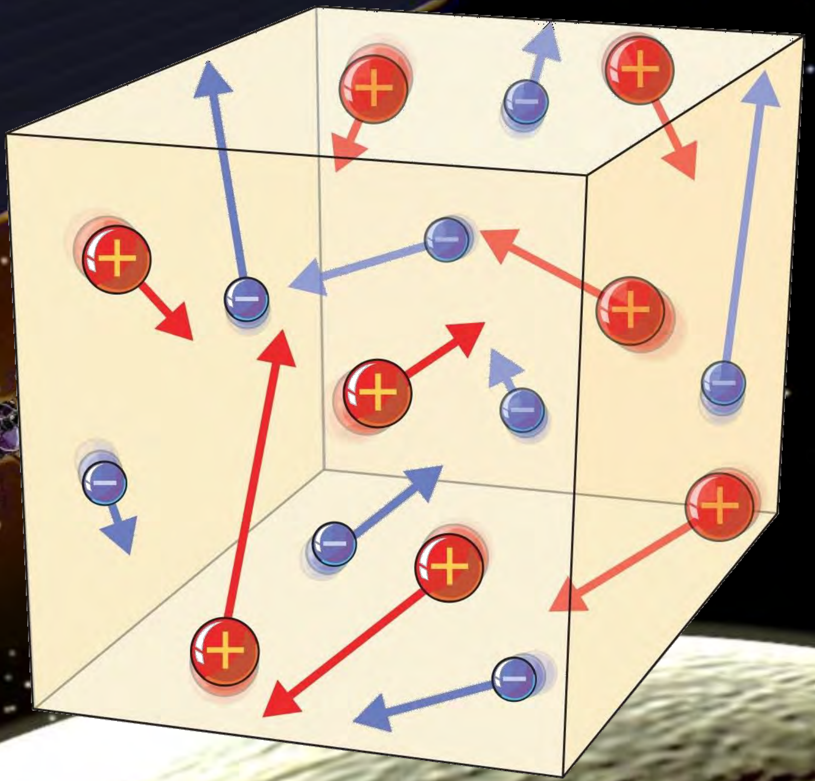
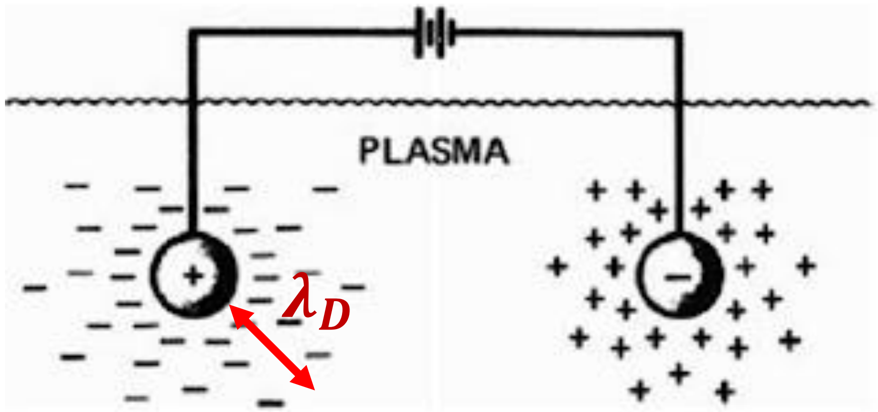
Space plasma

- 4th state of matter
- So hot: atoms split up into electrons & ions
- Quasi-neutral: same numbers of e & i
- Electrons & ions moving independently
- Dynamics influenced by electromagnetic forces
- Plasma current generates electromagnetic field
- Exhibiting “collective” behaviors



Collective effects in shortest spatio-temporal scales

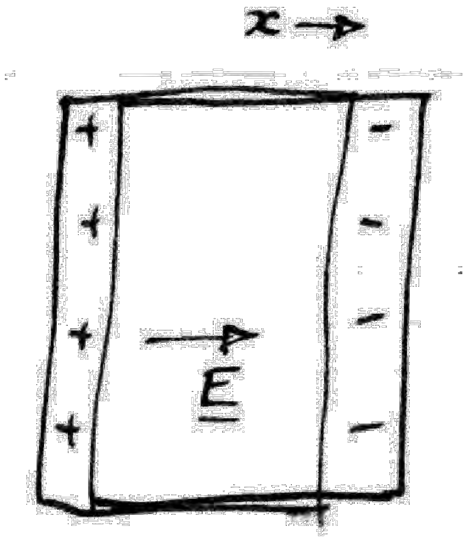
- Debye shielding



Debye length:

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}}$$

- Plasma oscillation



(electron) Plasma frequency:

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}}$$

Behavior as “plasma” manifests on spatio-temporal scales larger than these characteristic quantities.

Criteria for plasmas

Criterion 1: System scales should be greater than λ_D in space and than ${\omega_{pe}}^{-1}$ in time.

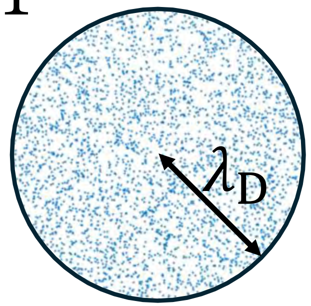
Criterion 2: Debye sphere should contain large number of particles.
…coming from the condition that the potential energy by a nearest particle should be much smaller than the particle's kinetic energy ("weakly-coupled").

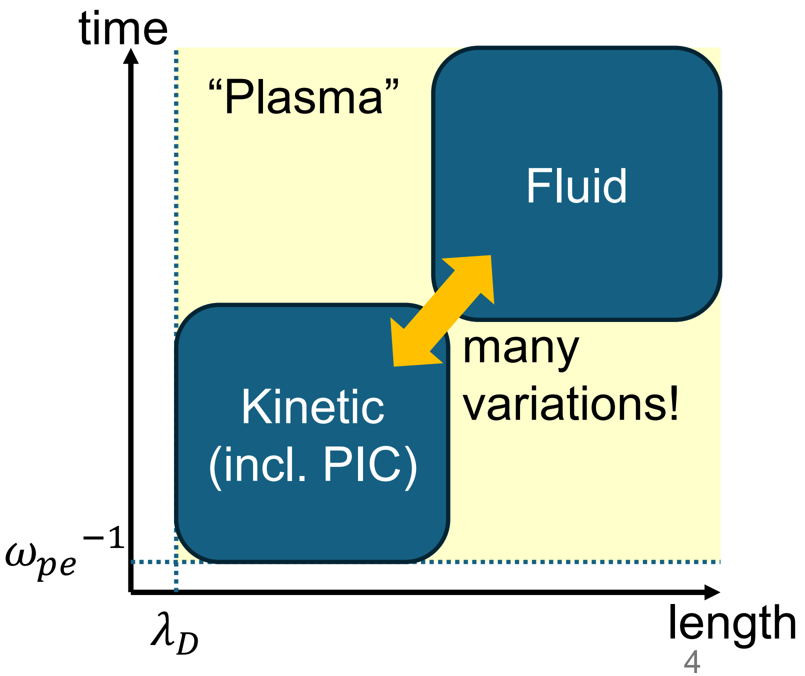
Potential energy: $\phi \sim \frac{e^2}{\varepsilon_0 r} \sim \frac{e^2}{\varepsilon_0} n^{\frac{1}{3}}$, Kinetic energy: $K = k_\mathrm{B} T_\mathrm{e}$

Weakly-coupled condition: $k_\mathrm{B} T_\mathrm{e} \gg \frac{e^2}{\varepsilon_0} n^{\frac{1}{3}}$ yields

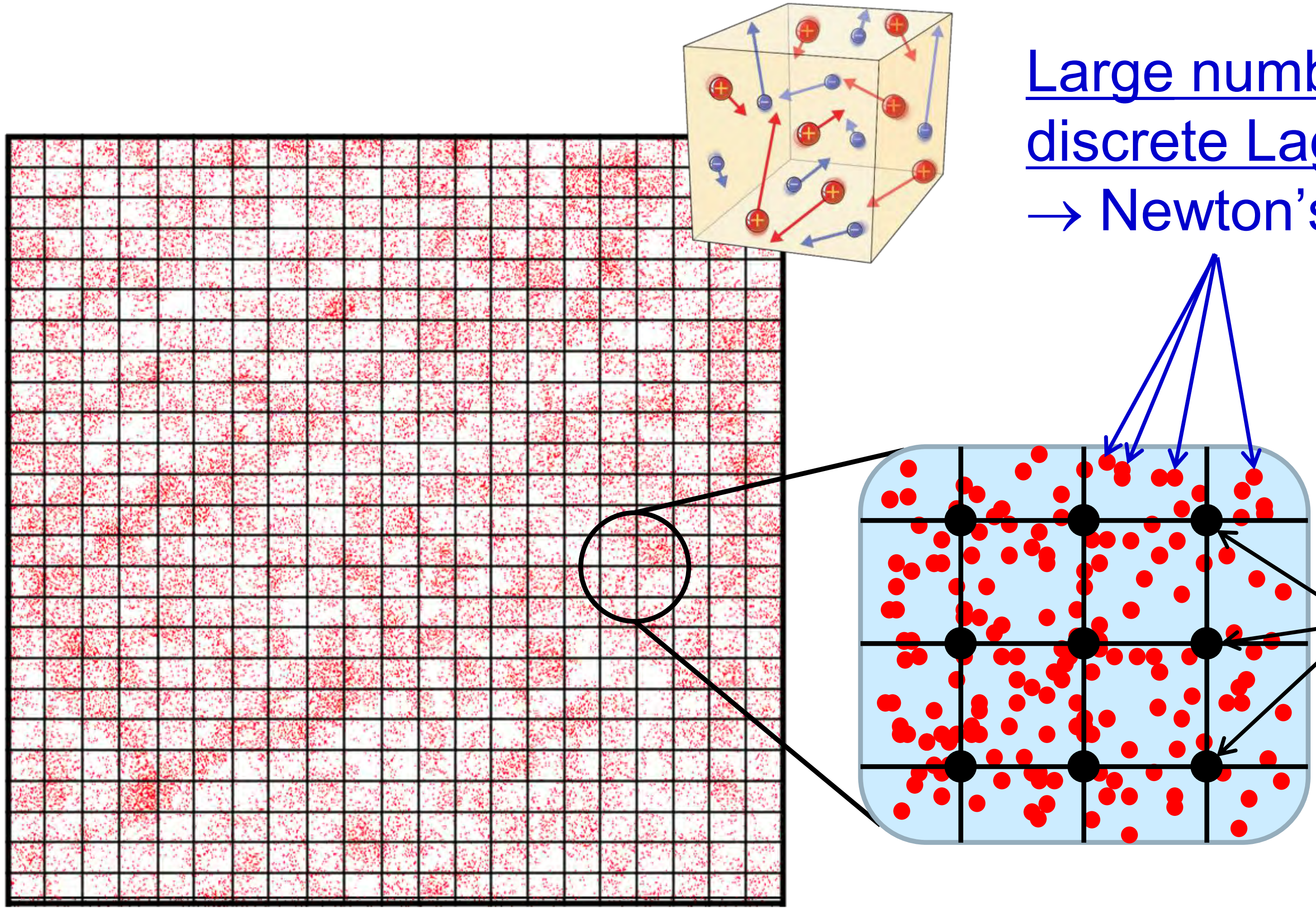
$$n^{\frac{2}{3}} \left(\frac{\varepsilon_0 k_\mathrm{B} T_\mathrm{e}}{n e^2} \right) = n^{\frac{2}{3}} {\lambda_D}^2 \gg 1$$

This also reads $n {\lambda_D}^3 \gg 1$,
where n: number density.





Particle-in-Cell (PIC) simulations



Large number of
discrete Lagrangian particles
→ Newton's equations of motion

$$m\frac{d\boldsymbol{v}_j}{dt} = q_j(\boldsymbol{E}(\boldsymbol{x}_j) + \boldsymbol{v}_j \times \boldsymbol{B}(\boldsymbol{x}_j))$$

$$\frac{d\boldsymbol{x}_j}{dt} = \boldsymbol{v}_j$$

EM-field (force)
on Eulerian mesh (grid)
→ Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

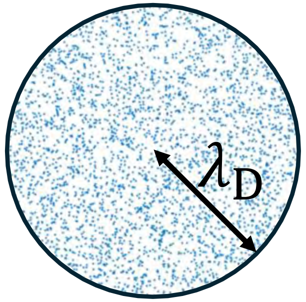
$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

Design of “plasma behavior” in PIC code

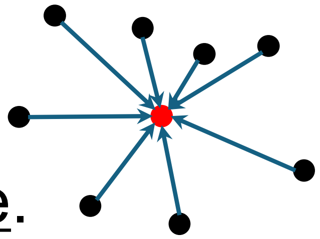
Plasma criterion 2: Debye sphere must contain large number of particles.

…request us to solve huge particles

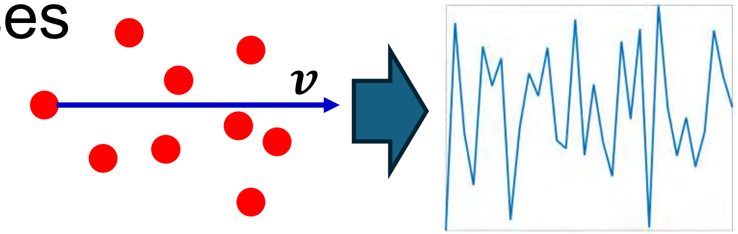
$\rightarrow$ Computationally too expensive!



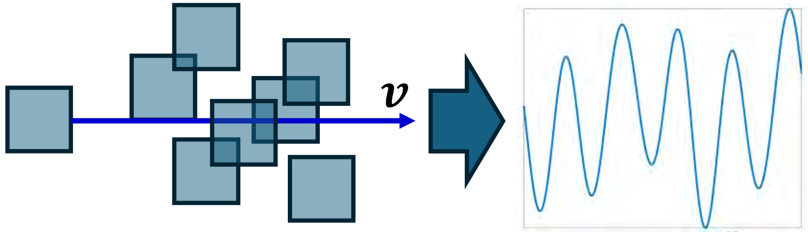
Solution 1: use of a computational particle
that combine many real-world particles into one.



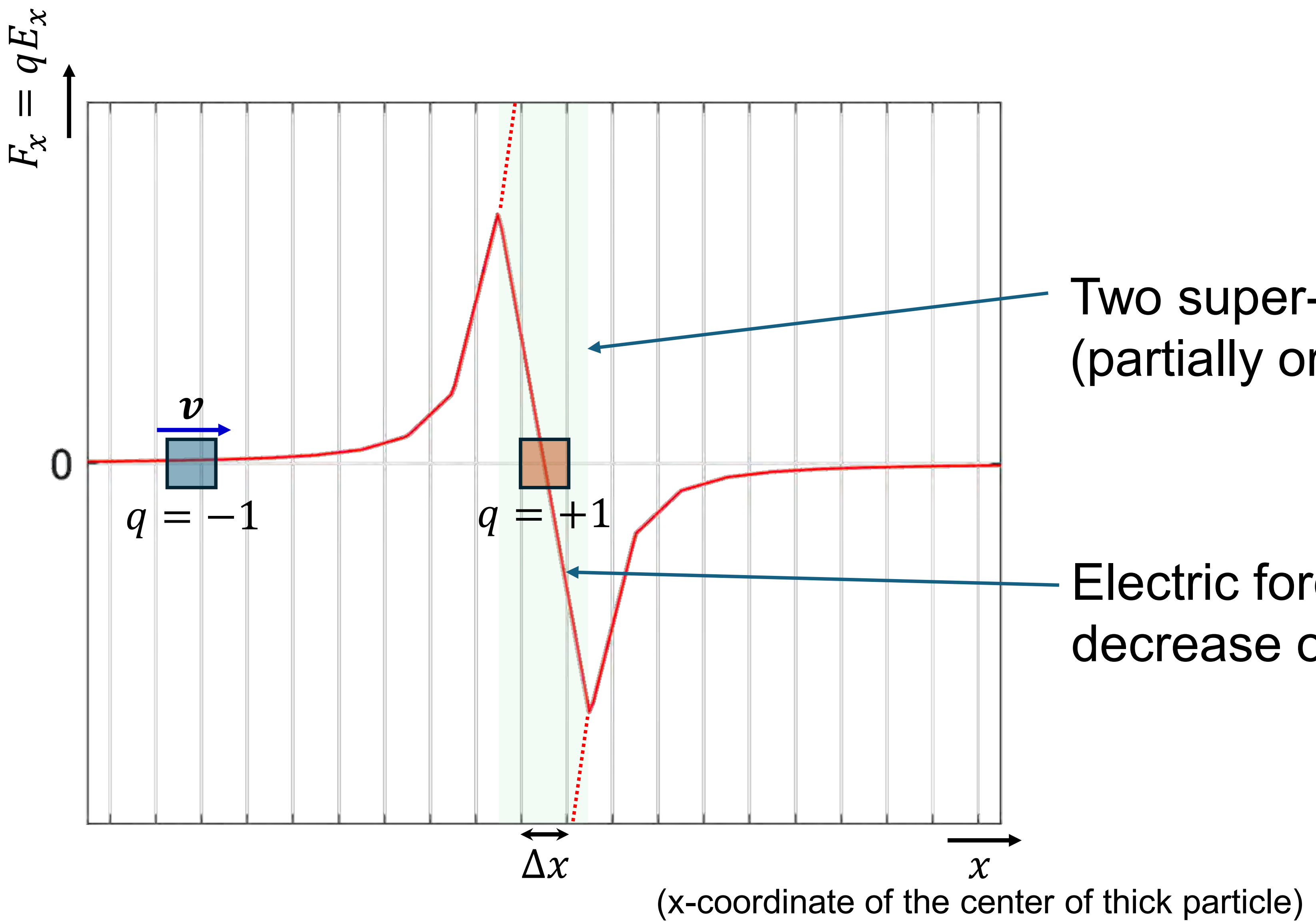
Caveat: a small number of particles with larger charge causes
too large electrostatic interactions between the particles.



Solution 2:
use of a “thick” particle or a “charge-cloud” particle,
to reduce short-range inter-particle collisions.
This is often referred to as a “super-particle”.



Collisionless nature of super-particles

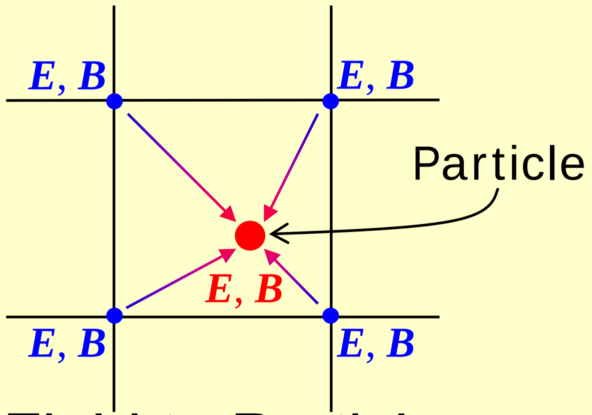


Two super-particles overlap (partially or totally).

Electric forces between them decrease or disappear.

Numerical procedures of PIC simulations

Initialization
- Variable definition
- Memory allocation
- Particle initialization
- Field initialization



Field to Particle

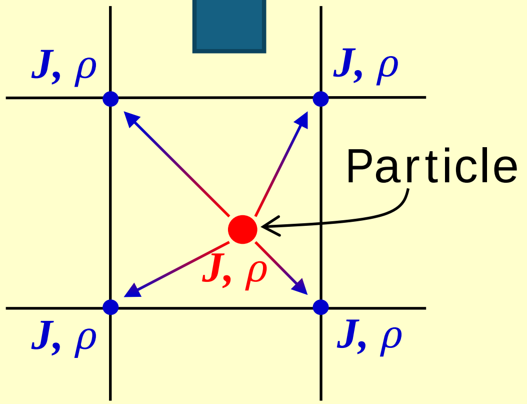
$$\frac{\mathrm{d}(m_i \mathbf{v}_i)}{\mathrm{d}t} = q_i(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B})$$

$$\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = \boldsymbol{v}_i$$

Update of particle velocities/positions

Main loop

Δt



Particle to Field

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{J}$$

or

$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$ in ES approx.

Update of EM-field

Job completion
Diagnostics

Maxwell's equations

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

where $\varepsilon_0 \mu_0 = \frac{1}{c^2}$

Centered difference scheme

Define $u_i = u(xi)$. Then, Taylor-expand $u_{i+1/2}$ and $u_{i-1/2}$ around x_i.

$$u((i+1/2)\Delta x) = u_{i+1/2} = u_i + \frac{\Delta x}{2}\left(\frac{\partial u}{\partial x}\right)_i + \frac{1}{2!}\left(\frac{\Delta x}{2}\right)^2\left(\frac{\partial^2 u}{\partial x^2}\right)_i + \frac{1}{3!}\left(\frac{\Delta x}{2}\right)^3\left(\frac{\partial^3 u}{\partial x^3}\right)_i + \cdots$$

$$u((i-1/2)\Delta x) = u_{i-1/2} = u_i - \frac{\Delta x}{2}\left(\frac{\partial u}{\partial x}\right)_i + \frac{1}{2!}\left(\frac{\Delta x}{2}\right)^2\left(\frac{\partial^2 u}{\partial x^2}\right)_i - \frac{1}{3!}\left(\frac{\Delta x}{2}\right)^3\left(\frac{\partial^3 u}{\partial x^3}\right)_i + \cdots$$

where, $\Delta x = x_i - x_{i-1}$.

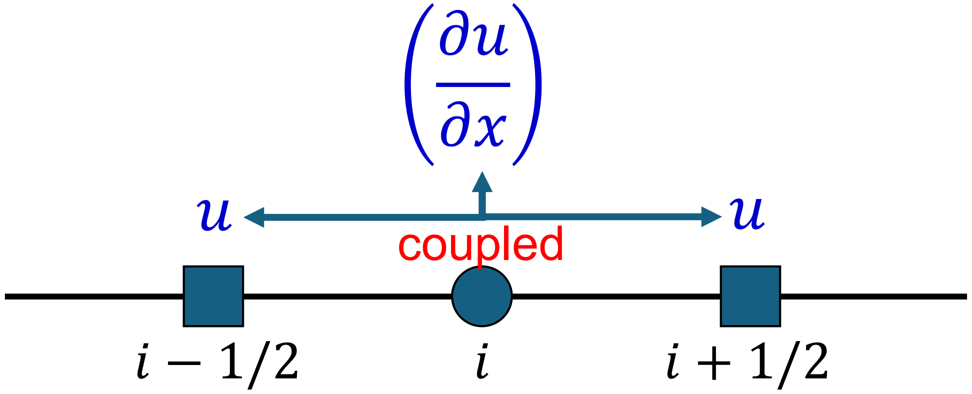
Subtraction of each other gives

$$u_{i+1/2} - u_{i-1/2} = \Delta x\left(\frac{\partial u}{\partial x}\right)_i + \frac{2}{3!}\left(\frac{\Delta x}{2}\right)^3\left(\frac{\partial^3 u}{\partial x^3}\right)_i + \cdots$$

Centered difference expression

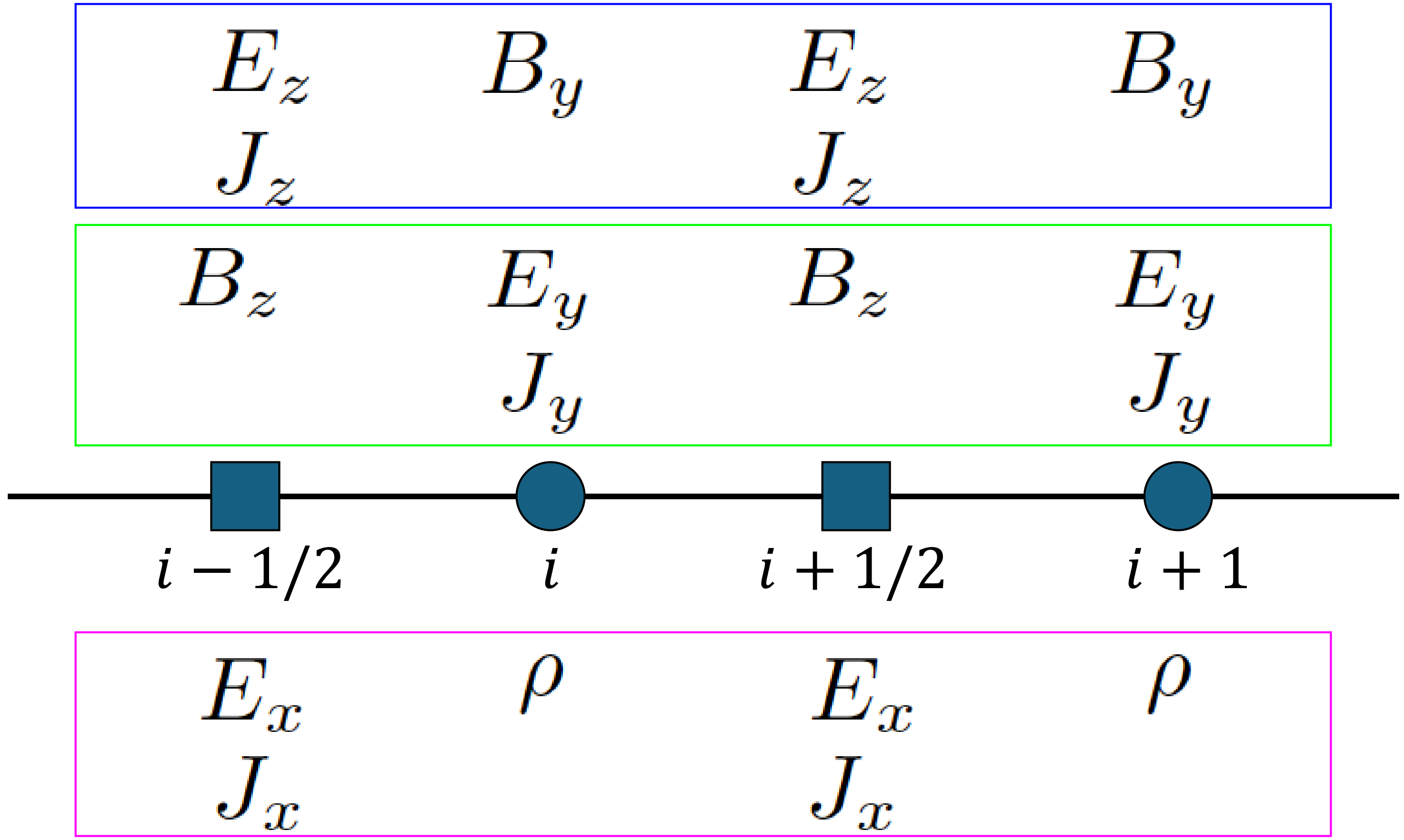
$$\left(\frac{\partial u}{\partial x}\right)_i = \frac{u_{i+1/2} - u_{i-1/2}}{\Delta x} + O(\Delta x^2)$$

$$\sim \frac{u_{i+1/2} - u_{i-1/2}}{\Delta x}$$



"Staggered" grid assignment

Example of grid assignment (1D along x-axis)



$$\frac{\partial E_z}{\partial t} = c^2 \frac{\partial B_y}{\partial x} - J_z$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

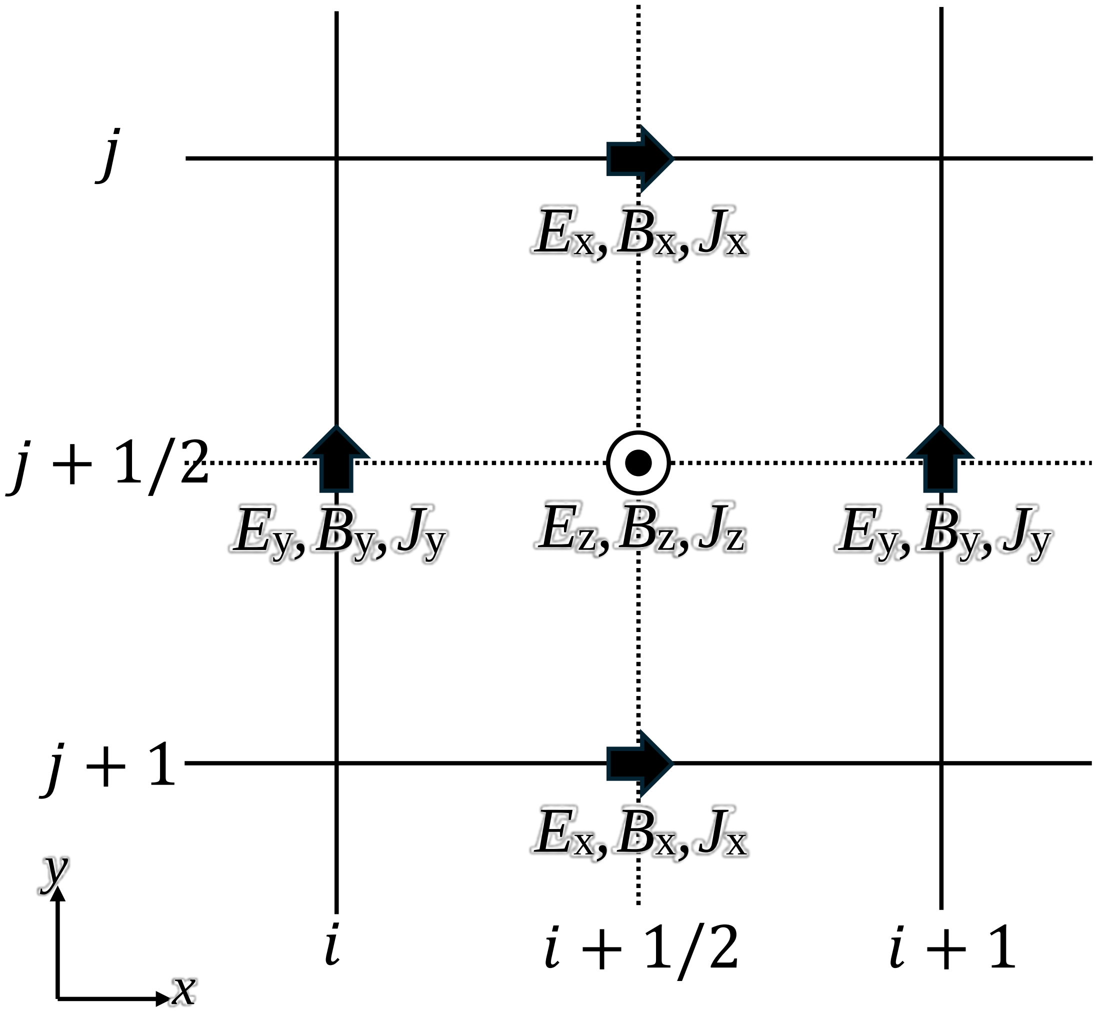
$$\frac{\partial E_y}{\partial t} = -c^2 \frac{\partial B_z}{\partial x} - J_y$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

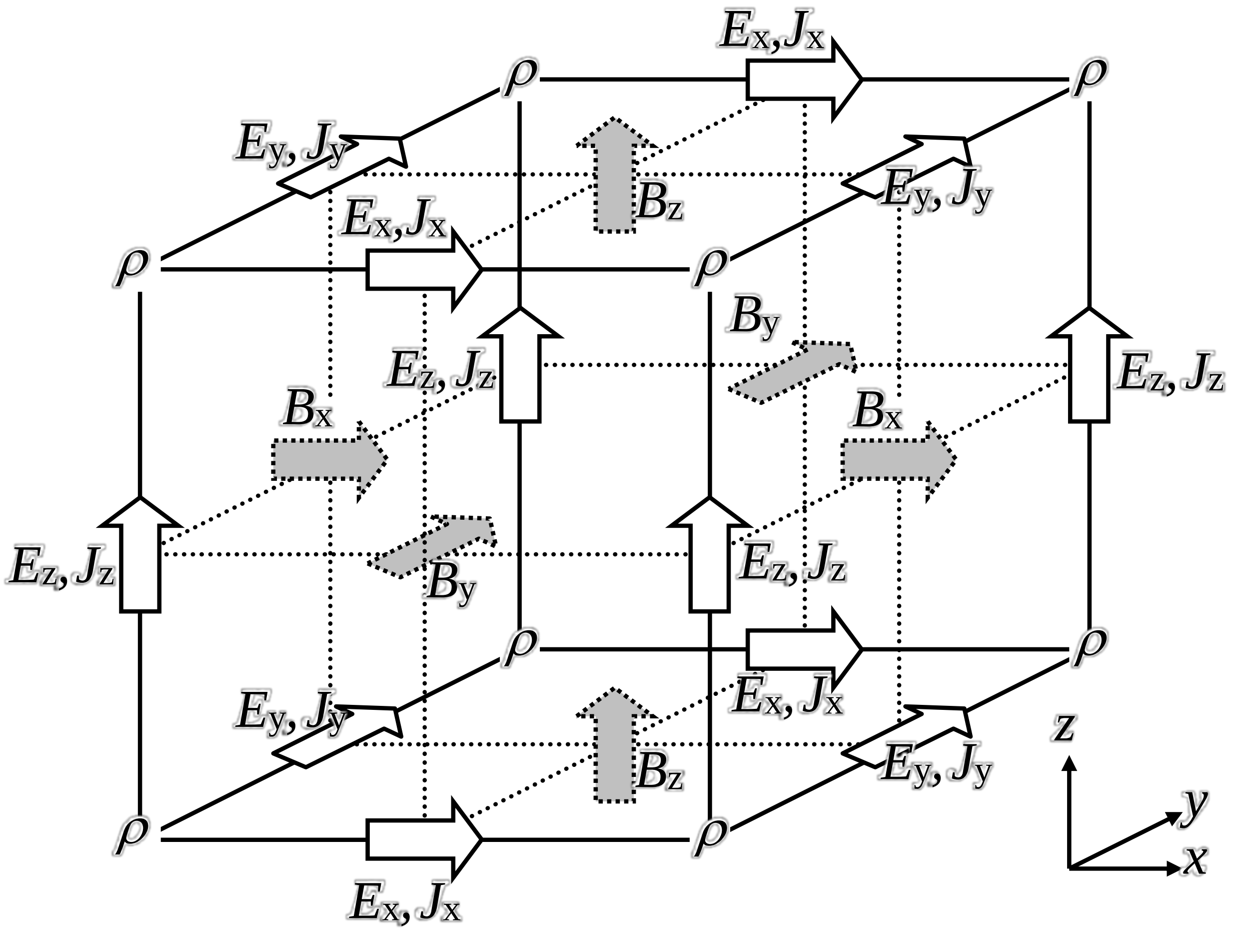
$$\frac{\partial E_x}{\partial t} = -J_x$$

Examples of grid assignment (2D & 3D)

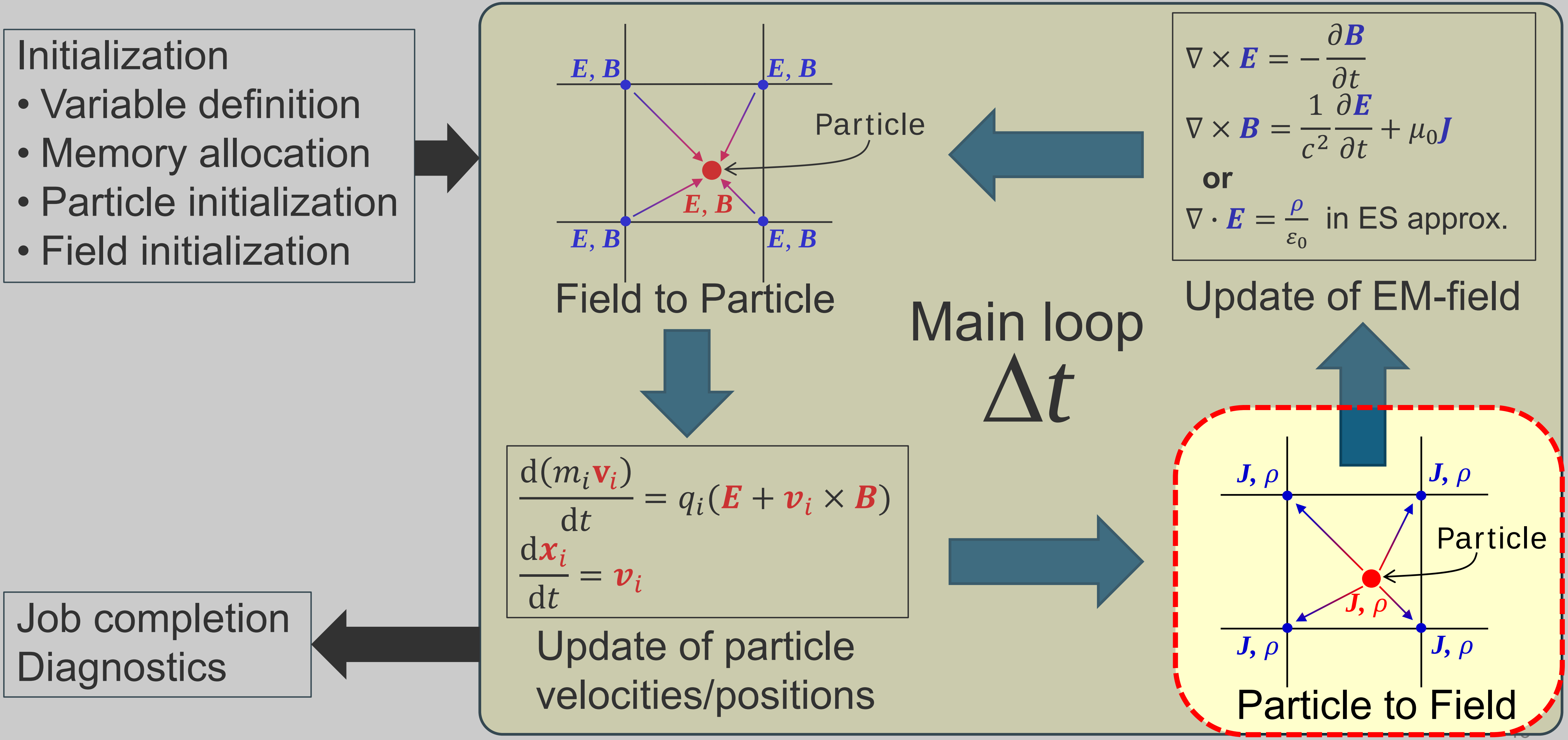
- 2D



- 3D



Numerical procedures of PIC simulations



Charge density

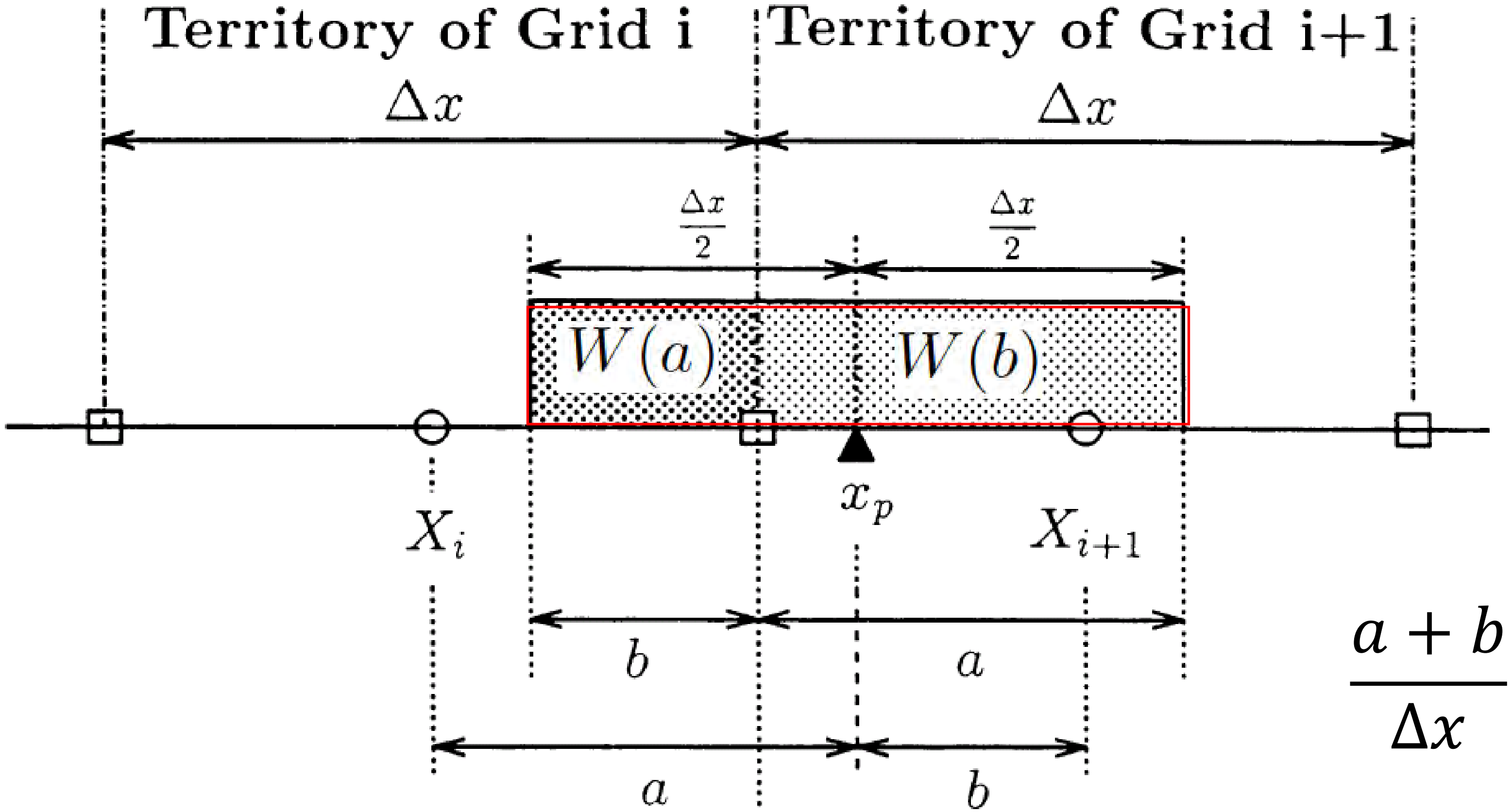
$$\rho_i = \frac{1}{\Delta x}\sum_{j}^{N_p} q_j W(x_j - X_i)$$

Np: Number of Particles

Shape Function

$$W(x) = 1 - \frac{|x|}{\Delta x}, \quad |x| \le \Delta x$$
$$= 0, \quad |x| > \Delta x$$

"thick particle"



$$\frac{a+b}{\Delta x} = 1$$

$$W(a) = \frac{b}{\Delta x}$$

$$W(b) = \frac{a}{\Delta x}$$

Current density in 1D system along x-axis

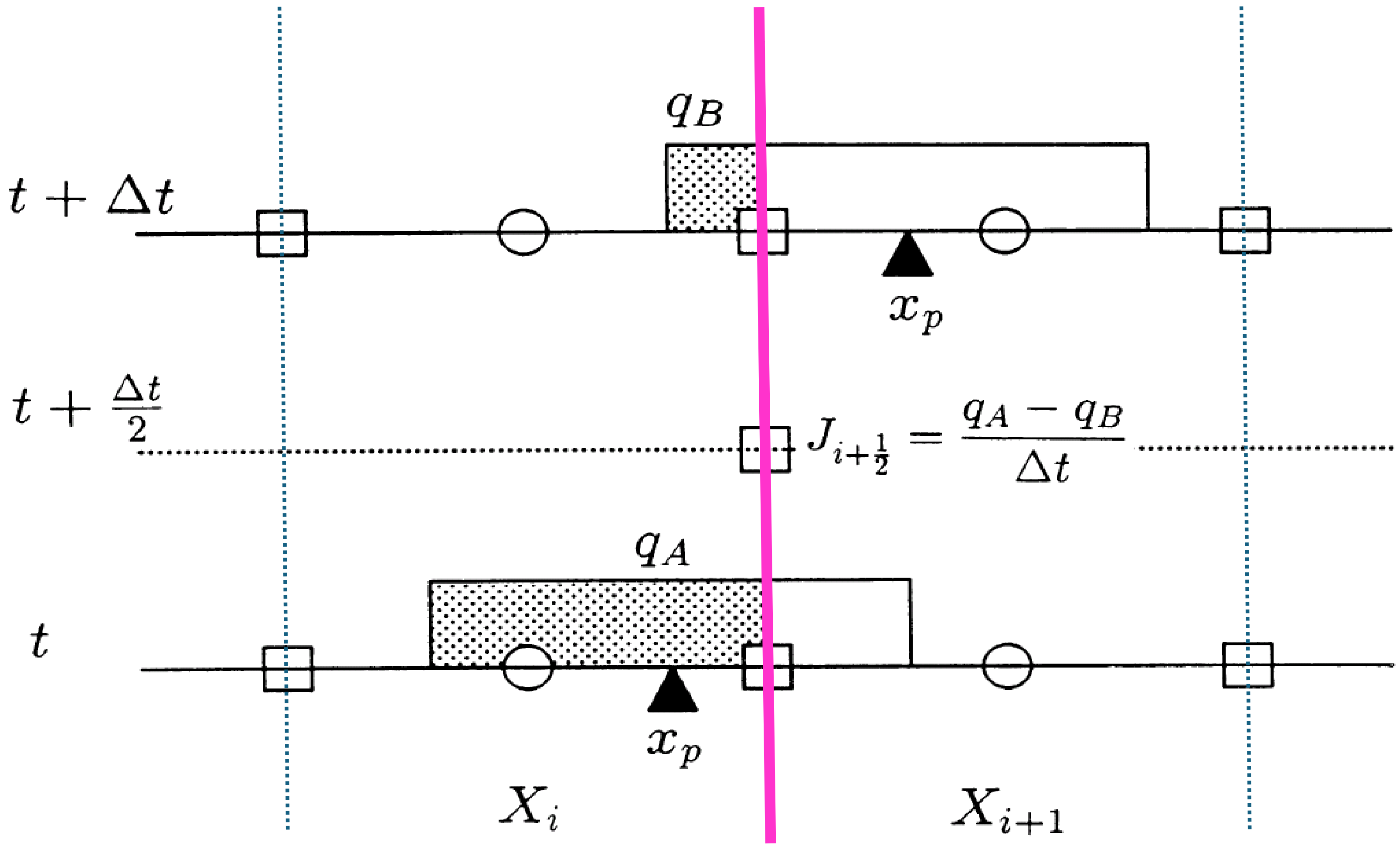
- J_y, J_z components: area-weighting method (same as charge density)
- J_x computed based on the area-weighting does NOT satisfy charge continuity equation: $\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\nabla \cdot \boldsymbol{J} = -\frac{\partial J_x}{\partial x}$ …(1)
  $\rightarrow$ cause an accumulative error in an electrostatic (ES) field.

◆Solution 1: Correcting an ES field every time step

1. Define charge ρ_c associated with error in ES field : $\rho_c = \rho - \nabla \cdot \boldsymbol{E}$
2. Solve Poisson's equation: $\nabla^2 \phi_c = -\rho_c$, for ϕ_c: electric potential
3. Compute ES field correction: $\boldsymbol{E}_c = -\nabla \phi_c$
4. Add $\boldsymbol{E}_c$ to $\boldsymbol{E}$.

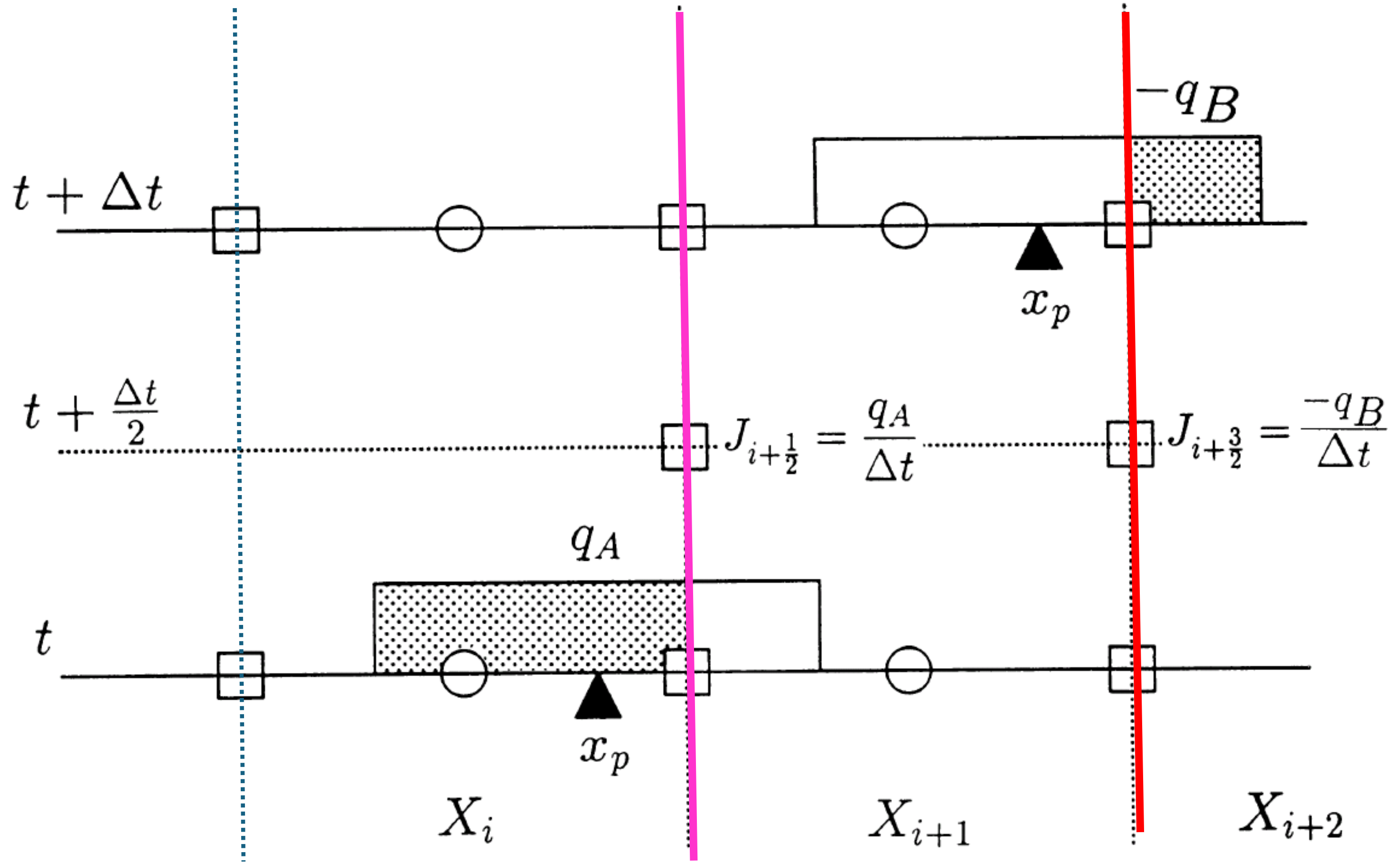
◆Solution 2: Using a "Charge Conservation Method (CCM)" to compute current, which satisfies (1) in the machine accuracy.

Charge conservation method in 1D (J_x): case 1



$$q_A = q\frac{X_{i+1} - x_p(t)}{\Delta x} \qquad q_B = q\frac{X_{i+1} - x_p(t + \Delta t)}{\Delta x}$$

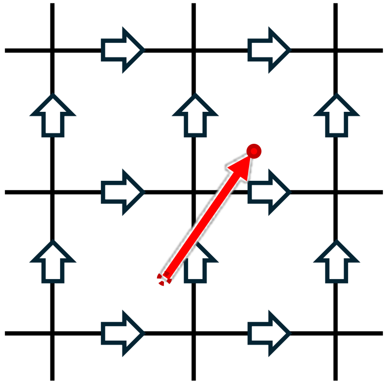
Charge conservation method in 1D (J_x): case 2



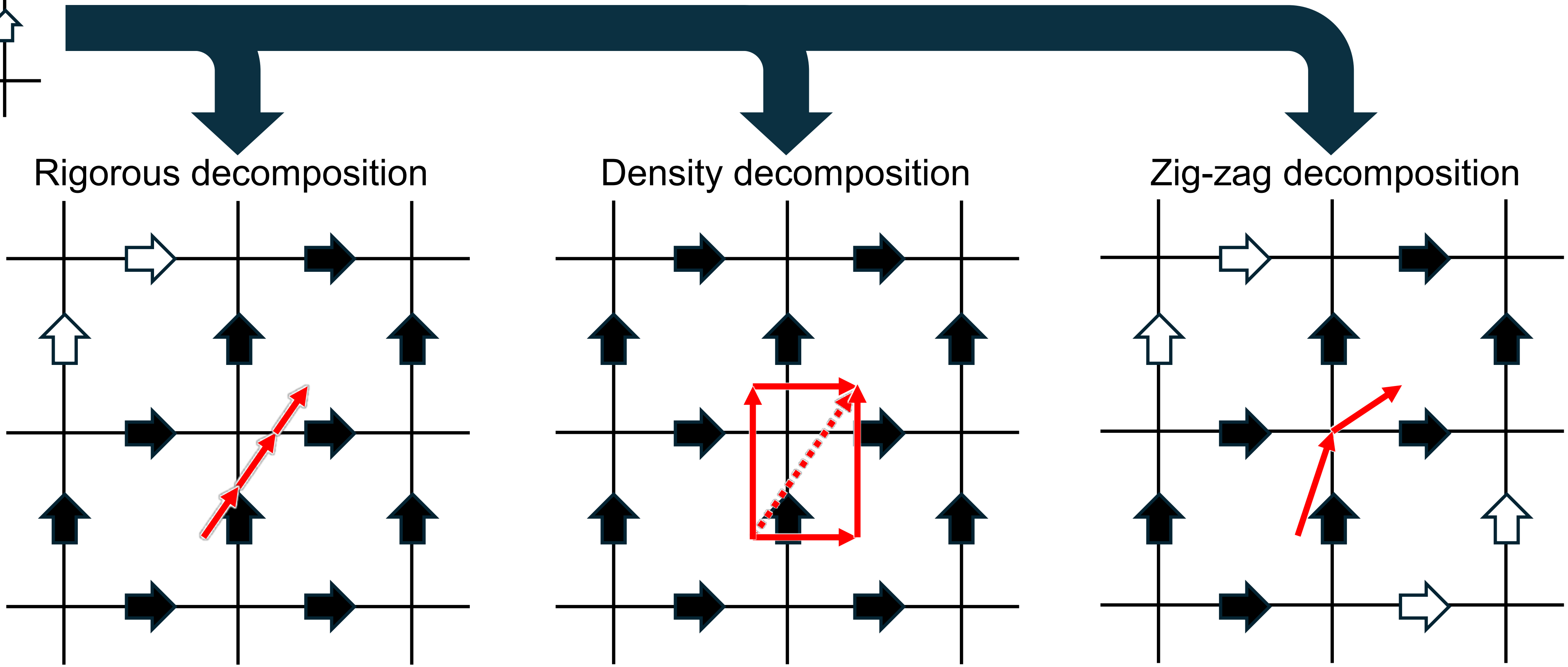
$$q_A = q\frac{X_{i+1} - x_p(t)}{\Delta x}$$

$$q_B = q\frac{X_{i+1} - x_p(t + \Delta t)}{\Delta x}$$

Charge conservation methods in higher dimensions

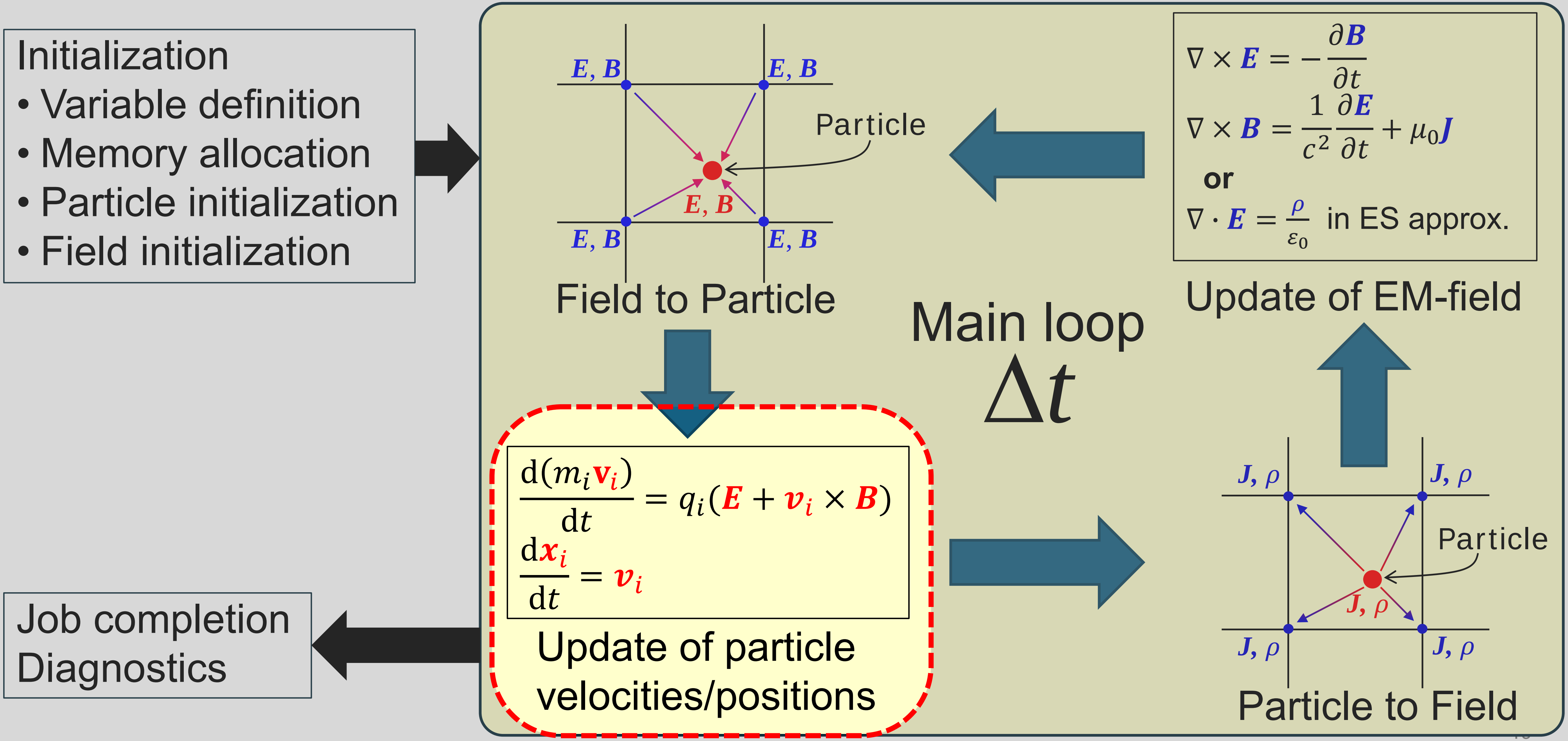


Trajectory decomposition needed in case the particle moves across cell edges; various approaches proposed.



[Villasenor & Buneman, 1992] [Esirkepov, 2001] [Umeda+, 2003]

Numerical procedures of PIC simulations



Update of velocity: Buneman-Boris method

$$\frac{\boldsymbol{v}^{t+\Delta t/2}-\boldsymbol{v}^{t-\Delta t/2}}{\Delta t}=\frac{q_s}{m_s}\left(\boldsymbol{E}^t+\frac{\boldsymbol{v}^{t+\Delta t/2}+\boldsymbol{v}^{t-\Delta/2}}{2}\times\boldsymbol{B}^t\right)$$

$$\boldsymbol{v}^{-}=\boldsymbol{v}^{t-\Delta t/2}+\frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2}\qquad \boldsymbol{v}^{+}=\boldsymbol{v}^{t+\Delta t/2}-\frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2}$$

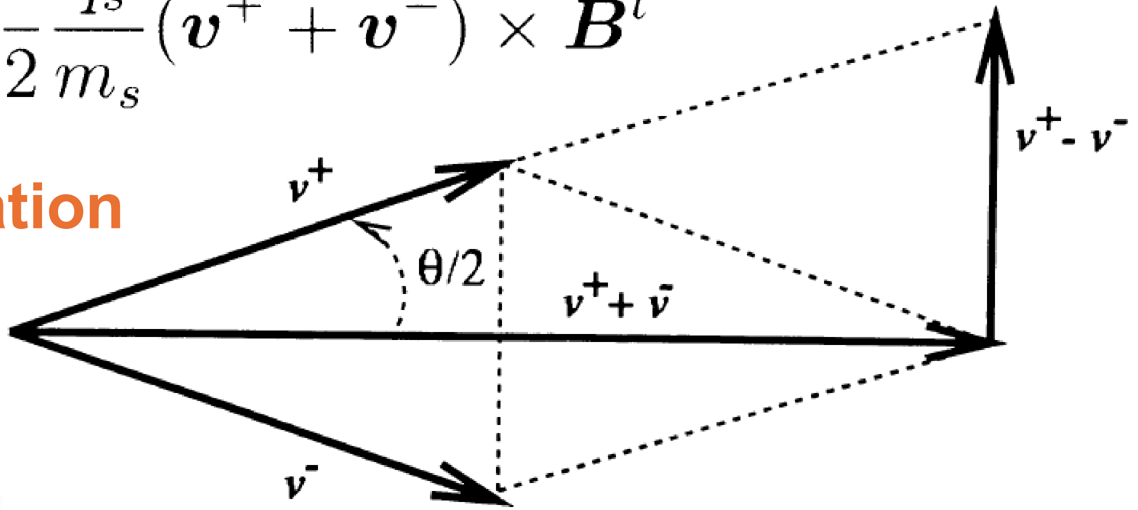
$$\frac{\boldsymbol{v}^{+}-\boldsymbol{v}^{-}}{\Delta t}=\frac{1}{2}\frac{q_s}{m_s}(\boldsymbol{v}^{+}+\boldsymbol{v}^{-})\times\boldsymbol{B}^t$$

Kinetic Energy Conservation

$$(\boldsymbol{v}^{+})^2=(\boldsymbol{v}^{-})^2$$

Small Phase Delay

$$\Omega_c=\frac{\tan^{-1}\omega_c\Delta t/2}{\Delta t/2}$$



$$\Omega_c/\omega_c=0.9967\text{ with }\omega_c\Delta t=0.2$$

Relativistic equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\boldsymbol{v}) = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

$$m = \gamma m_0$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

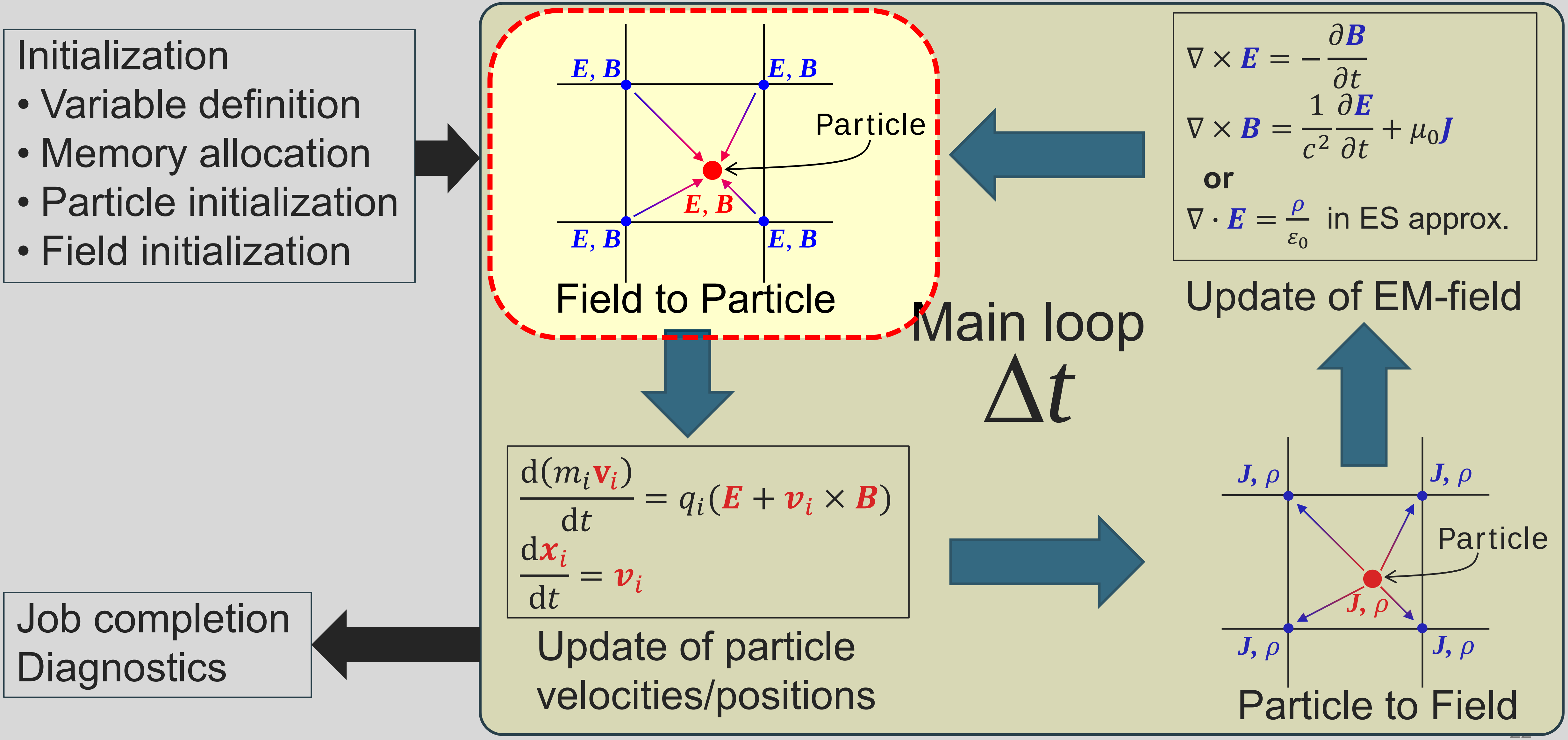
$$\boldsymbol{u} = \frac{c}{\sqrt{c^2 - |\boldsymbol{v}|^2}}\boldsymbol{v}$$

$$\boldsymbol{B}_u = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{B}$$

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{q}{m_0}\left(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}_{\boldsymbol{u}}\right)$$

$$\boldsymbol{v} = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{u}$$

Numerical procedures of PIC simulations

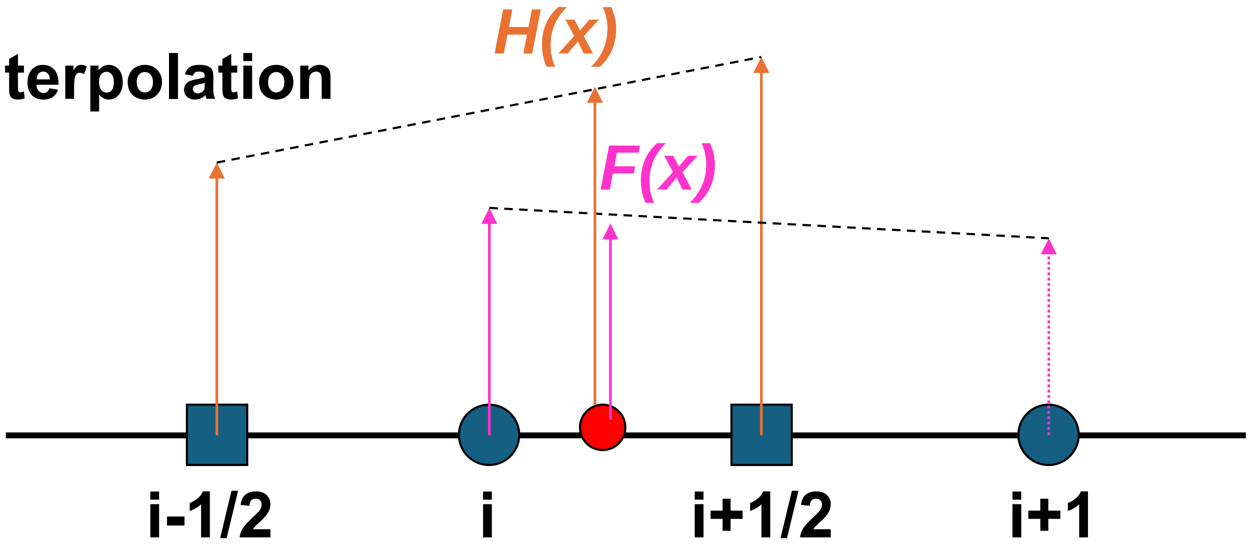


Field interpolation to particle position

$$F(x) = \sum_{i=1}^{N_x} F_i W(x - X_i)$$

$$H(x) = \sum_{i=1}^{N_x} H_{i+1/2} W(x - X_{i+1/2})$$

Linear Interpolation



Electrostatic self-force cancellation

$$\rho_i = \frac{1}{\Delta x}\sum_j^{N_p} q_j W(x_j - X_i)$$

$$\frac{E_{x,i+1/2} - E_{x,i-1/2}}{\Delta x} = \frac{\rho_i}{\varepsilon_0}$$

$$E_x(x) = \sum_{i=1}^{N_x} E_{x,i+1/2} W(x - X_{i+1/2})$$

→ **Self-force**

Relocation $$E_{x,i} = \frac{E_{x,i-1/2} + E_{x,i+1/2}}{2}$$

$$E_x(x) = \sum_{i=1}^{N_x} E_{x,i} W(x - X_i)$$

→ **No Self-force**

Magnetostatic self-force cancellation

Magnetostatic equation (Ampere's Law) $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$

$$\boldsymbol{J}_{yz,i+1/2} = \frac{1}{\Delta x} \sum_j^{N_p} q_j \boldsymbol{v}_{yz} W(x_j - X_{i+1/2})$$

Source Relocation J_y & B_z

$$J_{y,i} = \frac{J_{y,i-1/2} + J_{y,i+1/2}}{2}$$

$$\frac{B_{z,i+1/2} - B_{z,i-1/2}}{\Delta x} = -\mu_o J_{y,i}$$

Field Relocation J_z & B_y

$$\frac{B_{y,i+1} - B_{y,i}}{\Delta x} = \mu_o J_{z,i+1/2}$$

$$B_{y,i+1/2} = \frac{B_{y,i} + B_{y,i+1}}{2}$$

$$\boldsymbol{B}_{yz}(x) = \sum_{i=1}^{N_x} \boldsymbol{B}_{yz,i+1/2} W(x - X_{i+1/2})$$

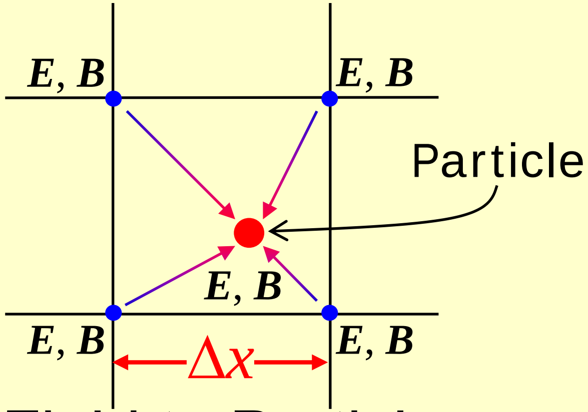
→ **No Self-force**

Numerical procedures of PIC simulations

Initialization

- Variable definition
- Memory allocation
- Particle initialization
- Field initialization

Main loop Δt

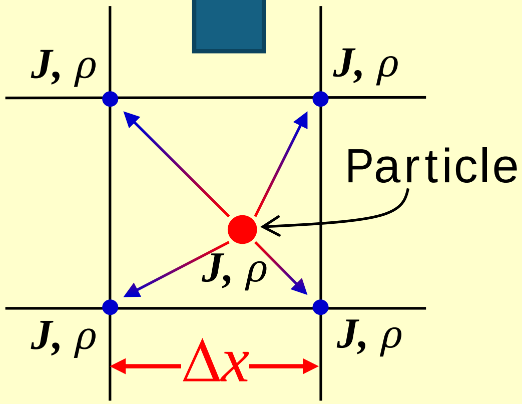


Field to Particle

$$\frac{\mathrm{d}(m_i \mathbf{v}_i)}{\mathrm{d}t} = q_i(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B})$$

$$\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = \boldsymbol{v}_i$$

Update of particle velocities/positions



Particle to Field

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{J}$$

or

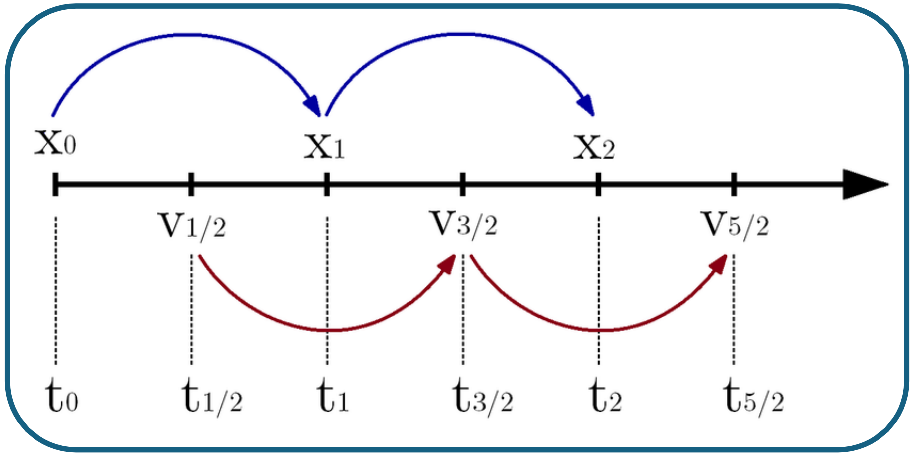
$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$ in ES approx.

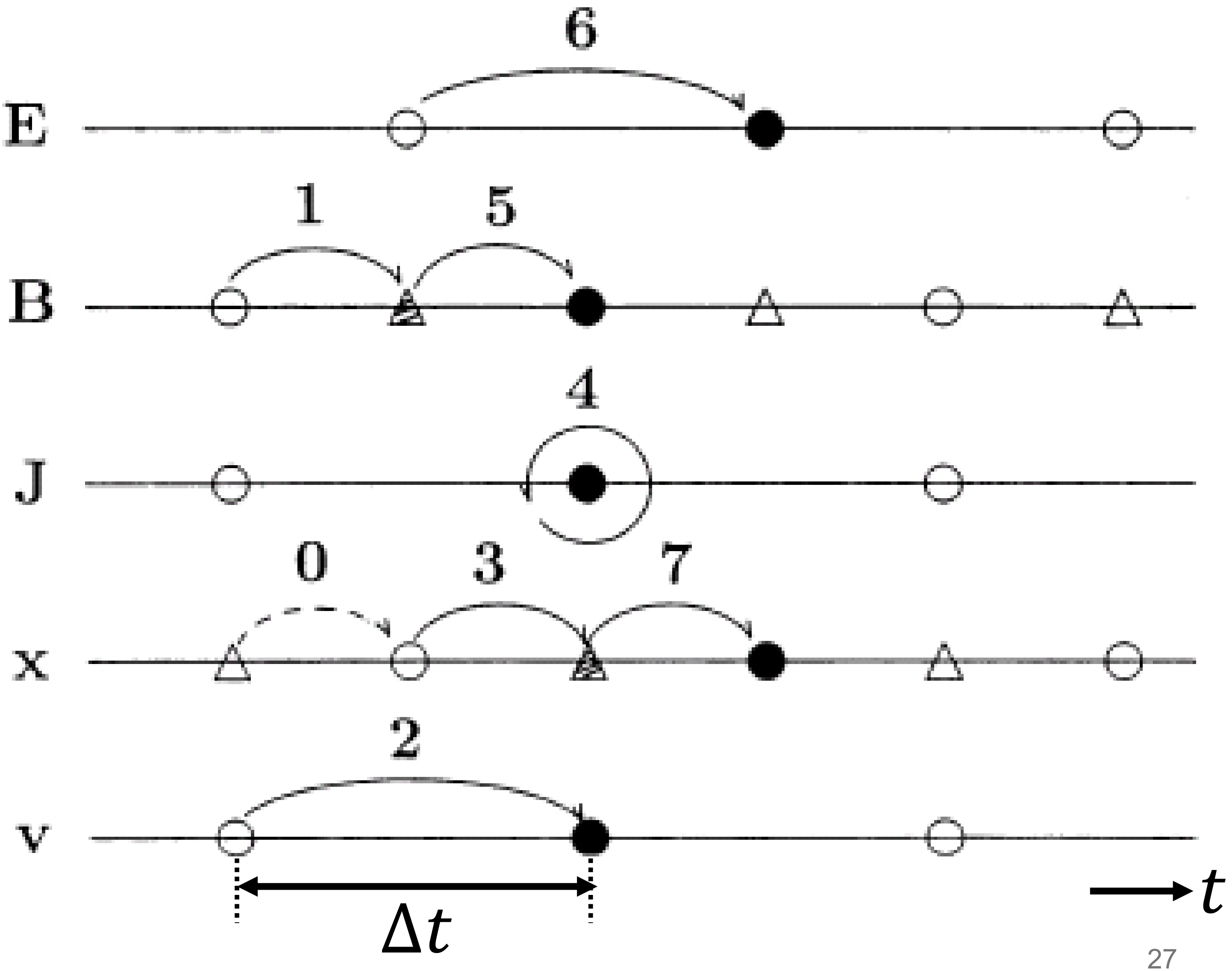
Update of EM-field

Job completion
Diagnostics

Time Step Chart

Leap-frog scheme





Effect of centered differentiation

$$E(X_i, t) = E_o \exp(ikX_i - i\omega t)$$

$$\frac{\partial E(X_i,t)}{\partial x} = \frac{E(X_i + \Delta x/2, t) - E(X_i - \Delta x/2, t)}{\Delta x}$$

$$= \frac{1}{\Delta x}\left[\exp(ik\Delta x/2) - \exp(-ik\Delta x/2)\right] E(X_i,t)$$

$$= i\frac{\sin(k\Delta x/2)}{\Delta x/2} E(X_i,t) = iKE(X_i,t)$$

$$k \Rightarrow K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

$$\omega \Rightarrow \Omega = \frac{\sin(\omega\Delta t/2)}{\Delta t/2}$$

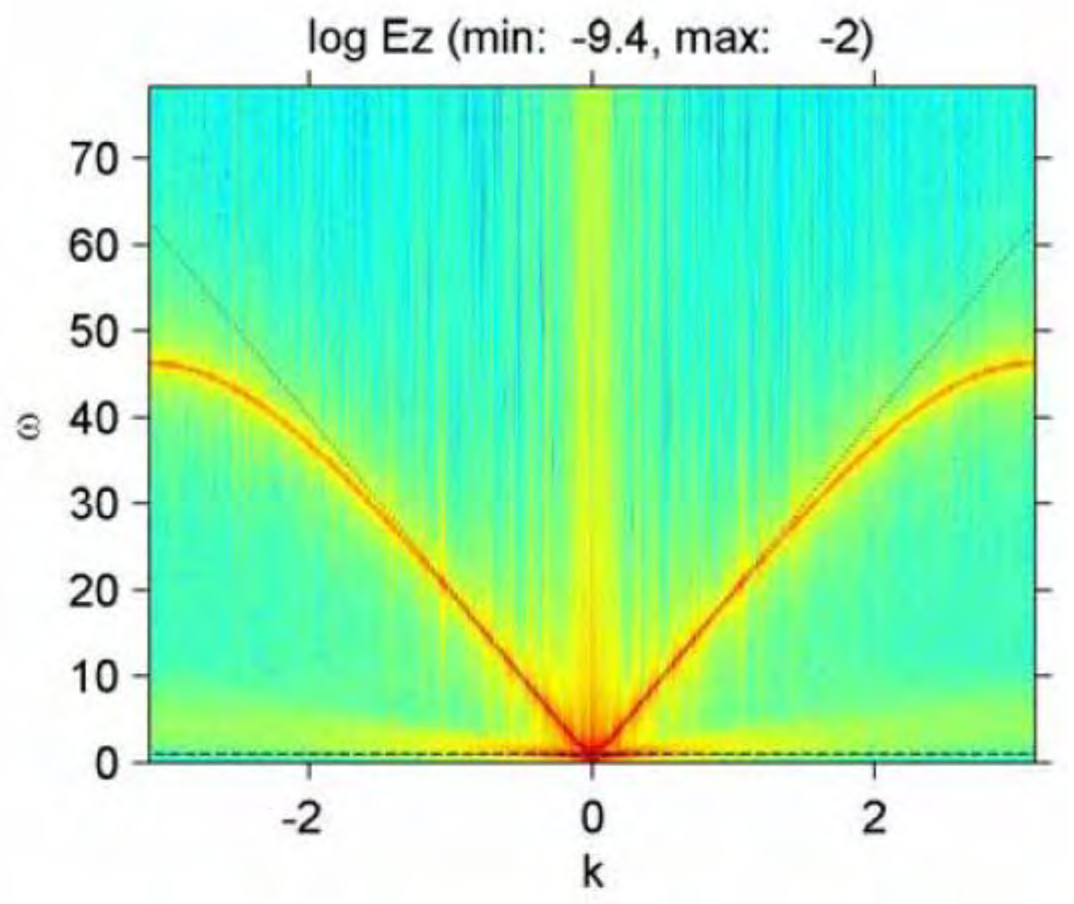
Modified dispersion relation of light mode

Electromagnetic modes in vacuum

$$\omega^2 = c^2 k^2$$

In centered difference scheme,

$$\Omega^2 = c^2 K^2$$



$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}, \qquad K = \frac{\sin(k \Delta x/2)}{\Delta x/2}$$

Condition for numerical stability 1

Centered Difference Scheme in space and time

$$\Omega^2 = c^2 K^2 \qquad \Omega = \frac{\sin(\omega\Delta t/2)}{\Delta t/2}, \qquad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

For $k = \dfrac{\pi}{\Delta x}$ we have $\sin(\dfrac{\omega\Delta t}{2}) = \dfrac{\Delta t}{\Delta x}c < 1$

Courant Condition

$$c\Delta t < \Delta x$$

Condition for numerical stability 2

Shortest wavelength scale to solved in the explicit PIC is

$$|k| \sim \frac{1}{\lambda_D}$$

where λ_D is the Debye length.

Replacing *k* with *K*

$$|\sin(k\Delta x/2))| \sim \frac{\Delta x}{2\lambda_D} < 1$$

Explicit PIC should satisfy the following stability condition

$$\Delta x < 2\lambda_D$$

Violation of the condition leads to numerical (unphysical) heating.

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Initialization
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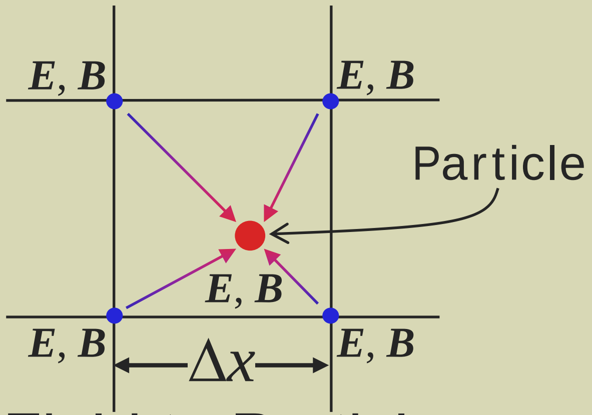
Main loop Δt

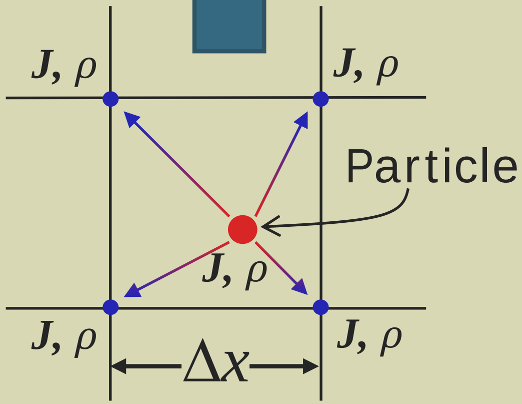


Field to Particle

$$\frac{\mathrm{d}(m_i \mathbf{v}_i)}{\mathrm{d}t} = q_i(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B})$$

$$\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = \boldsymbol{v}_i$$

Update of particle velocities/positions



Particle to Field

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{J}$$

or

$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$ in ES approx.

Update of EM-field

Job completion
Diagnostics

Initial & boundary conditions

Initialization

- Particle loading
  - Positions:
  - Velocities: Maxwellian, shifted Maxwellian, loss-cone, ring, etc.
- Initial electrostatic field: Poisson's equation

Outer boundary condition

- Periodic boundaries
- Reflecting boundaries
- Open boundaries
  - Particle injection from outer edge
  - Non-reflective field boundary: masking method

Inner boundary condition

- for object-plasma interaction study

Numerical solutions of Poisson's equation

$\nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0,\ \boldsymbol{E} = -\nabla\phi$ reduce to $\nabla^2\phi = -\rho/\varepsilon_0$ (Poisson's equation).

In 1D,

$$\frac{\partial^2\phi}{\partial x^2} = -\frac{\rho}{\varepsilon_0}$$

In Fourier space,

$$k^2\hat{\phi} = \frac{\hat{\rho}}{\varepsilon_0}$$

In centered difference scheme,

$$K^2\hat{\phi}_k = \frac{\hat{\rho}_k}{\varepsilon_0},\ \text{where}\ K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

FFT-based Poisson solver,

ρ_i → FFT (DFT) → $\hat{\rho}_k$ → $K^2\hat{\phi}_k = \frac{\hat{\rho}_k}{\varepsilon_0}$ → $\hat{\phi}_k$ → FFT (DFT) → ϕ_i

In periodic system,

$$K^2\hat{\phi}_0 = \frac{\hat{\rho}_0}{\varepsilon_0} = 0\ \text{for}\ k = 0 \longrightarrow \sum_i \frac{\rho_i}{N_x} = 0$$

"Charge neutral"

Cancellation of uniform current

Consider periodic 1D system along x-axis:

$$\frac{\partial \boldsymbol{J}_u}{\partial t} = \frac{n_e e^2}{m_e}\boldsymbol{E}_u \qquad \frac{\partial \boldsymbol{E}_u}{\partial t} = -\boldsymbol{J}_u$$

…provide a solution in the following form:

$$\boldsymbol{J}_u = \boldsymbol{J}_o \exp\left(i\omega_{pe} t\right) \qquad \boldsymbol{E}_u = \frac{i}{\omega_{pe}}\boldsymbol{J}_o \exp\left(i\omega_{pe} t\right)$$

The uniform component must be subtracted.
(Periodic system should also be "current neutral".)

$$\boldsymbol{J}_u = \frac{1}{N_x}\sum_{i=1}^{N_x}\boldsymbol{J}_i \qquad \boldsymbol{J}_{i,sub} = \boldsymbol{J}_i - \boldsymbol{J}_u$$

Initial & boundary conditions

Initialization

- Particle loading
  - Positions:
  - Velocities: Maxwellian, shifted Maxwellian, loss-cone, ring, etc.
- Initial electrostatic field: Poisson's equation

Outer boundary condition

- Periodic boundaries
- Reflecting boundaries
- Open boundaries
  - Particle injection from outer edge
  - Non-reflective field boundary: masking method

Inner boundary condition

- for object-plasma interaction study

Inner boundary conditions for object-in-plasma

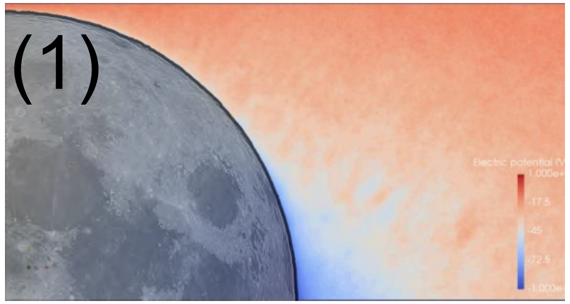
<u>Solid objects in space</u>
- Celestial bodies …(1)
- Dust grains …(2)
- Spacecraft / instrument …(3)

…and so on.

<u>Effects at **object surface**</u>

Inner boundaries

- Particle loss
- Particle emission
- Charge deposition
- Surface potential
- Conducting current
- EM scattering

(1)


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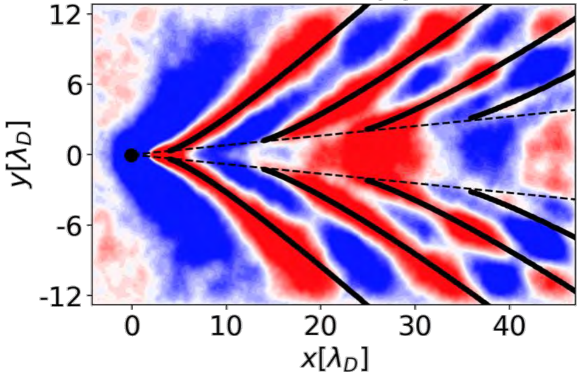

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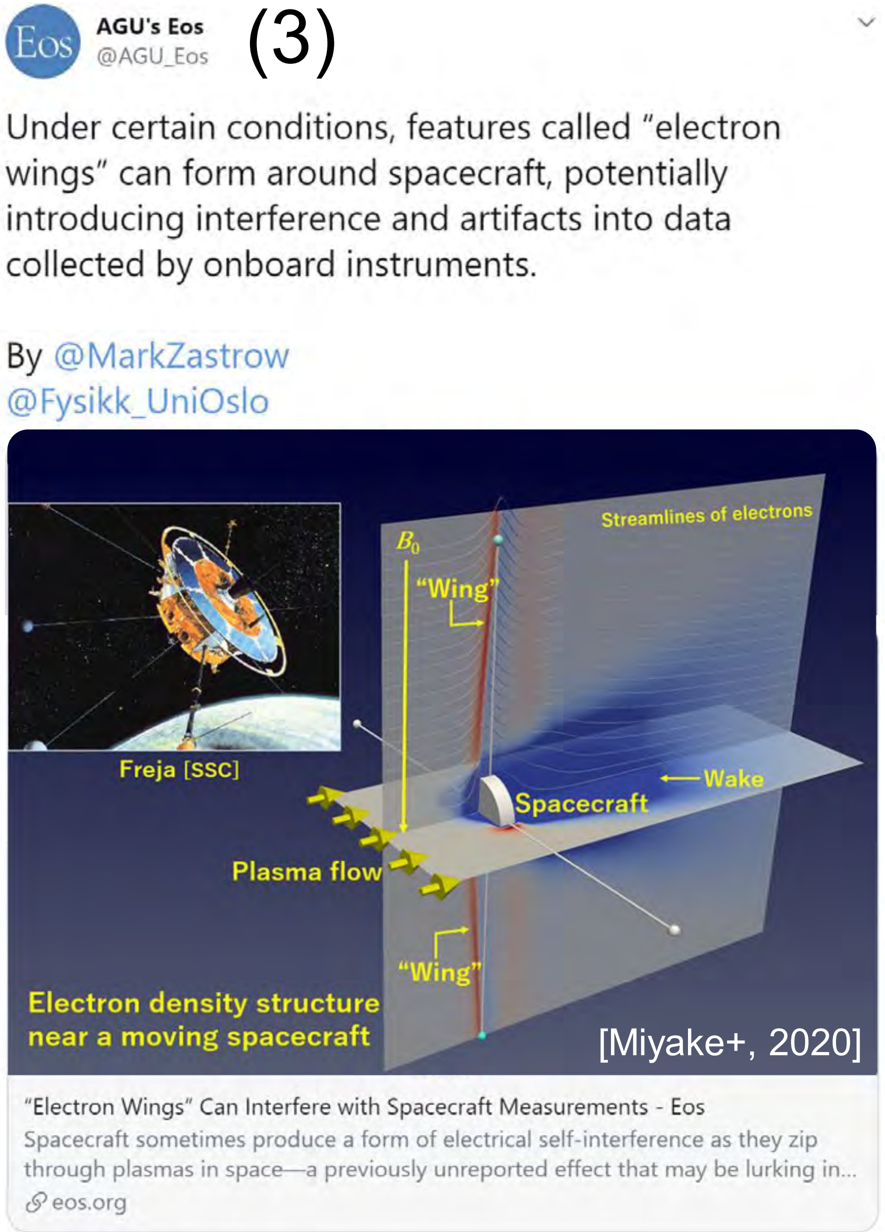


[Miyake+, 2020]

Capacitance matrix

capacitance

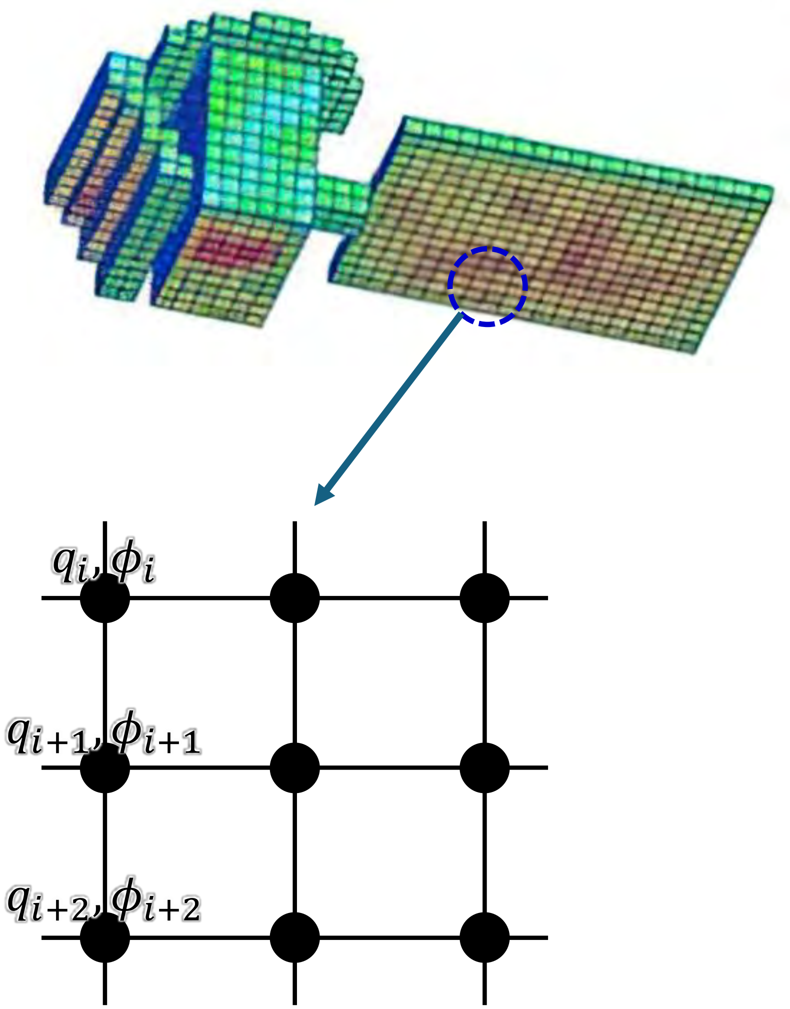
$$Q = {\color{red}C}\Phi$$

charge potential

charges potentials

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_n \end{pmatrix} = \begin{bmatrix} c_{11} & \cdots & c_{n1} \\ c_{12} & & c_{n2} \\ c_{13} & & c_{n3} \\ \vdots & \ddots & \vdots \\ c_{1n} & \cdots & c_{nn} \end{bmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_n \end{pmatrix}$$

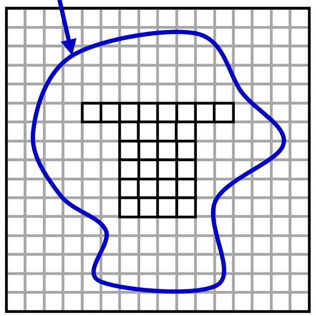
capacitance matrix



Control of on-grid potential/charge

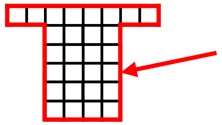
1. Solve Poisson's eq.

$\rho_{\mathrm{orig}} \Longrightarrow \phi_{\mathrm{orig}}$



2. Compute $\Delta\rho_i$

$$\Delta\phi_j = \phi_{\mathrm{target\_}j} - \phi_{\mathrm{orig}_j}$$



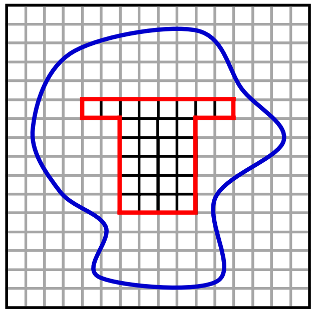
$$\Delta q_i = \sum_j c_{ij}\Delta\phi_j$$

$i, j = 1 \cdots N_{\mathrm{grid\_for\_object}}$

c_{ij}: capacitance matrix element

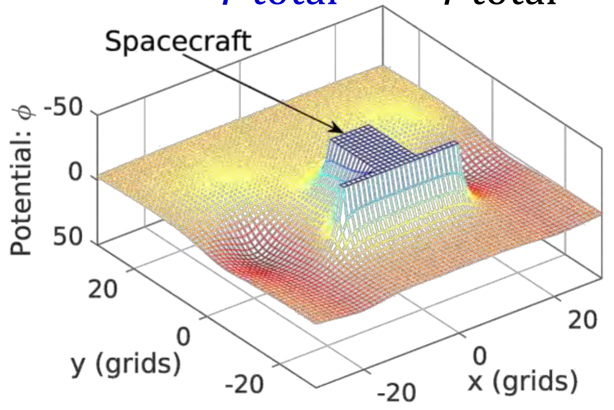
3. Add charge correction

$\rho_{\mathrm{total}} \leftarrow \rho_{\mathrm{orig}} + \Delta q/(\Delta x)^{N_{\mathrm{dim}}}$



4. Solve Poisson's eq.

$\rho_{\mathrm{total}} \Longrightarrow \phi_{\mathrm{total}}$

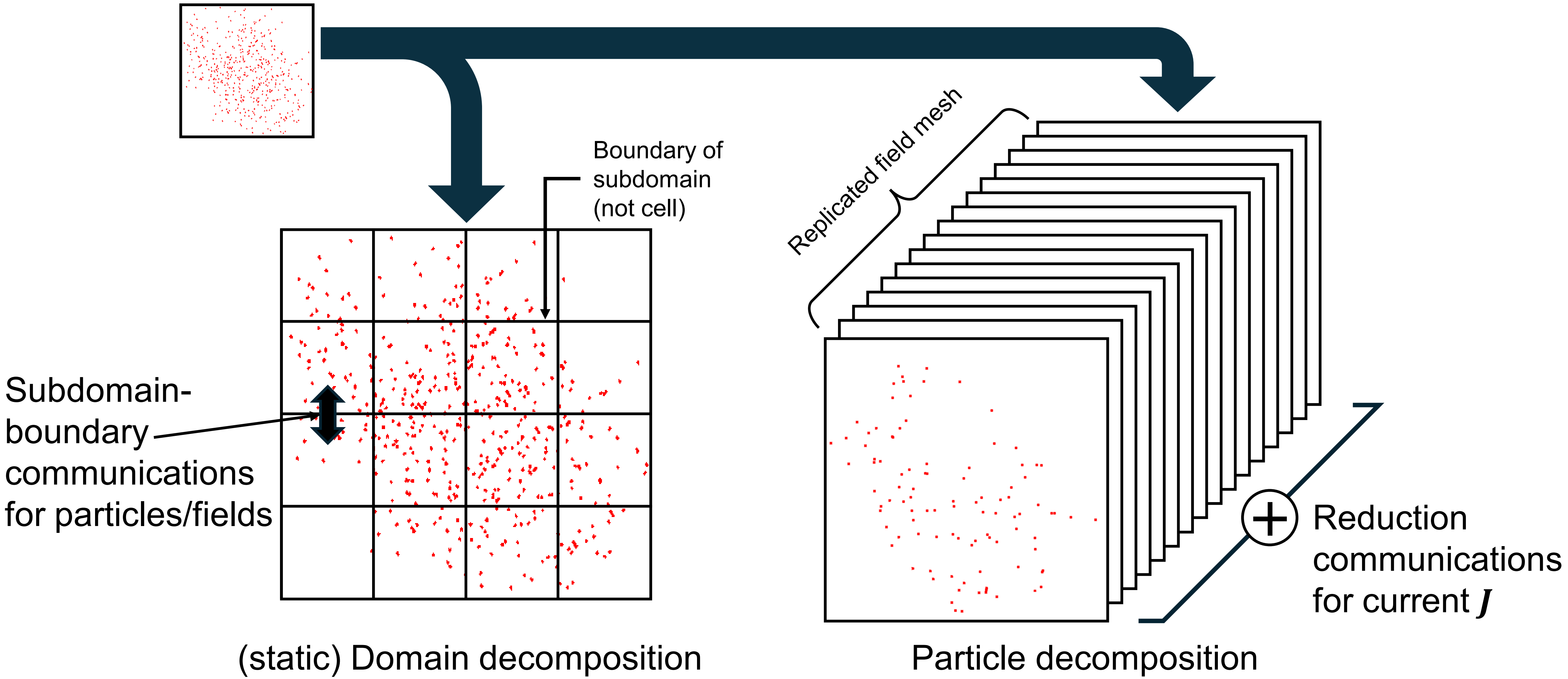


Some applications…

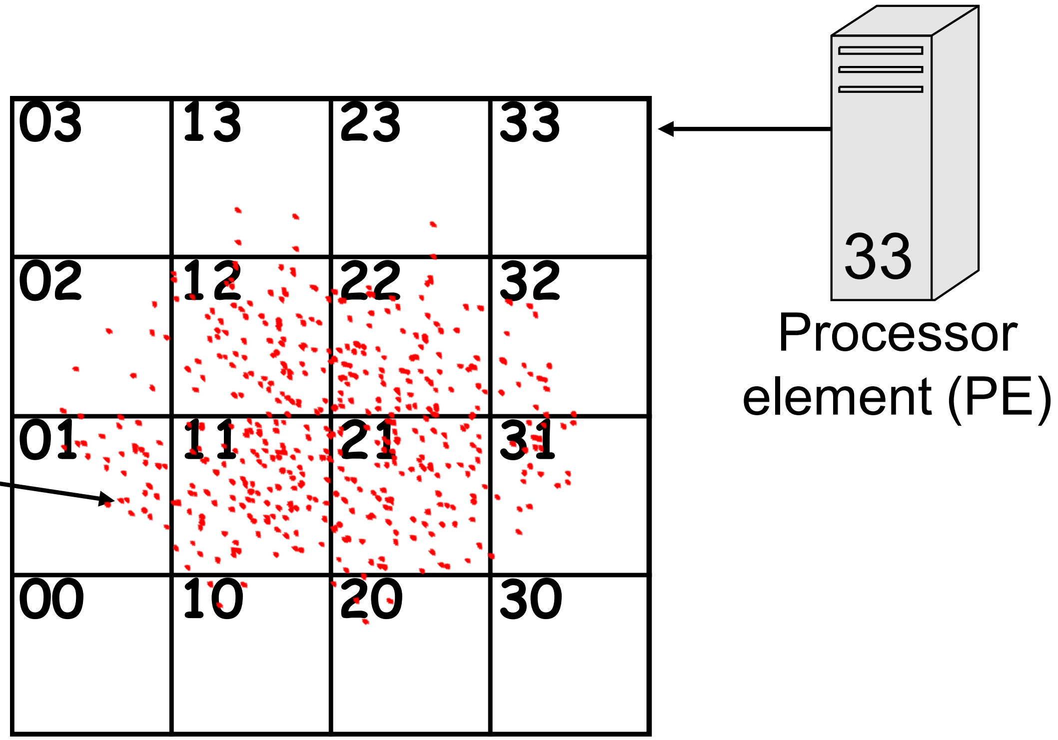
1. Known target potentials on all grids on object surface: Solve $Q = C\Phi$.
   Application: electrodes with applied potential
2. Equi-potential over object surface, but its potential value unknown:
   Solve $Q = C\Phi$ with $\sum_i \Delta q_i = 0$.
   Application: conducting objects with floating potential (e.g., spacecraft)

PIC simulations on Supercomputers

Fundamental strategies of parallelization

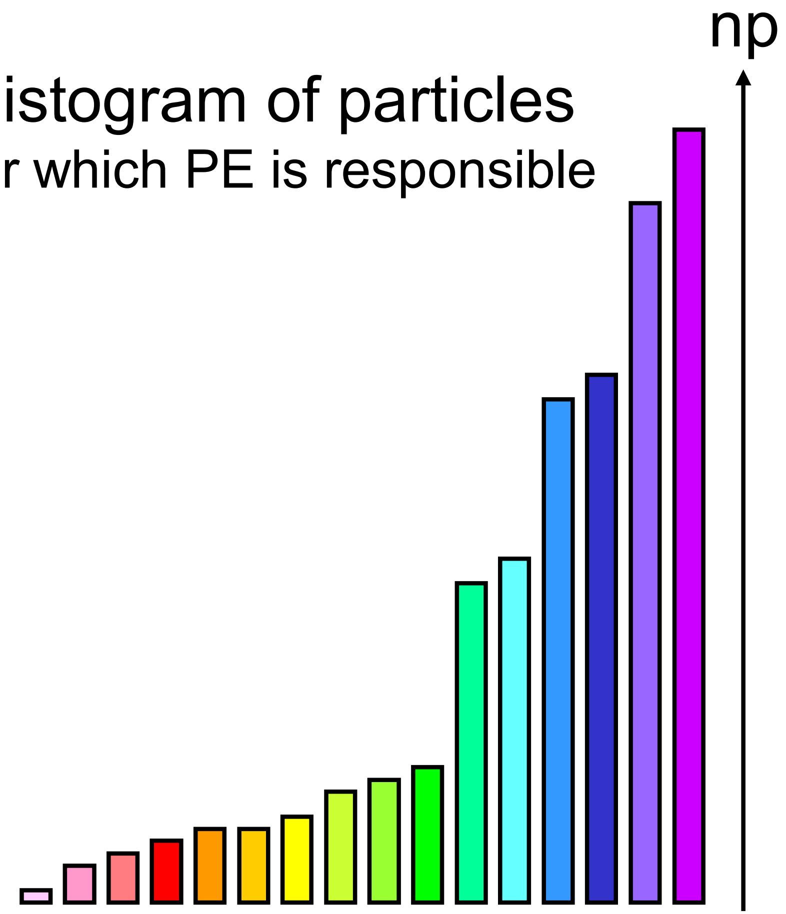


Load balancing consideration



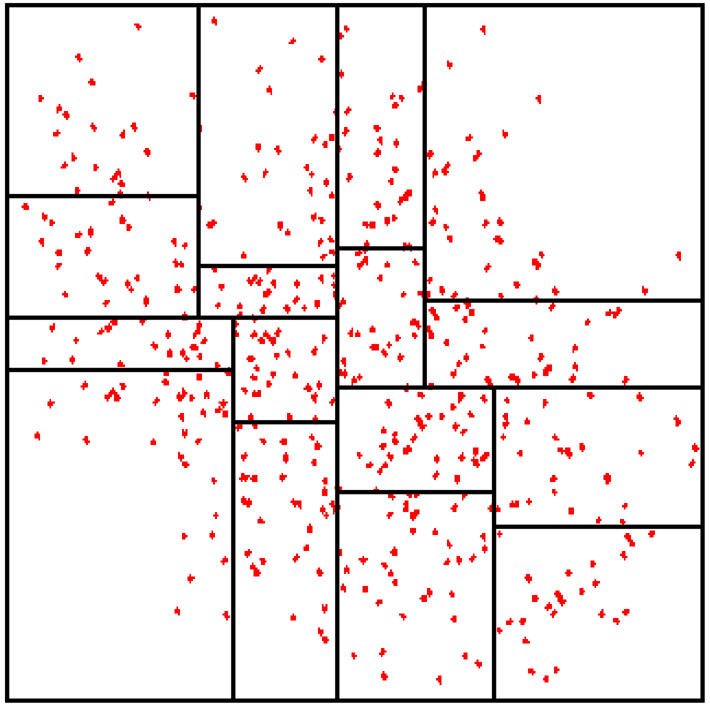
Particle population localized

Static domain decomposition



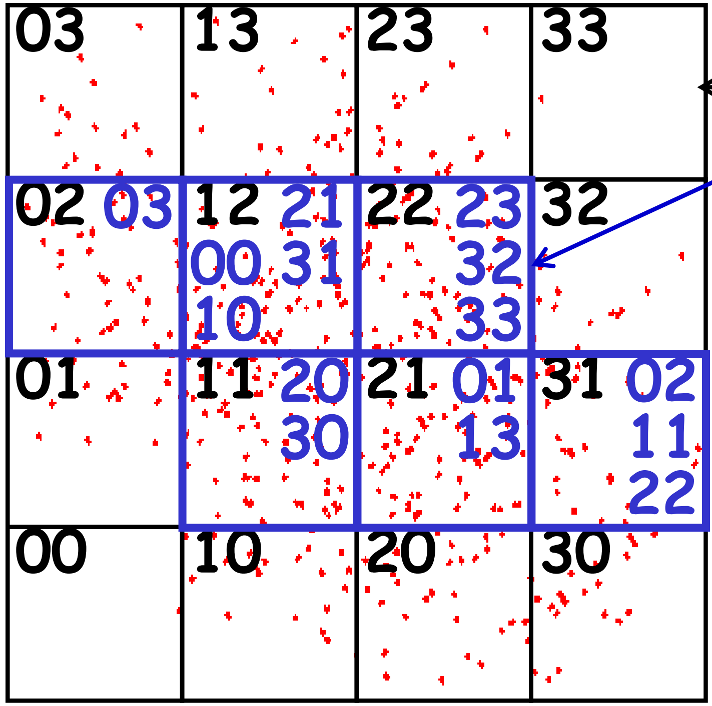
Advanced strategies for parallelization

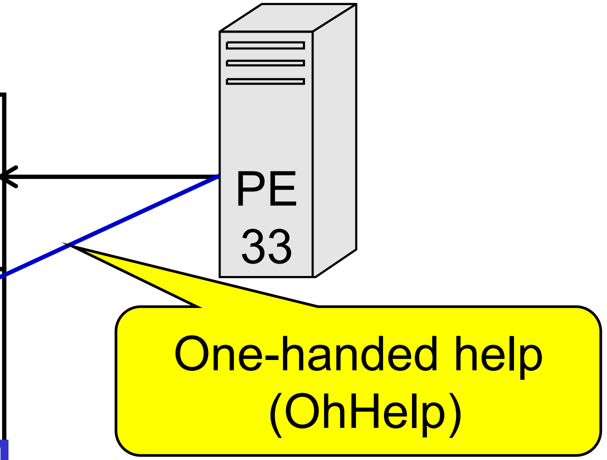
Orthogonal recursive bisection
[Thackera+, 2003]

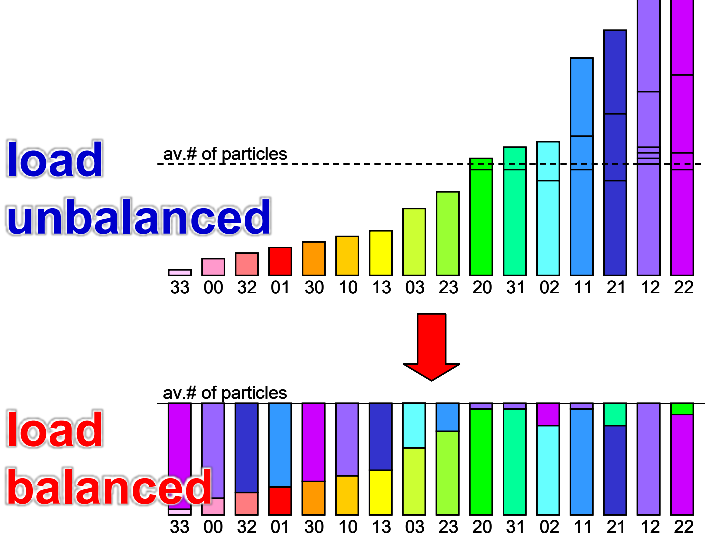


Dynamic domain decomposition

OhHelp
[Nakashima+, 2009]







Static domain decomposition
+
Dynamic PE assignment

References from past ISSSs

Advanced Methods for Space Simulations (ISSS-7)
"One-dimensional Electromagnetic Particle Code: KEMPO1"
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Computer Space Plasma Physics:
Simulation Technique and Software (ISSS-4)
"KEMPO1: Technical Guide to one-dimensional electromagnetic particle code"
Edited by H. Matsumoto and Y. Omura (1993)

Computer Simulation of Space Plasmas (ISSS-1)
"Particle simulation of electromagnetic waves and its application to space plasmas"
Edited by H. Matsumoto and T. Sato (1985)