PIC Simulation Hands-on Session

Yoshiharu Omura & Yohei Miyake
Research Institute for Sustainable Humanosphere, Kyoto University
omura@rish.kyoto-u.ac.jp

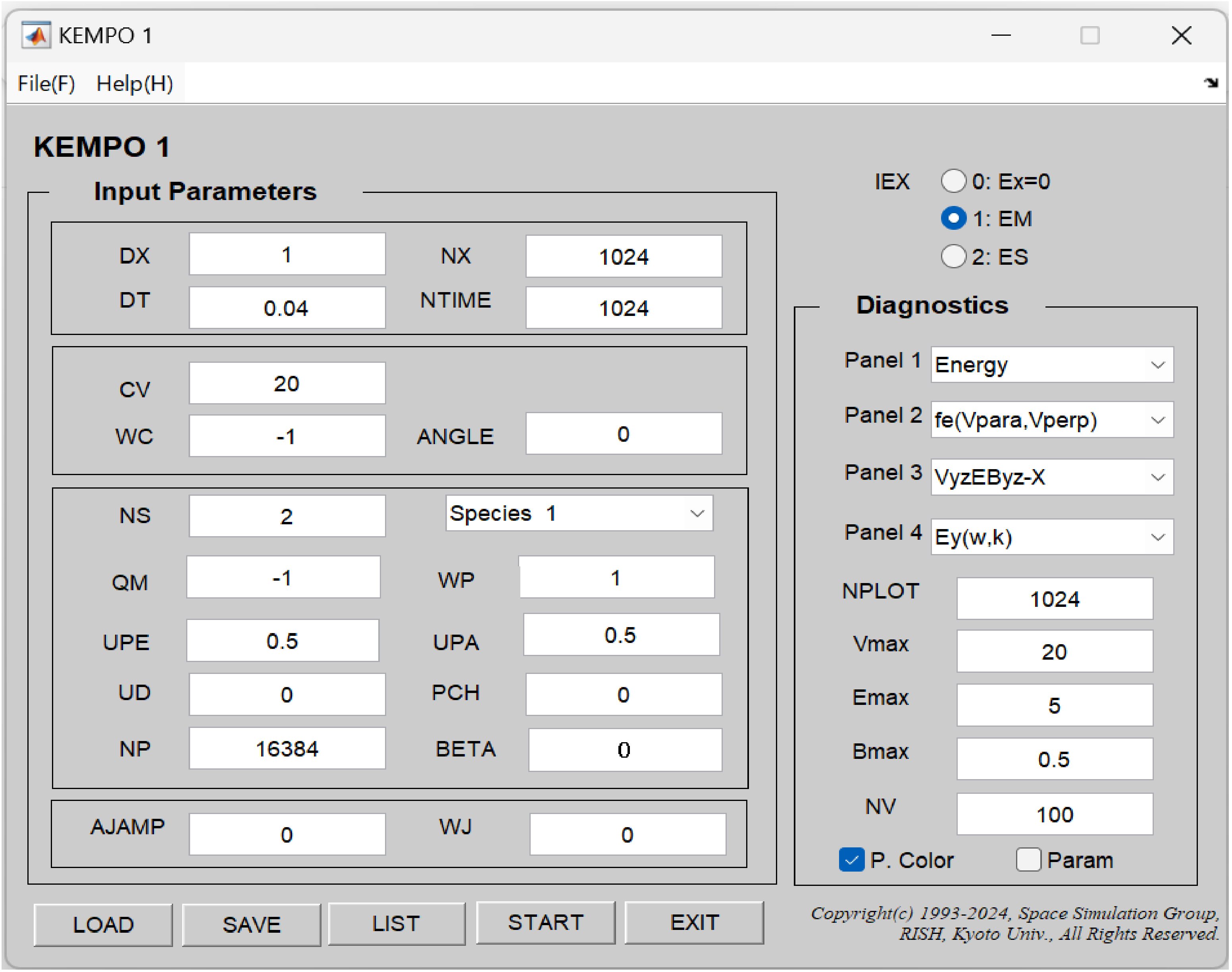
15th International School for Space Simulation, Garching, Germany, August 1, 2024

Software for Space Plasma Simulations

- 1D PIC Code: KEMPO1 (MATLAB)
- 2D PIC Code: KEMPO2 (MATLAB)

https://space.rish.kyoto-u.ac.jp/software/

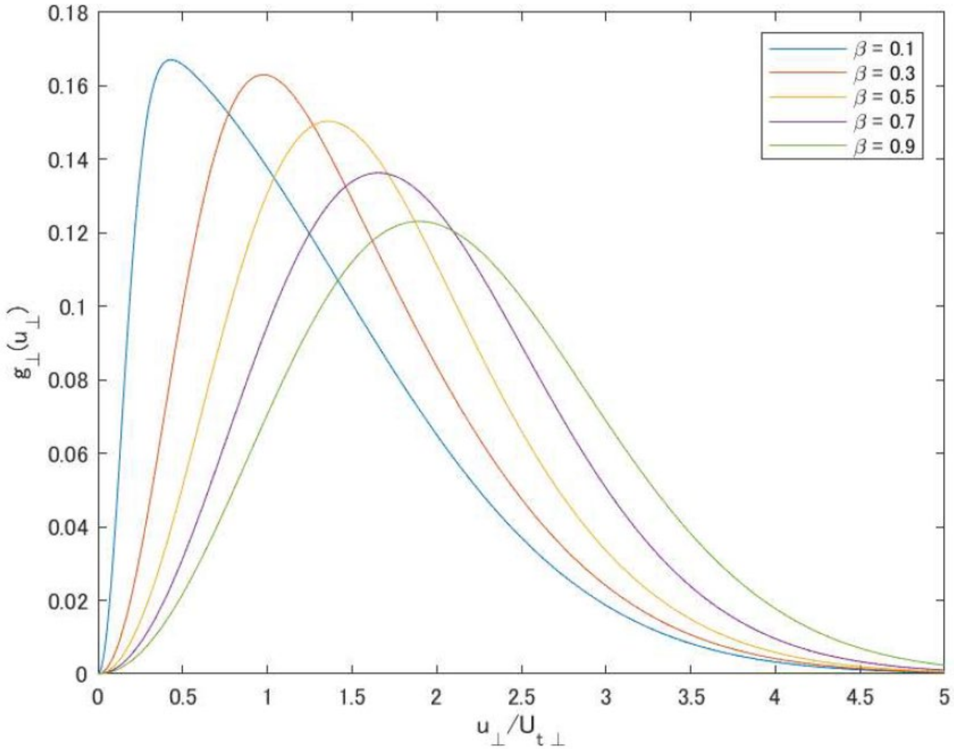
KEMPO1: GUI for Input Parameters

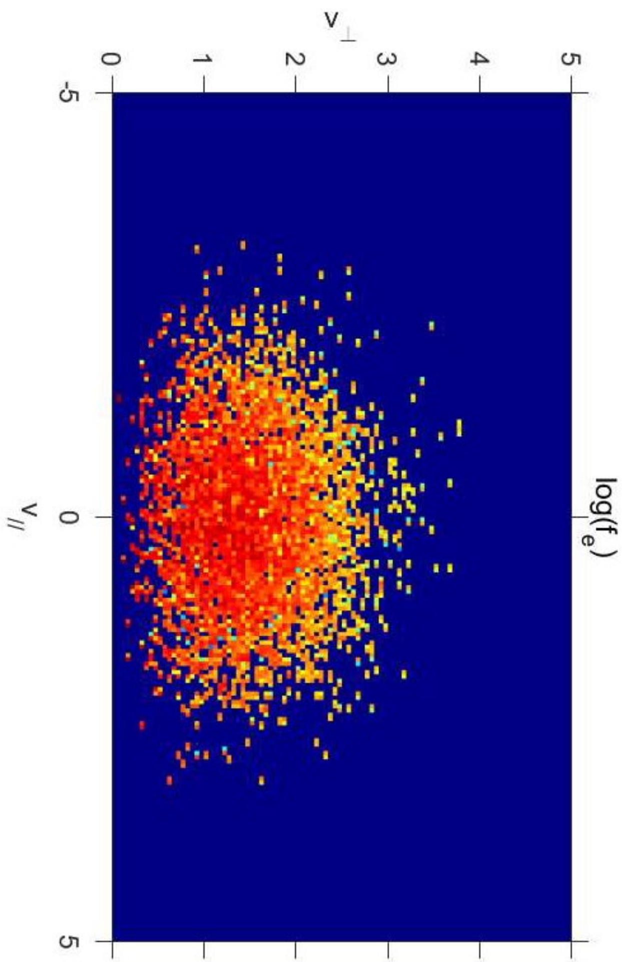


Subtracted Maxwellian Distribution

$$g_\perp(u_\perp) = \frac{1}{2\pi U_{t\perp}^2 (1-\beta)} \left[\exp\left(-\frac{u_\perp^2}{2U_{t\perp}^2}\right) - \exp\left(-\frac{u_\perp^2}{2\beta U_{t\perp}^2}\right) \right]$$

$$f(u_\|, u_\perp) = g_\|(u_\|) g_\perp(u_\perp)$$





Ref: Zenitani and Nakano, JGR Space Phys., 2023

Help (H)

↓

Parameters(P)

Figure 4

Parameter	Description
DX	Grid spacing
DT	Time step
CV	Speed of light
WC	Cyclotron frequency of species #1
ANGLE	Angle between the static magnetic field **Bo** and wave vector **k**
NX	Number of grid points
NTIME	Number of time steps in a simulation run
NS	Number of species
QM(i)	Charge to mass ratio of species #i
WP(i)	Plasma frequency of species #i
UPE(i)	Perpendicular thermal momentum of species #i
UPA(i)	Parallel thermal momentum of species #i
UD(i)	Drift momentum of species #i
PCH(i)	Pitch angle of species #i drifting with UD(i)
BETA(i)	Parameter of subtracted Maxwellian distribution of species #i (0<beta<1)
NP(i)	Number of particles of species #i
AJAMP	Amplitude of external current density (=0 : uniform current cancellation)
WJ	Freqeuncy of external current density (=0: no external current)
IEX	Field solution option 0: No Ex; 1: EM mode; 2: ES mode;
NPLOT	Number of diagnostics in a simulation run
Vmax, Emax, Bmax	Maximum values of velocities, electric and magnetic fields
NV	Number of bins for calculating the velocity distribution functions
P.Color	Particle dots are colored when checked
Parm	Light and drift velocities are drawn in w-k diagrams

* **SPACE-key**: Suspend and resume execution of the simulation; **Esc-key**: Terminate execution

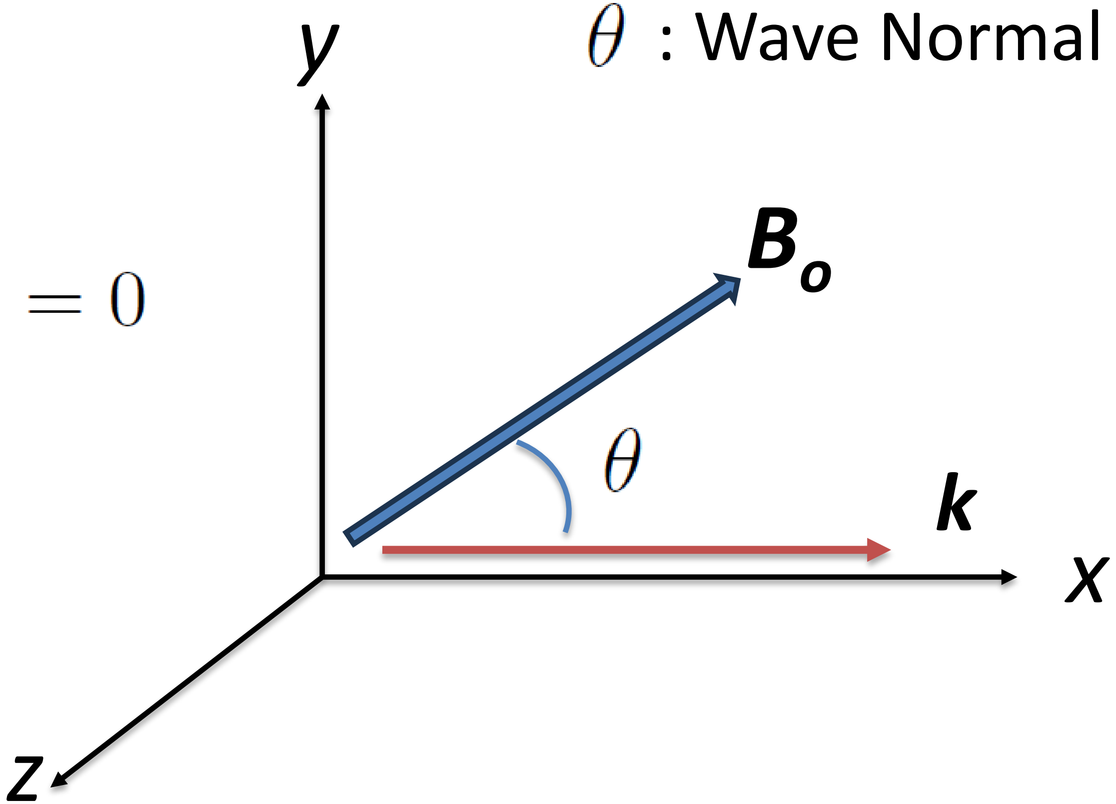
OK

Coordinates in KEMPO1

1D: Particle-In-Cell Code

θ : Wave Normal Angle

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$



Relative Unit System

We assume in KEMPO1

$$\varepsilon_0 = 1, \qquad \mu_0 = \frac{1}{c^2}$$

Reference Values for Space and Time

Speed of light: *c*

Characteristic frequency: ω max (ω_p , ω_c)

We choose Δt so that $\omega \Delta t << 2\pi$

We choose Δx so that $c\Delta t < \Delta x$

Charge Neutrality Condition

Number Density $$n_i = \frac{N_p}{N_x \Delta_x}$$

Nx: Number of Grid Points

Particle Charge $$q_i = \frac{\rho_i}{n_i} = \frac{\omega_{pi}^2}{(q_i/m_i)n_i}$$

q_i/m_i **is an arbitrary value.**

For simplicity, -1 for electrons.

1/16, 1/256 or 1/1836 for protons.

$$\sum_i \rho_i = \sum_i q_i n_i = \boxed{\sum_i \frac{\omega_{pi}^2}{q_i/m_i} = 0}$$

Renormalization

Simulation Unit System ← **Relative Unit System**

distance	$x_S = (1/\Delta x)x_R$
time	$t_S = (2/\Delta t)t_R$
velocity	$v_S = (\Delta t/2)(1/\Delta x)v_R$
electric field	$E_S = (\Delta t/2)^2(1/\Delta x)E_R$
magnetic field	$B_S = (\Delta t/2)B_R$
charge density	$\rho_S = (\Delta t/2)^2\rho_R$
current density	$J_S = (\Delta t/2)^3(1/\Delta x)J_R$
energy density	$\sigma_S = (\Delta t/2)^4(1/\Delta x)^2\sigma_R$
number density	$n_S = \Delta x n_R$
charge	$q_S = (\Delta t/2)^2(1/\Delta x)q_R$
mass	$m_S = (\Delta t/2)^2(1/\Delta x)m_R$

Coding with $\Delta x = 1$ and $\Delta t / 2 = 1$

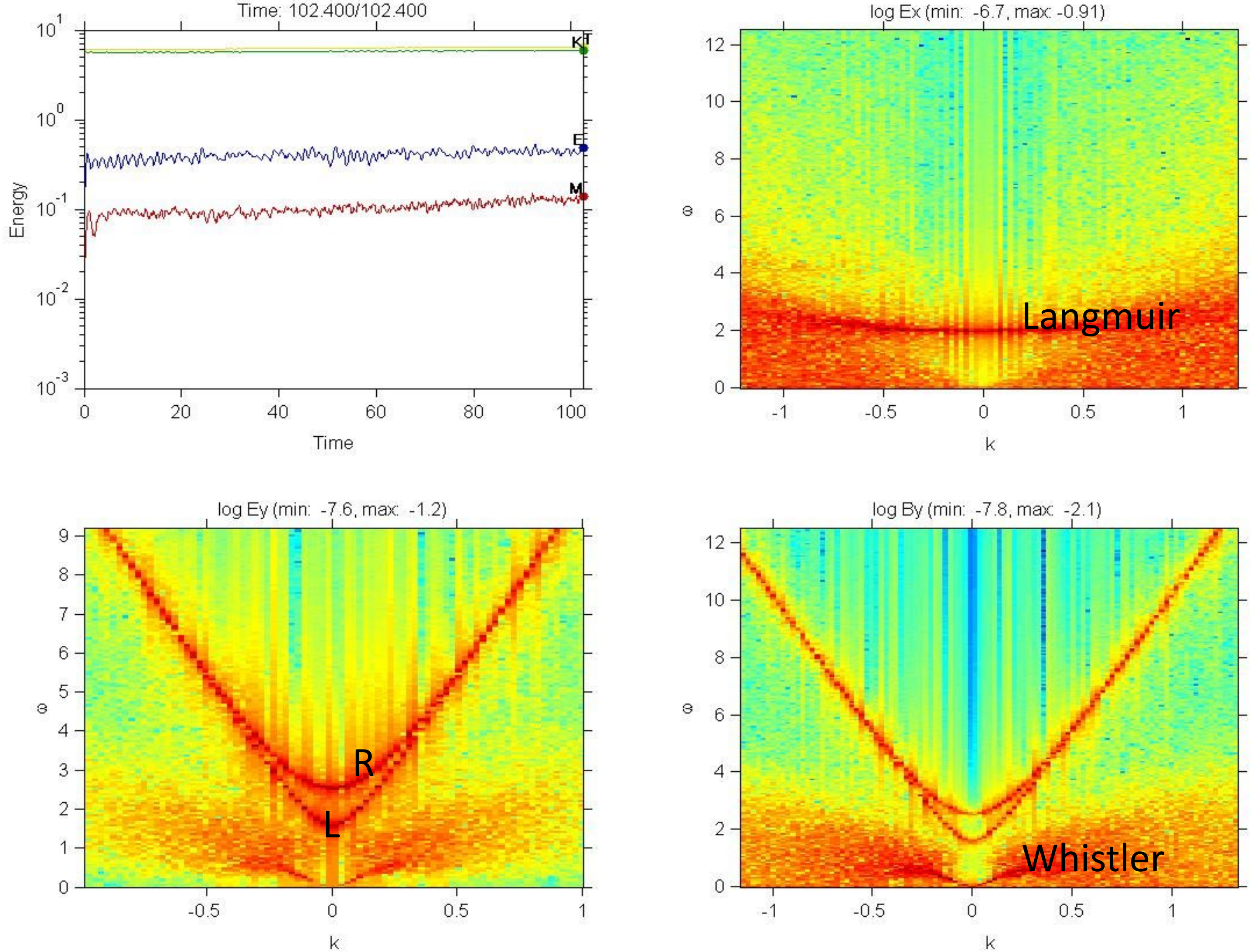
Maxwell's Equations

```
ey(X2) = ey(X2) -tcs*(bz(X2)-bz(X1)) -2.0*ajy(X2);
ez(X2) = ez(X2) +tcs*(by(X3)-by(X2)) -2.0*ajz(X2);

by(X2) = by(X2) + ez(X2) - ez(X1);
bz(X2) = bz(X2) - ey(X3) + ey(X2);
```

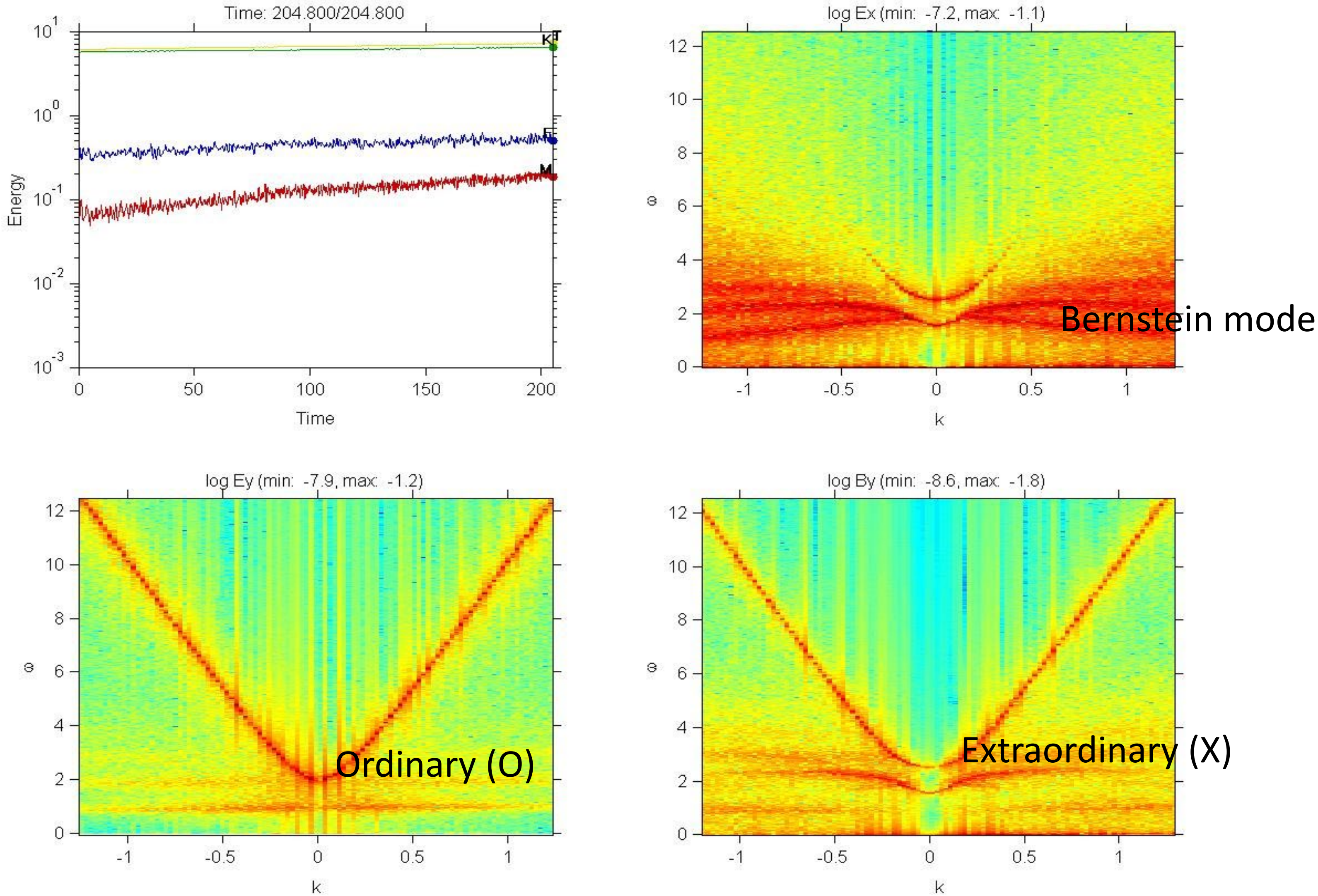
Equations of Motion

```
x = x +vx;

ux = ux + boris*(uyt*bz1 -uzt*by1) +ex1;
uy = uy + boris*(uzt*bx1 -uxt*bz1) +ey1;
uz = uz + boris*(uxt*by1 -uyt*bx1) +ez1;
```

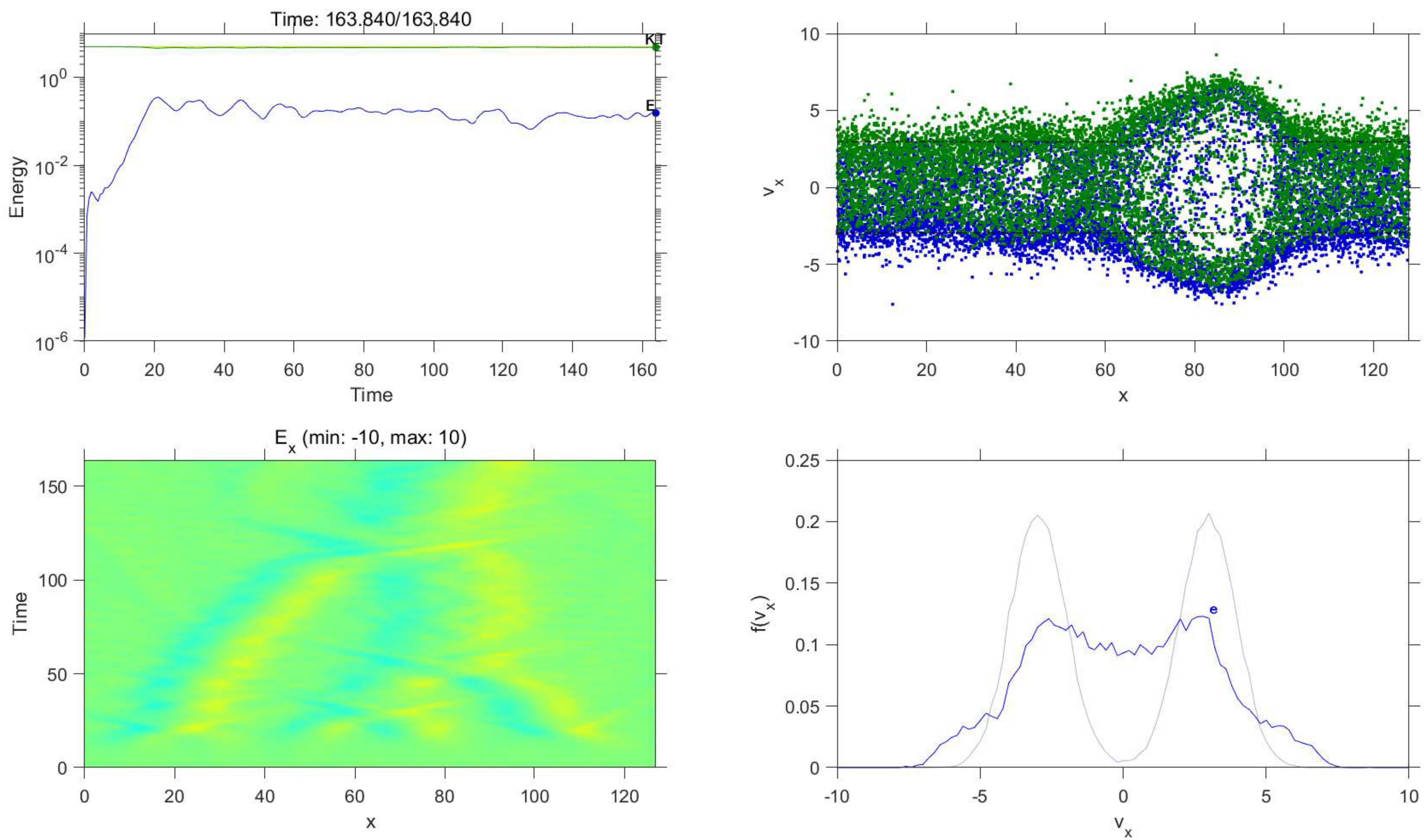
Normal Modes in the Parallel Direction



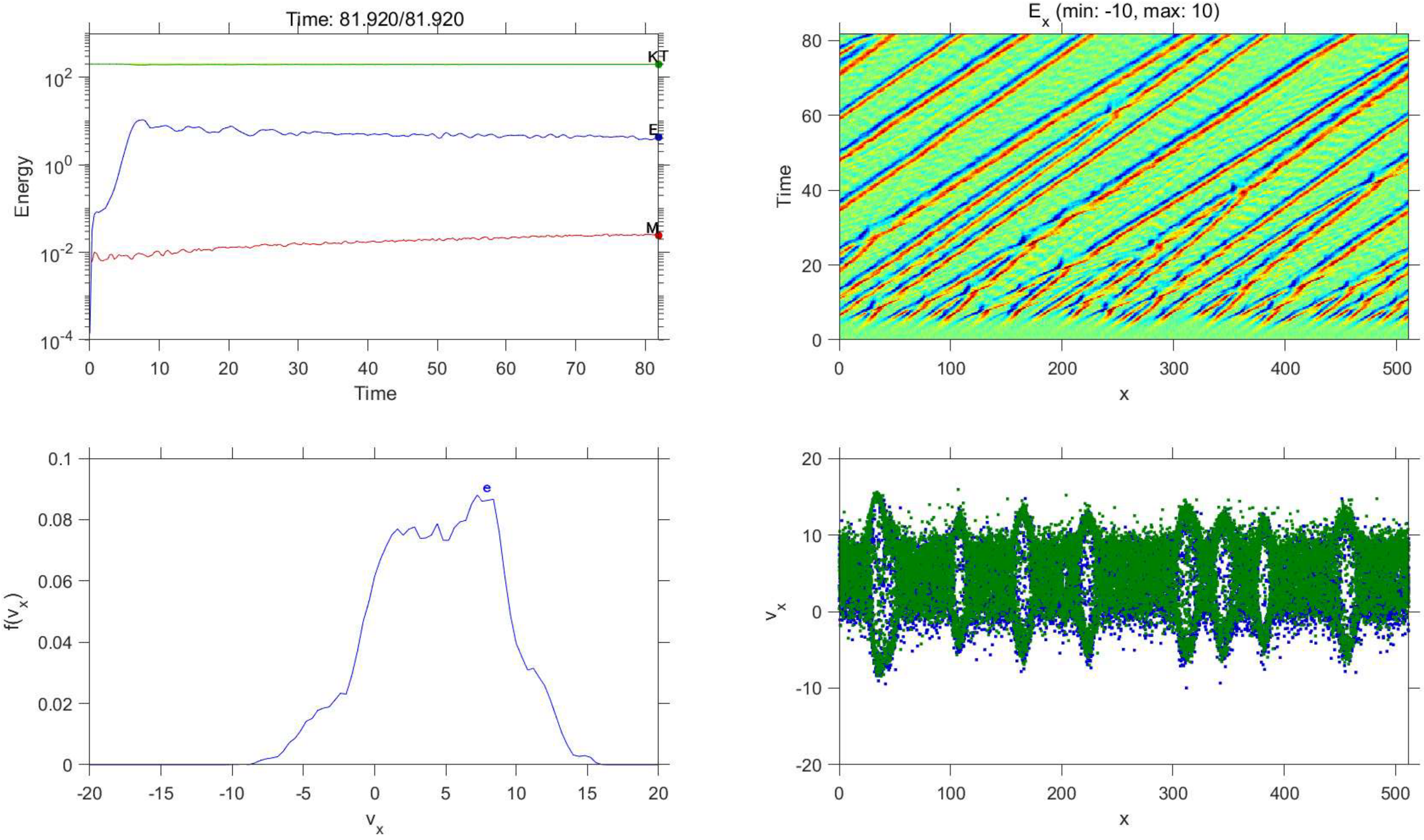
Normal Modes in the Perpendicular Direction



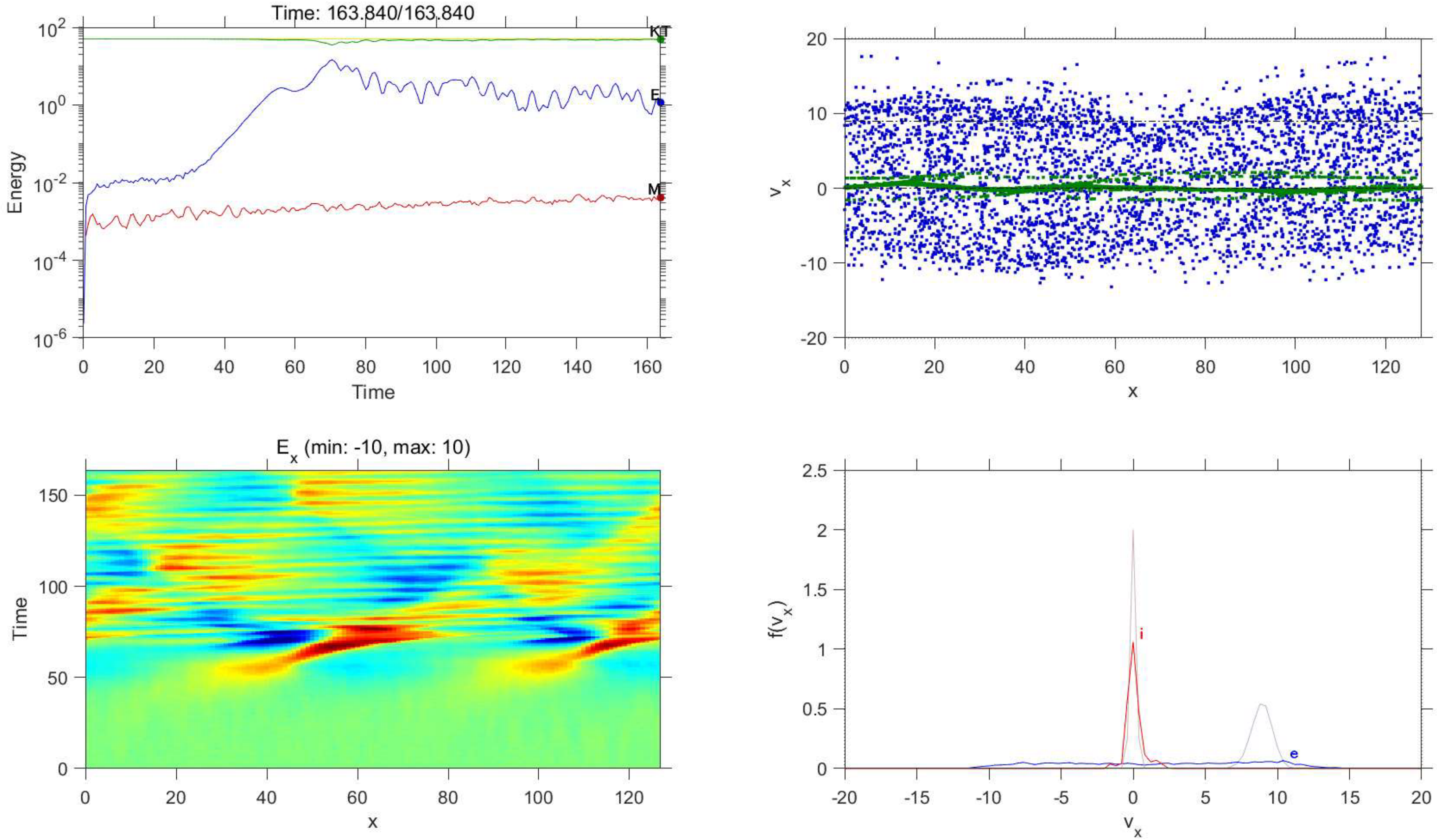
Electron Two-Stream Instability and Formation of Electron Holes (BGK mode)



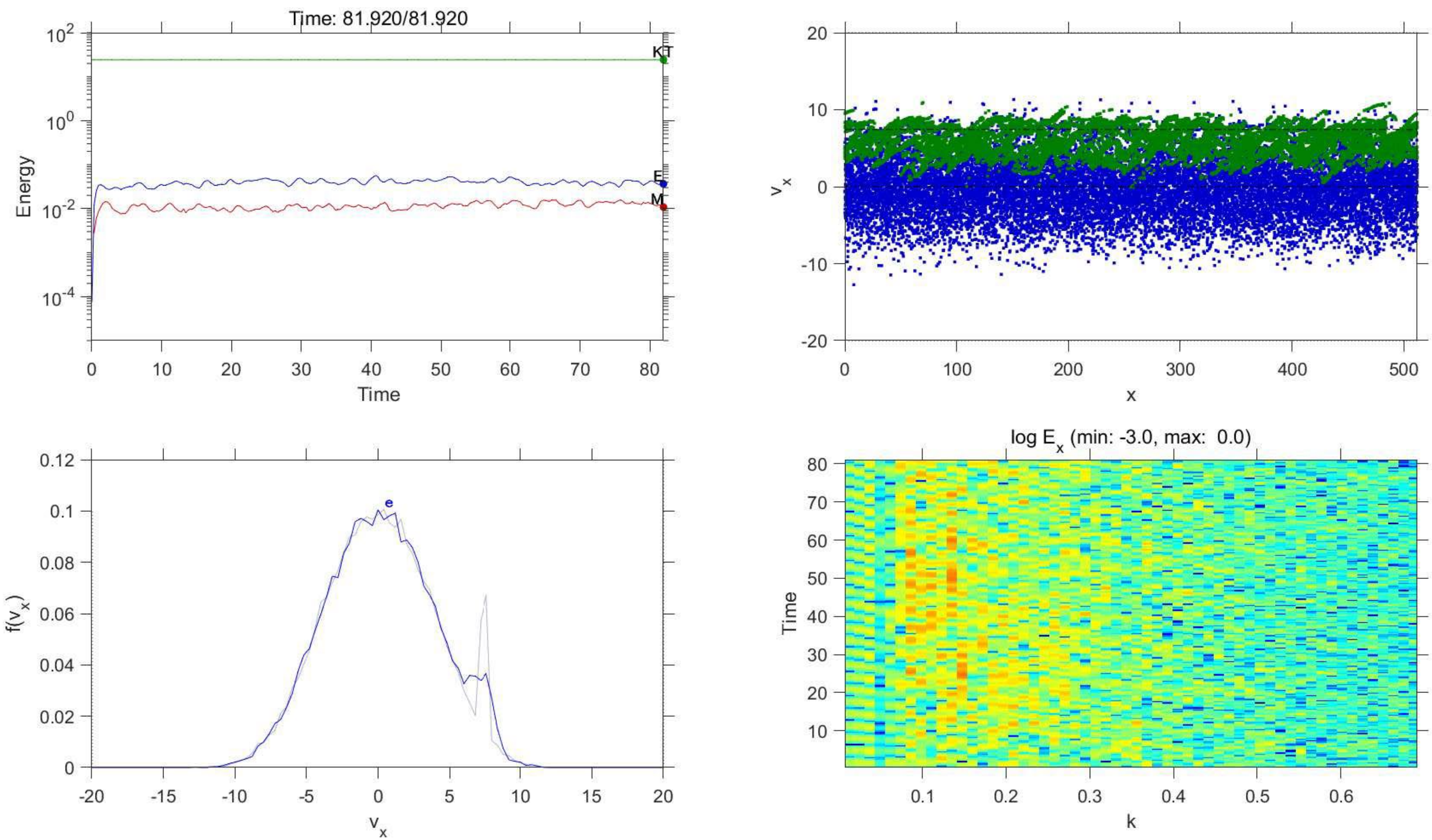
Electron Two-Stream Instability and Evolution of Electrostatic Solitary Waves



Buneman Instability and Excitation of Ion Acoustic Waves

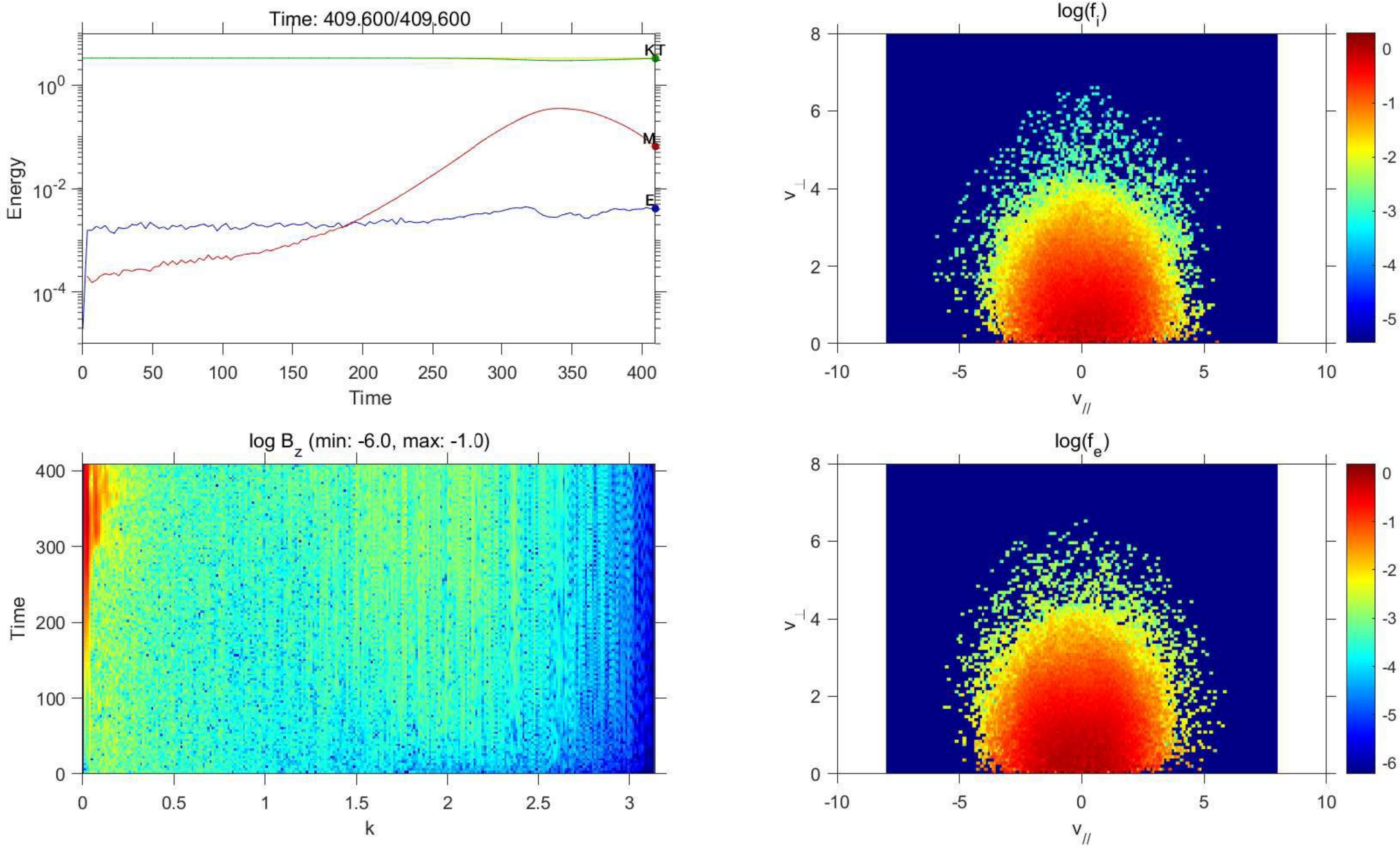


Bump-on-tail Instability



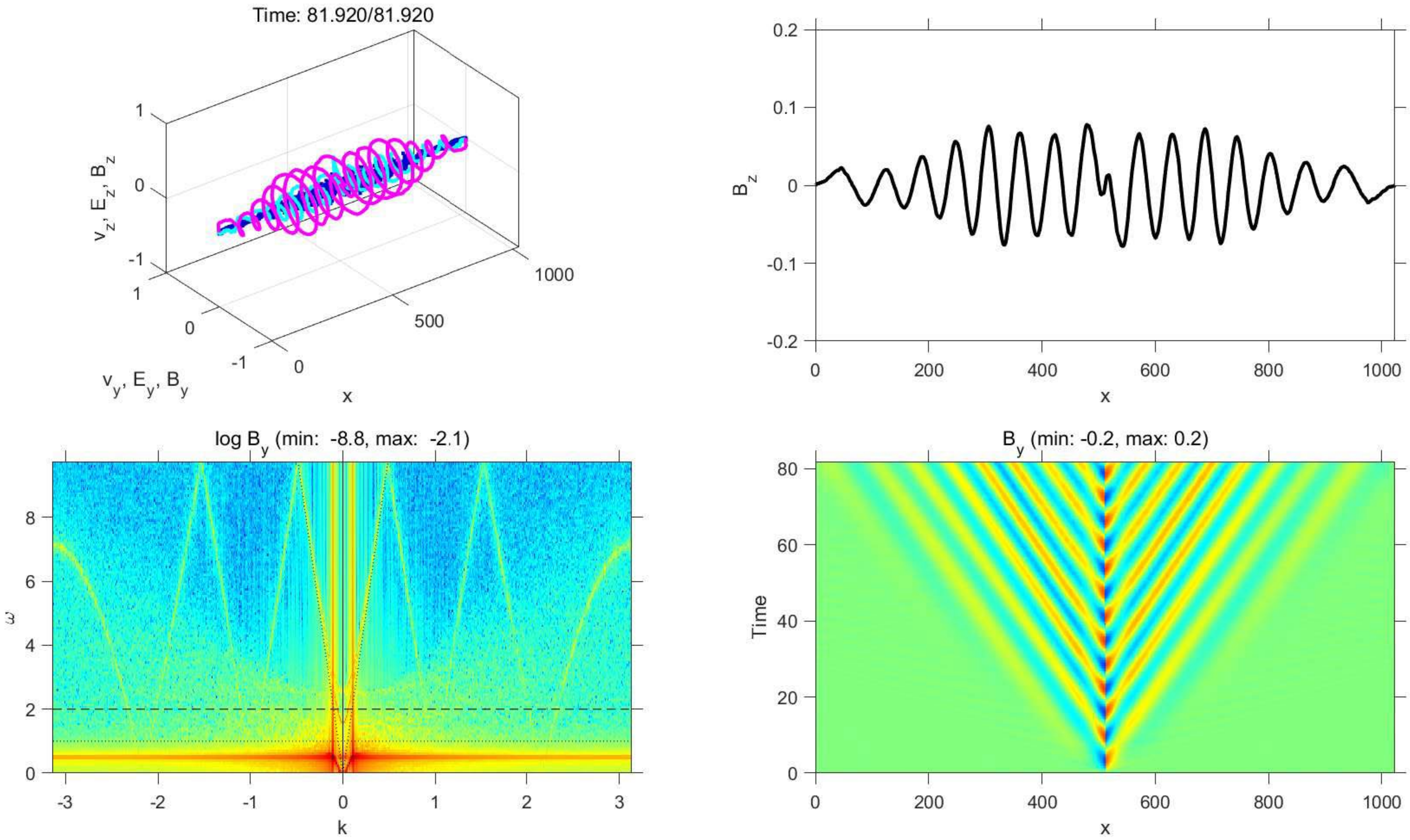
Electron-Positron Firehose Instability

$$P_{||} > P_{\perp}$$

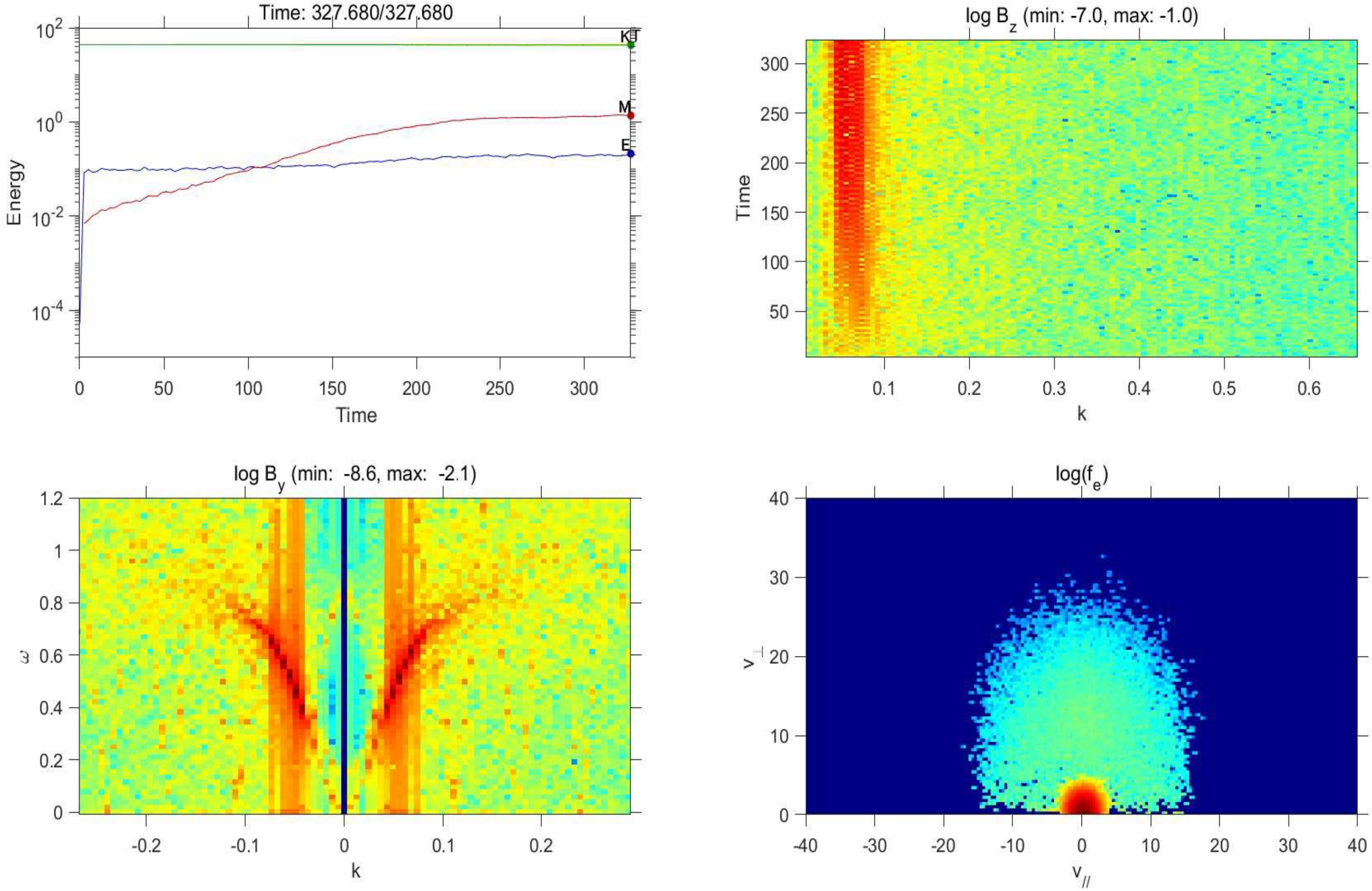


[Jao and Hau, Physical Review E, 2020]

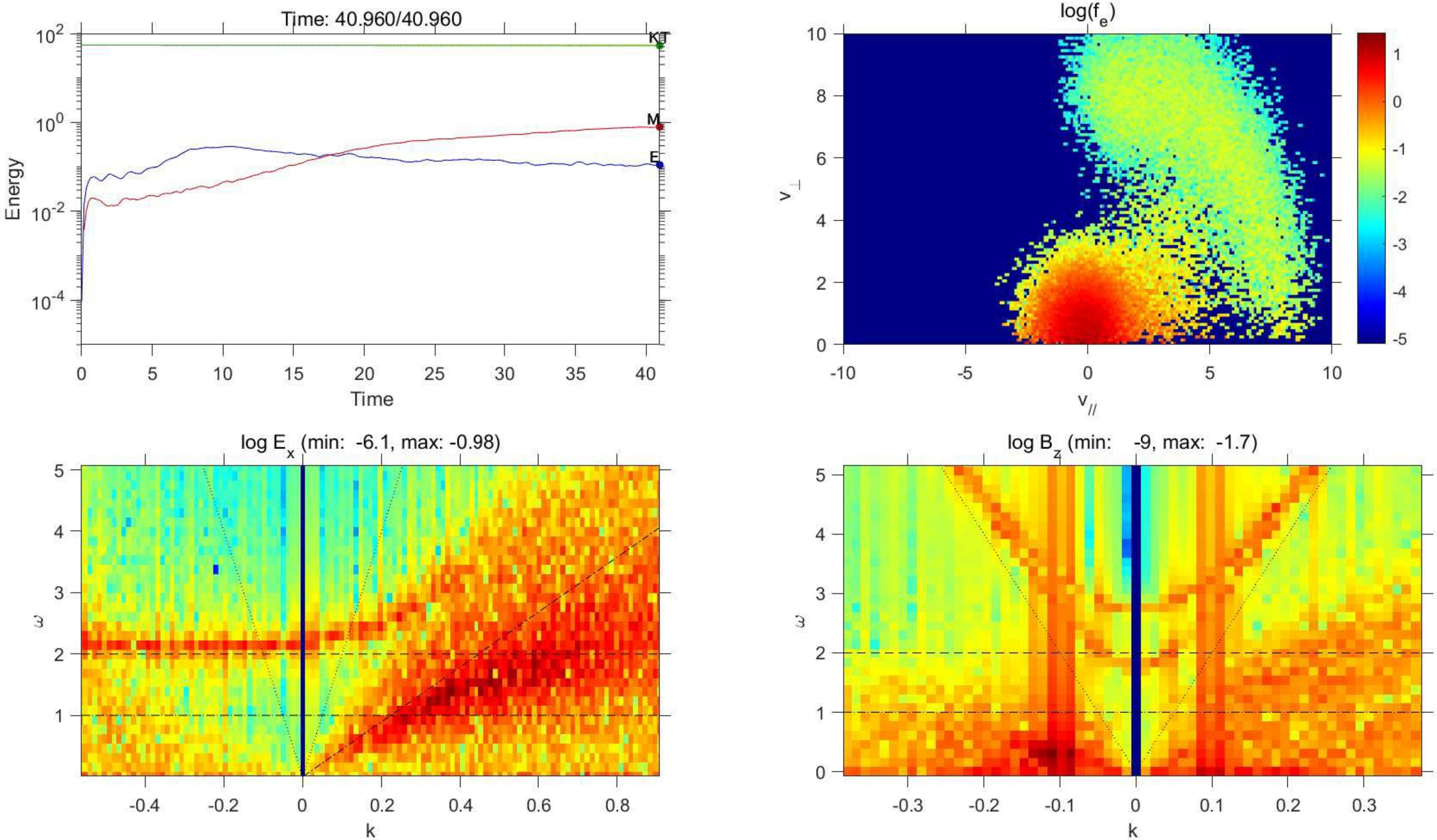
Whistler Mode Wave Radiation



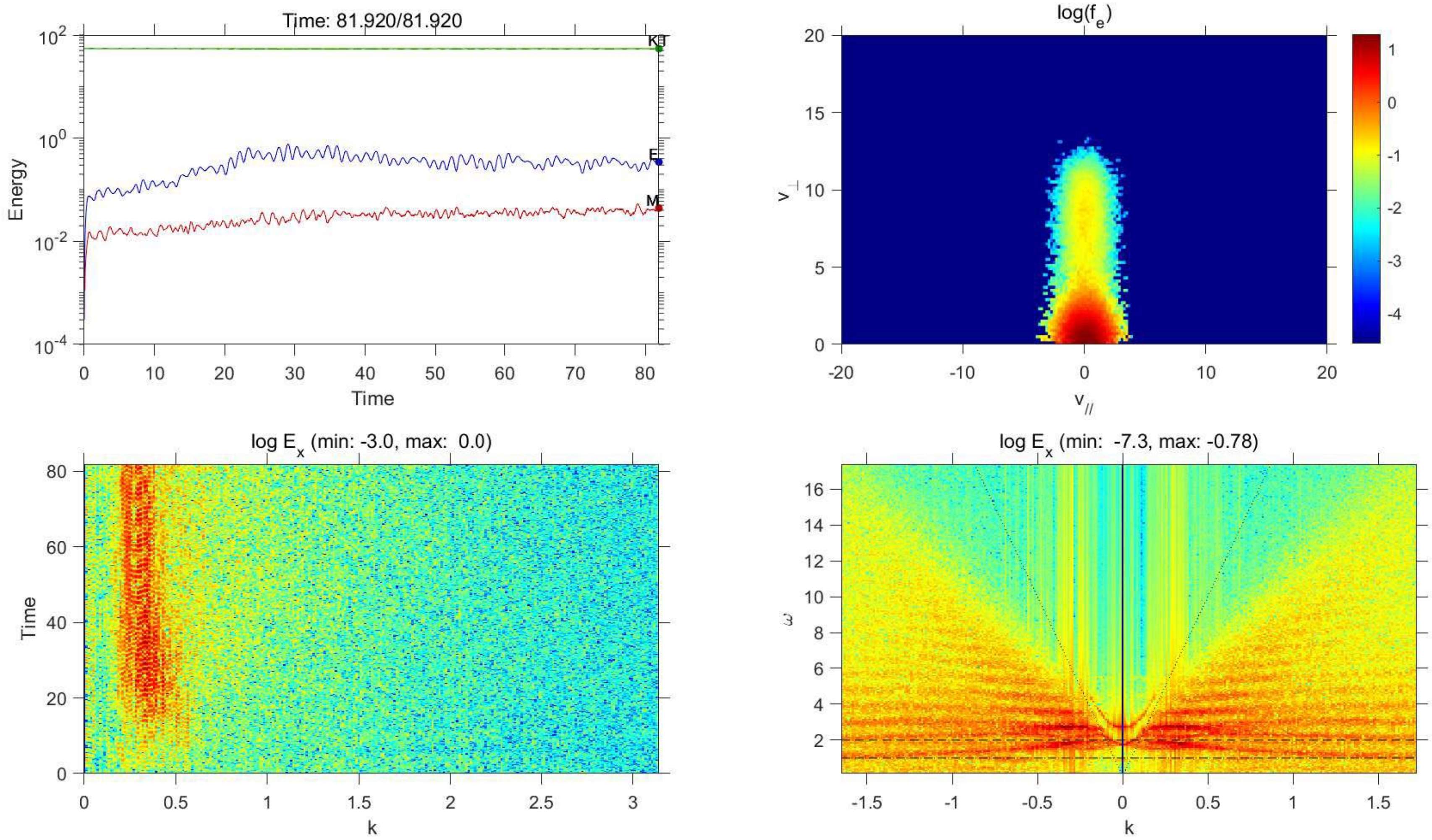
Whistler-mode Instability
Driven by Temperature Anisotropy of Hot Electrons
(subtracted Maxwellian distribution)



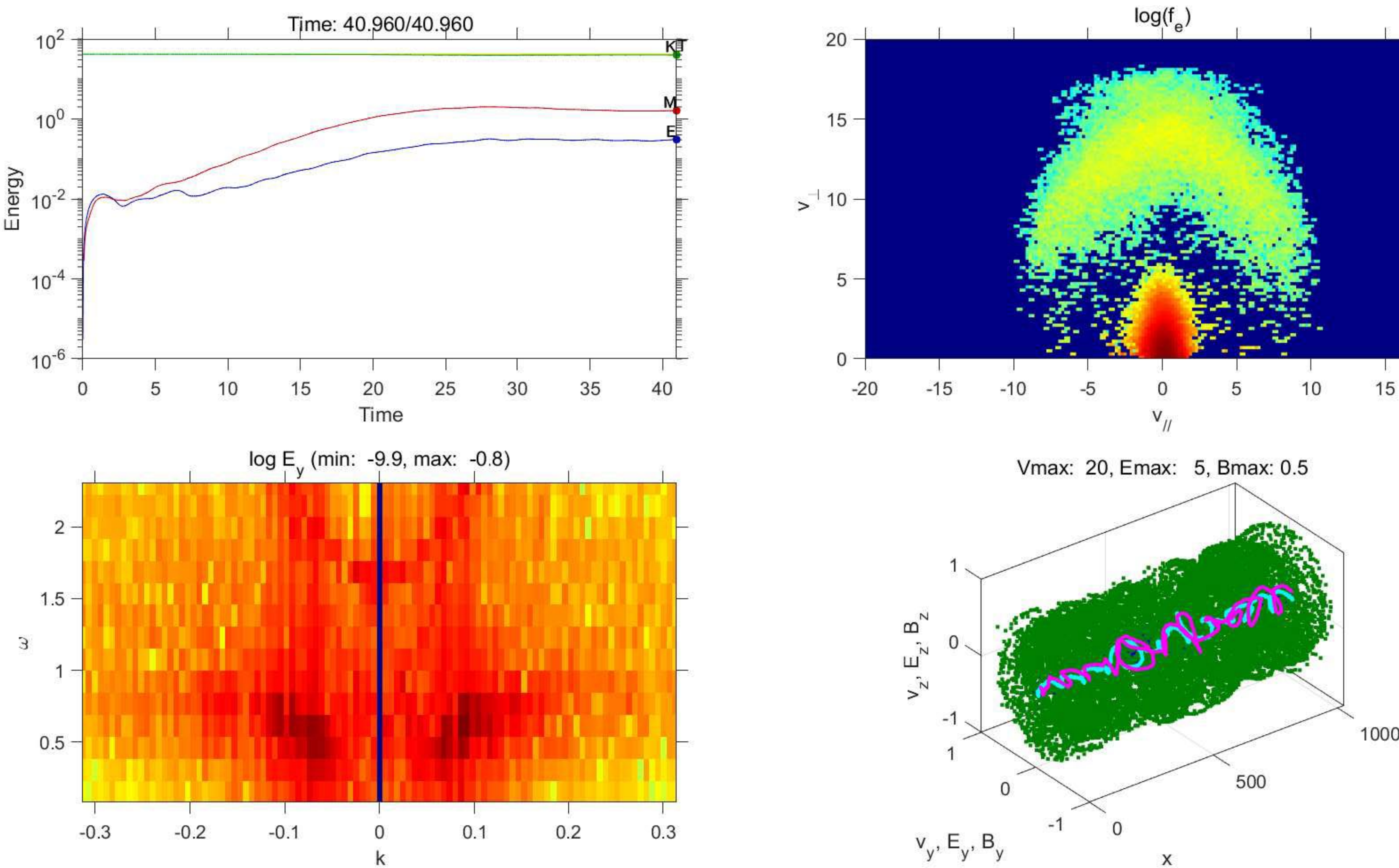
Whistler-mode Waves Excited by Electron Beam



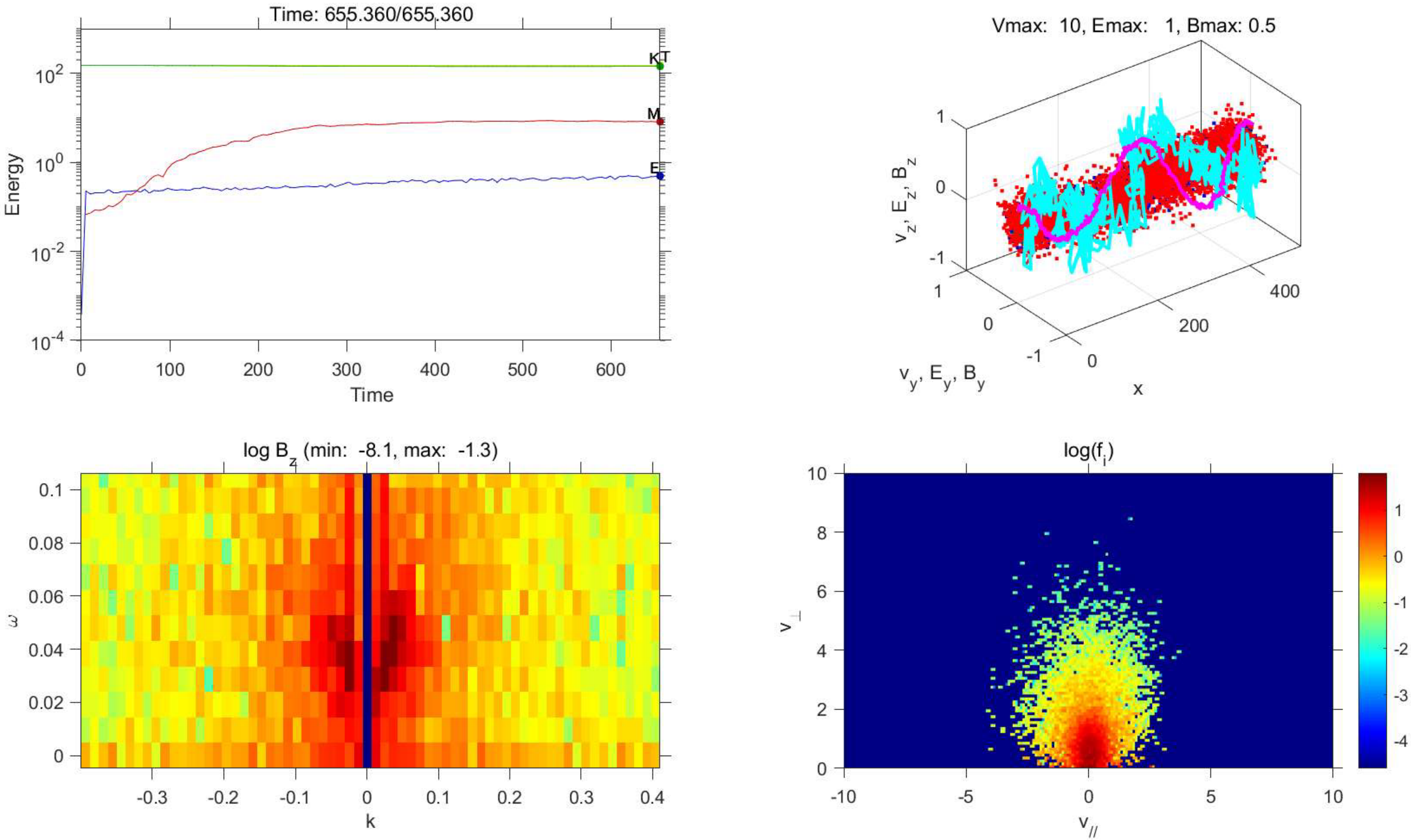
Electrostatic Cyclotron Harmonic Waves Excited by Ring Beam Electrons



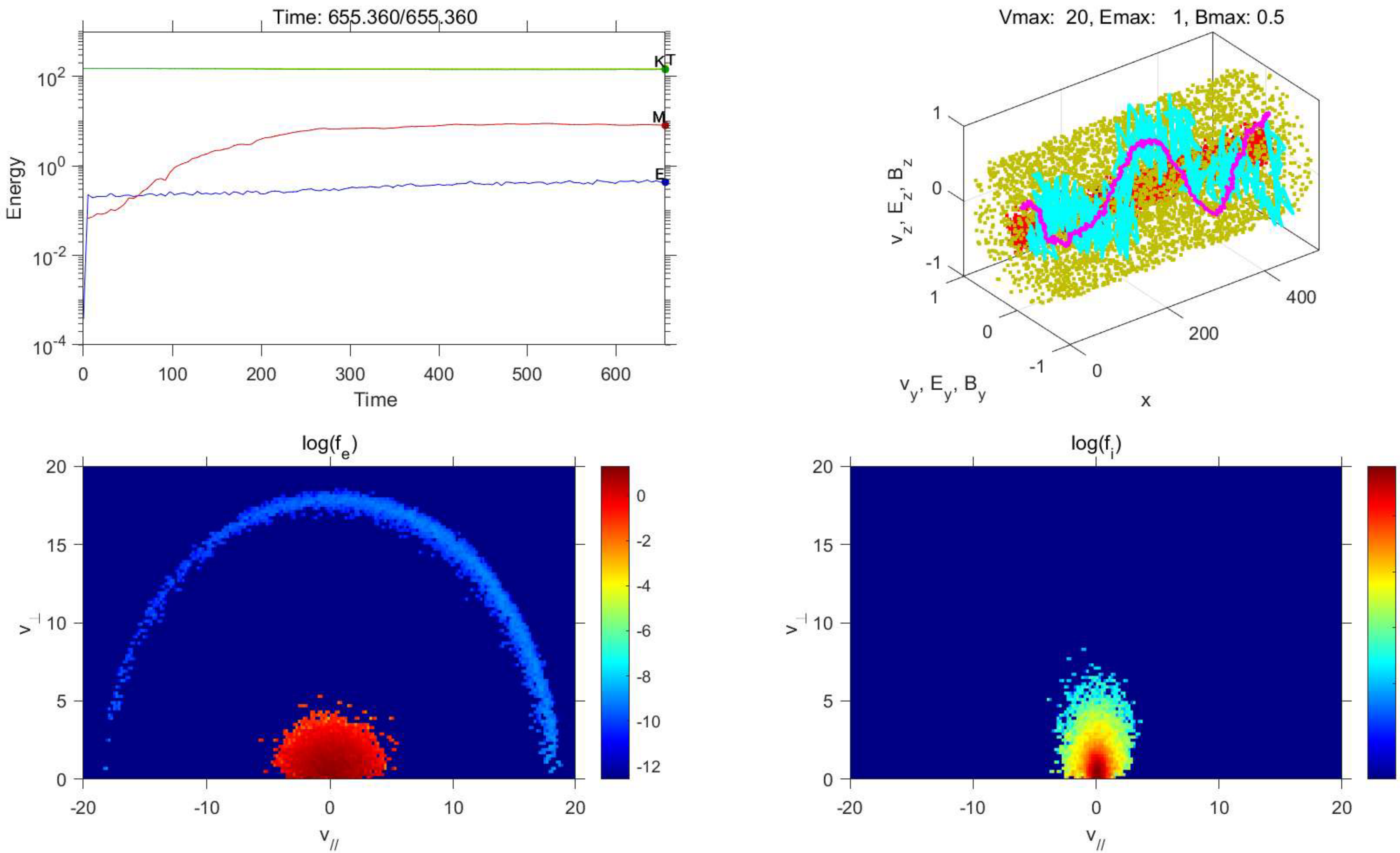
Relativistic Ring-Beam Instability (Parallel Propagation)



EMIC Wave Instability driven by Temperature Anisotropy of Protons (subtracted Maxwellian distribution)

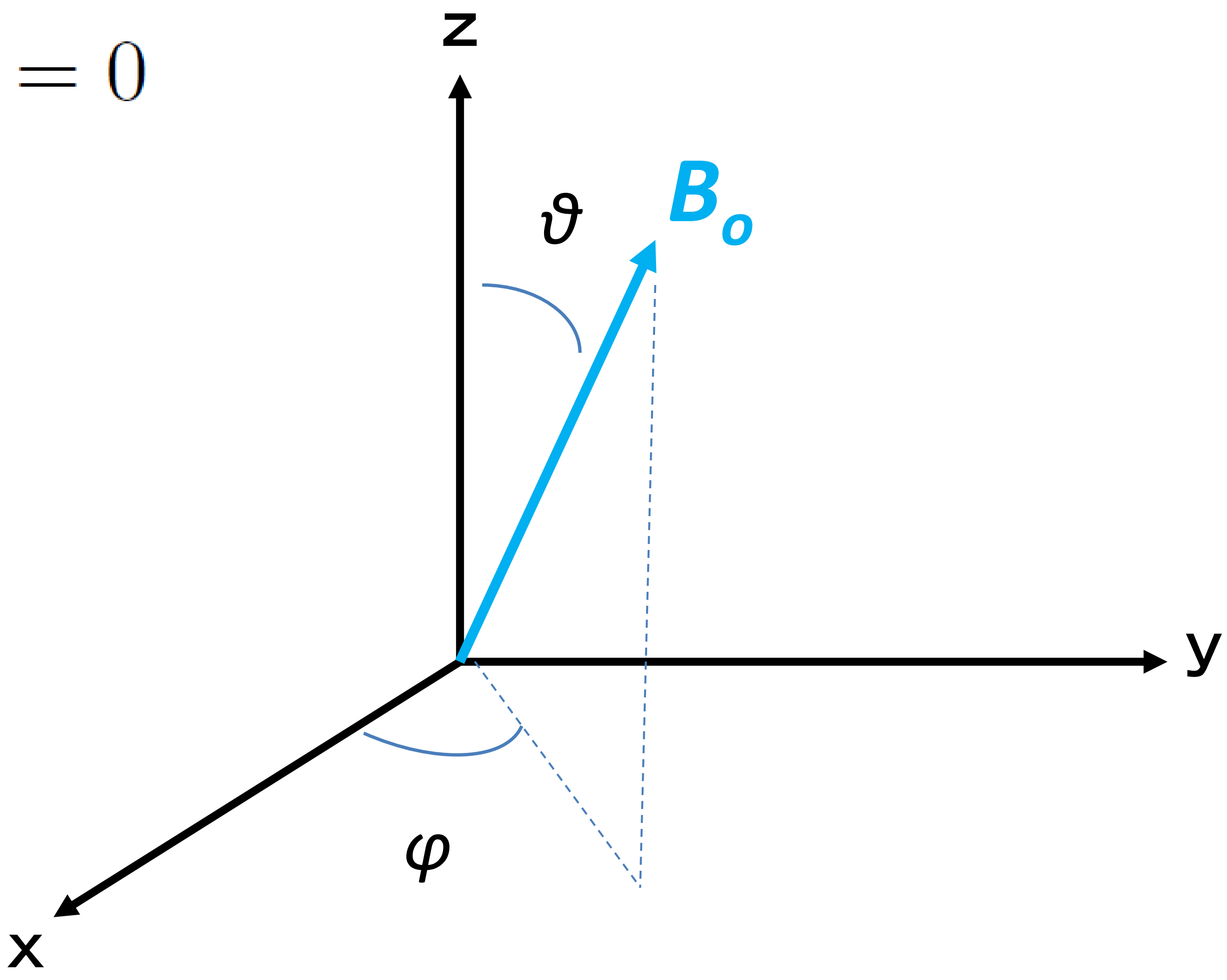


Pitch Angle Scattering of Relativistic Electrons by EMIC Waves due to Anisotropic Protons



KEMPO2

$$\frac{\partial}{\partial z} = 0$$



KEMPO2: Main Program

Method 1:
Electrostatic Field Correction Method

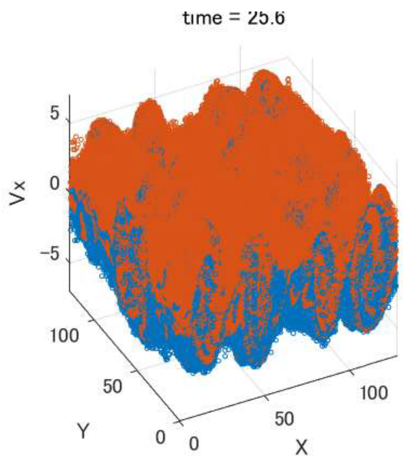
```
charge;
potential;
for itime = 1:ntime
    jtime = jtime +1;
    bfield;
    rvelocity;
    position;
    current;
    position;
    bfield;
    efield;
    charge;
    potential;
    diagnostics;
end
```

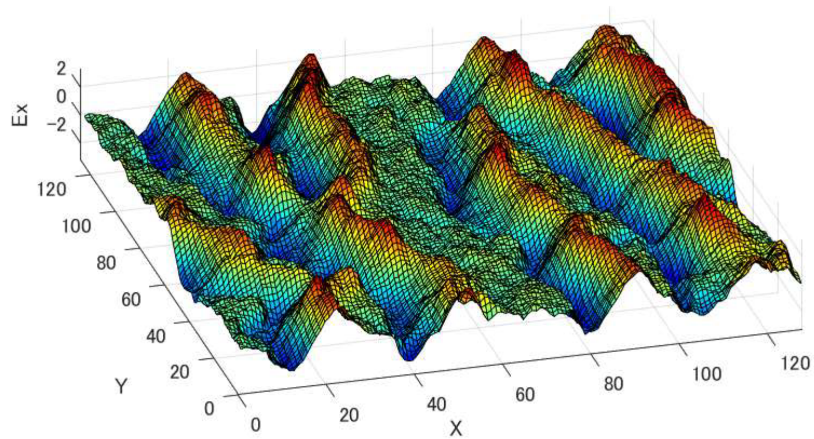
Method 2:
Charge Conservation (zigzag)

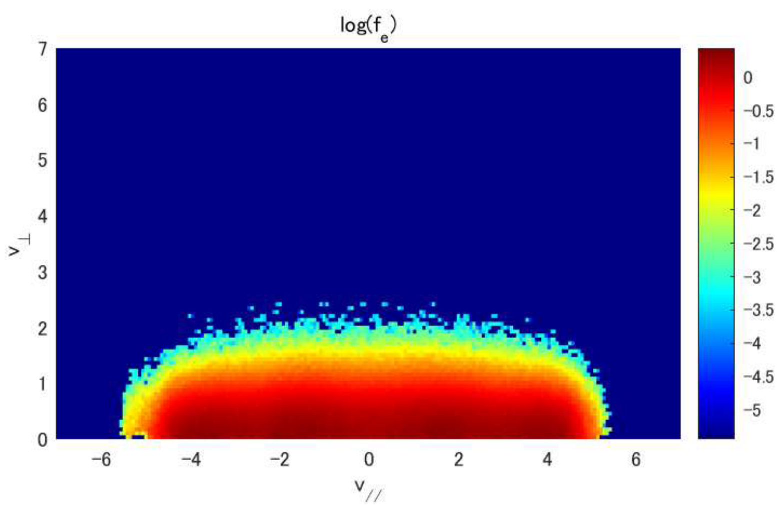
```
charge;
 potential;
for itime = 1:ntime
    jtime = jtime +1;
    bfield;
    rvelocity;
    position;
    current2;
    position;
    bfield;
    efield;
    diagnostics;
end
```

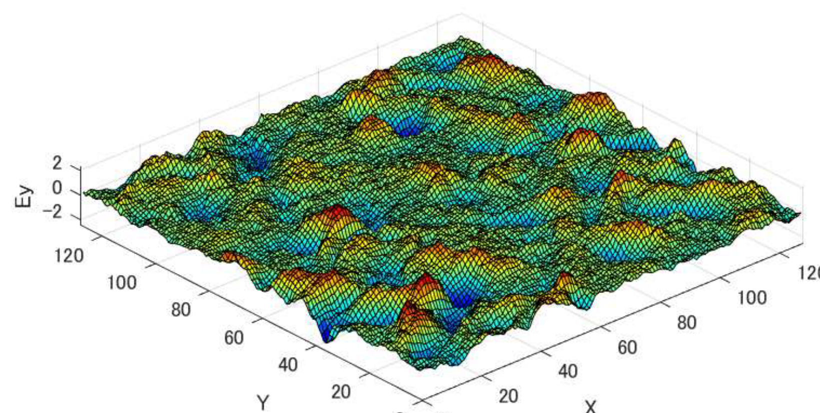
% Computation times for Methods 1 and 2 are nearly the same.

Electron Two-stream Instability





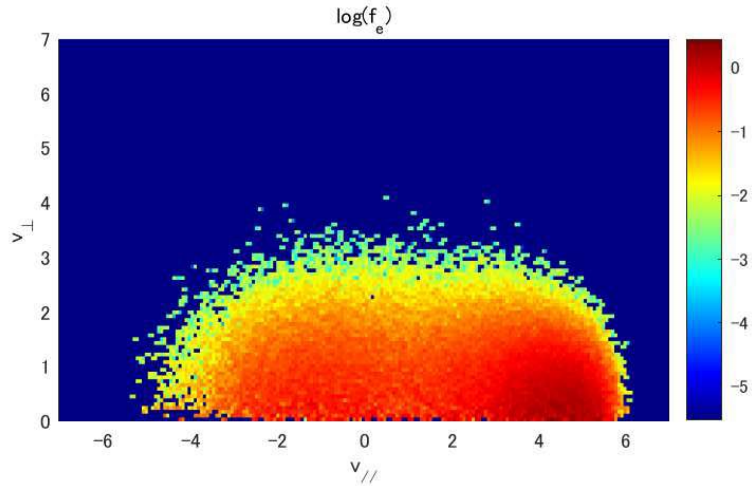


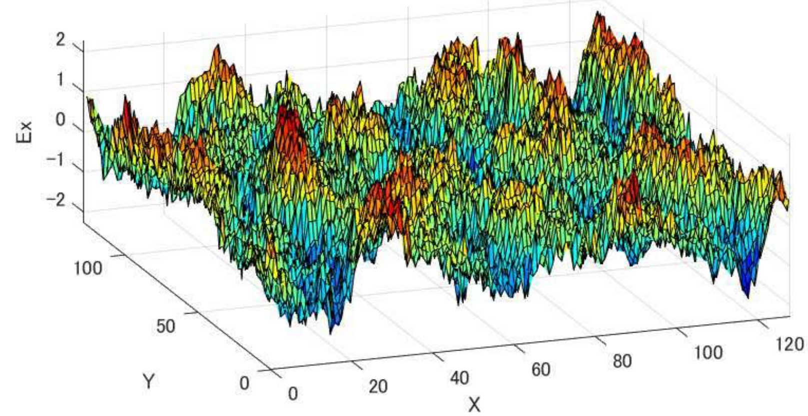


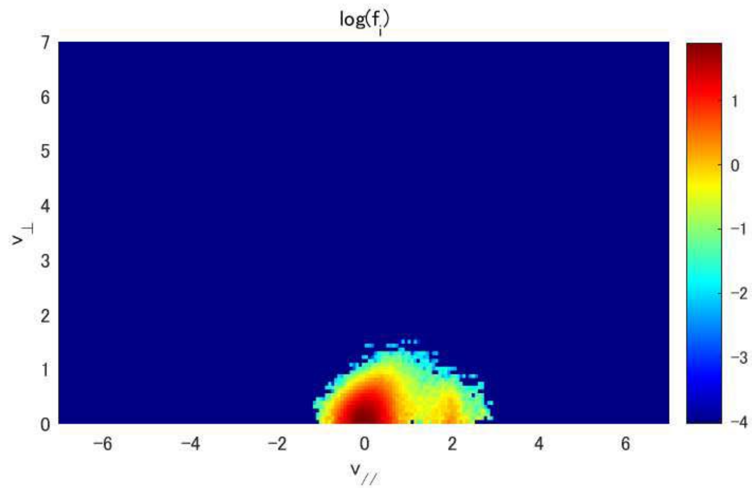
wp1 = 1, wp2 = 1, wc = -1

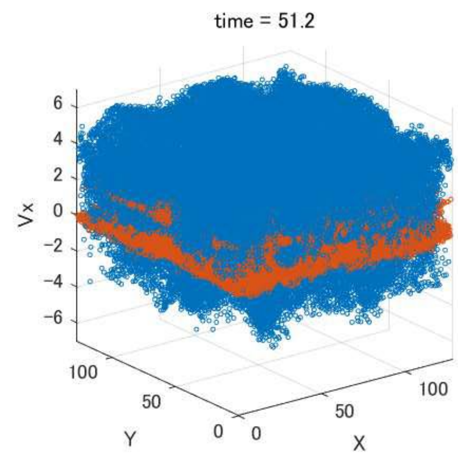
Buneman Instability

$m_i / m_e = 25$

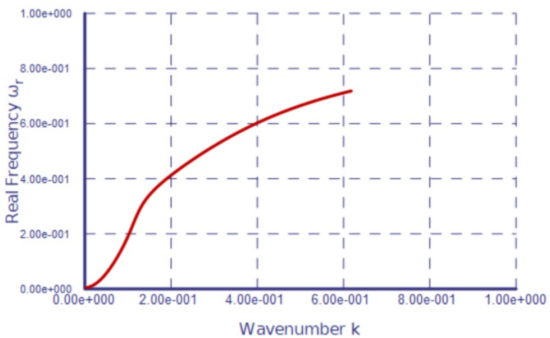


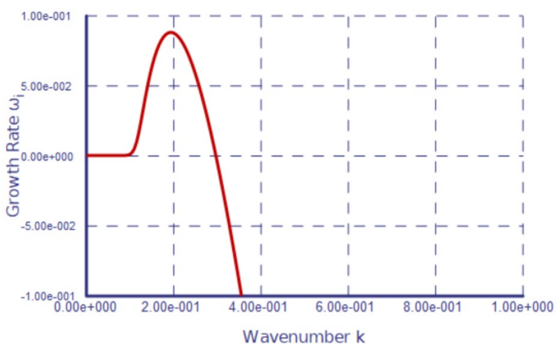


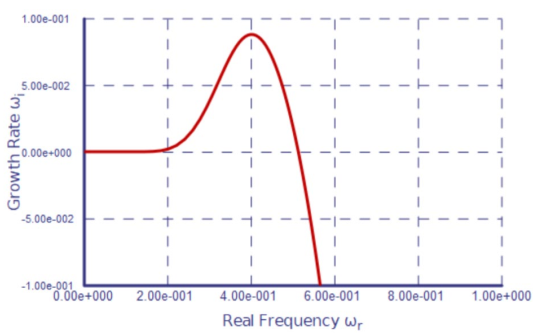


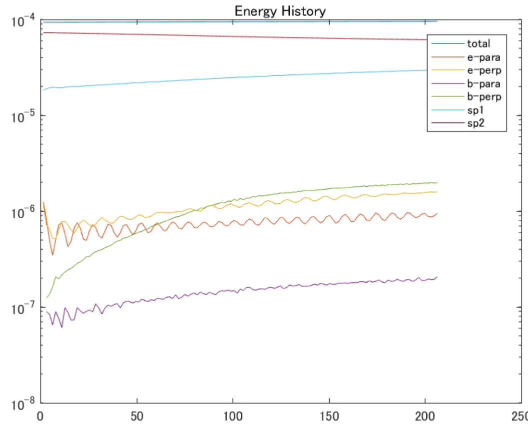


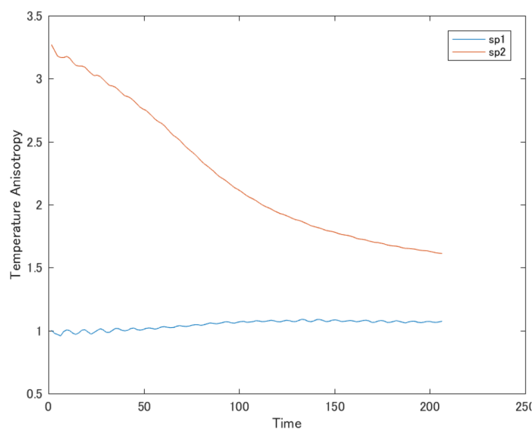
Whistler Mode Instability
Driven by Temperature Anisotropy



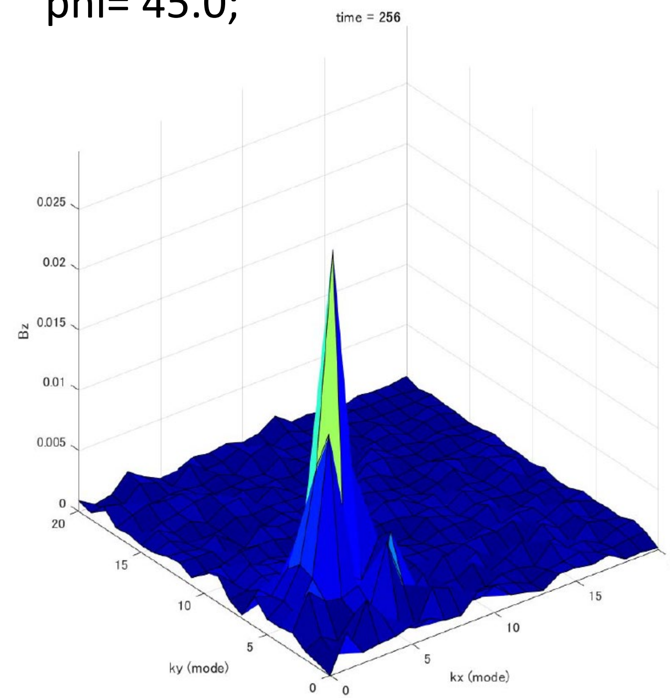






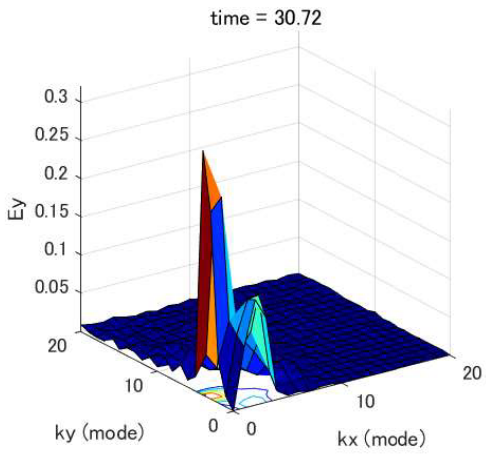


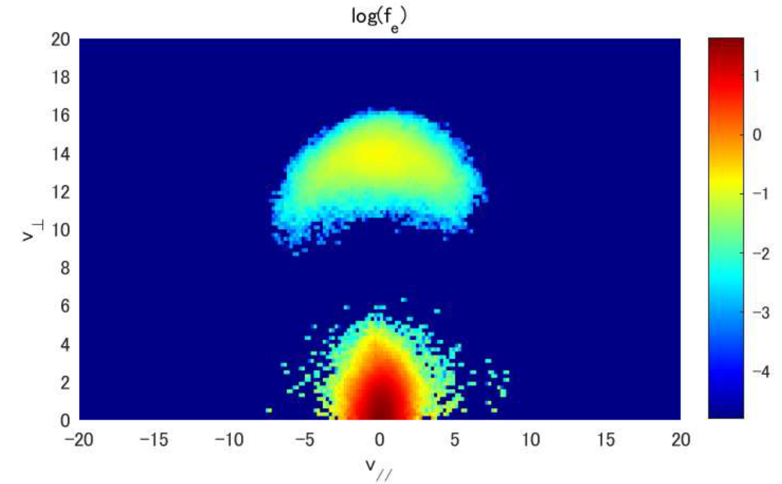
% direction of the static magnetic magnetic field
theta= 90.0;
phi= 45.0;

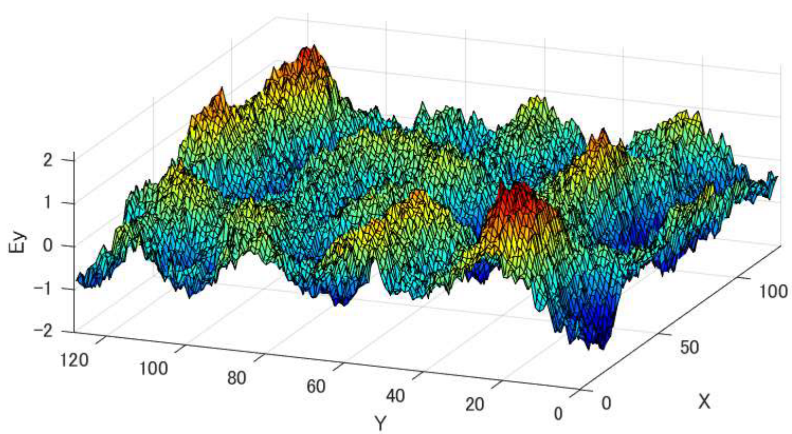


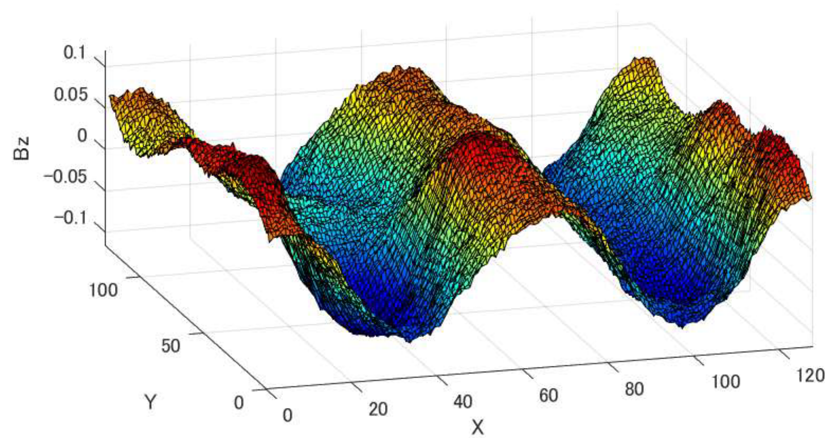

```
vs = sum(vx(m).^2 + vy(m).^2 + vz(m).^2);
vspara = sum(((vx(m)*bx0+vy(m)*by0+vz(m)*bz0)/b0).^2);
%Temperature Anisotropy  T_perp/T_para
    At(it,k) = 0.5*(vs/vspara -1);
```

Relativistic Ring-Beam Instability
Generating Whistler-mode and ECH Waves

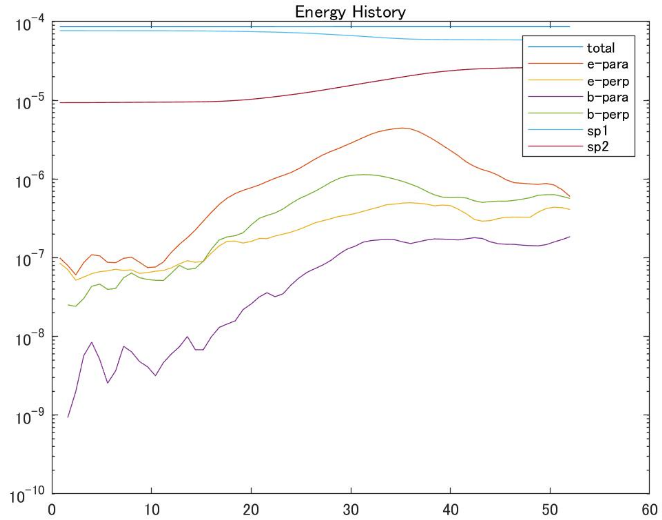


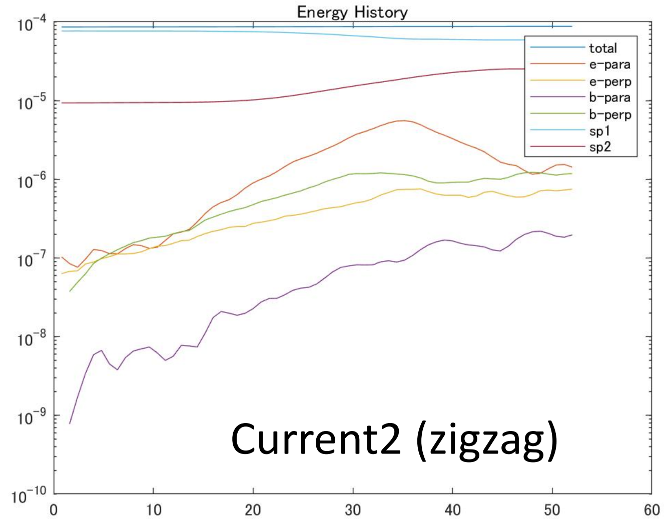




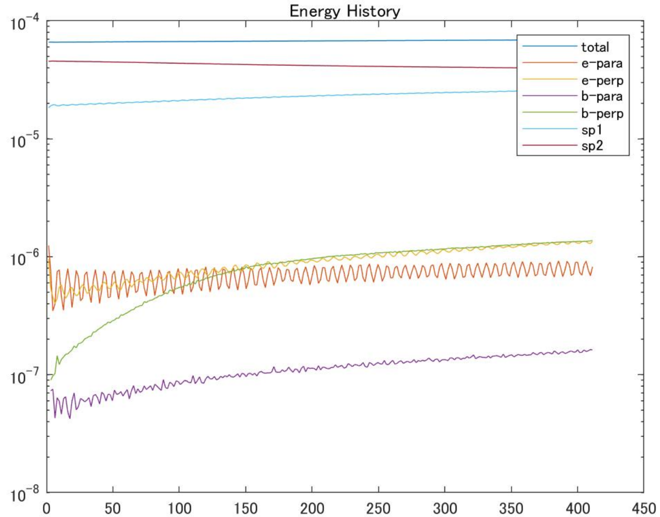


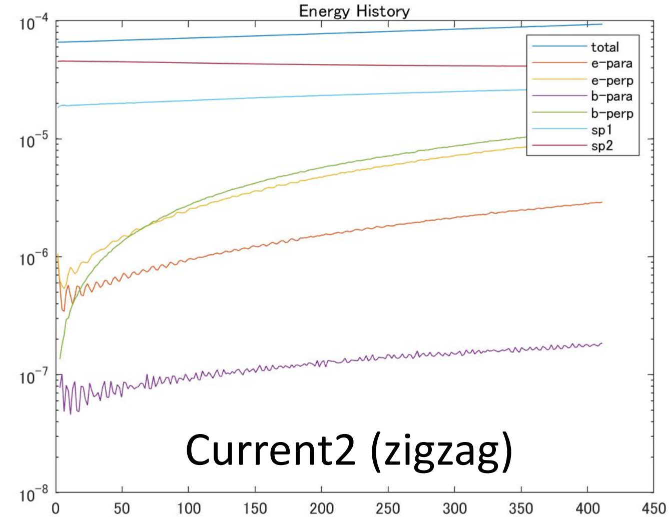
Buneman Instability





Whistler Instability





Two-Dimensional Simulation System (x, y)

$$\frac{\partial}{\partial z} = 0 \qquad \nabla = \boldsymbol{e}_x \frac{\partial}{\partial x} + \boldsymbol{e}_y \frac{\partial}{\partial y}$$

$$\boldsymbol{E} = \boldsymbol{E}_{xy} + \boldsymbol{E}_z$$

$$\boldsymbol{B} = \boldsymbol{B}_{xy} + \boldsymbol{B}_z$$

$$\boldsymbol{J} = \boldsymbol{J}_{xy} + \boldsymbol{J}_z$$

Courant Condition

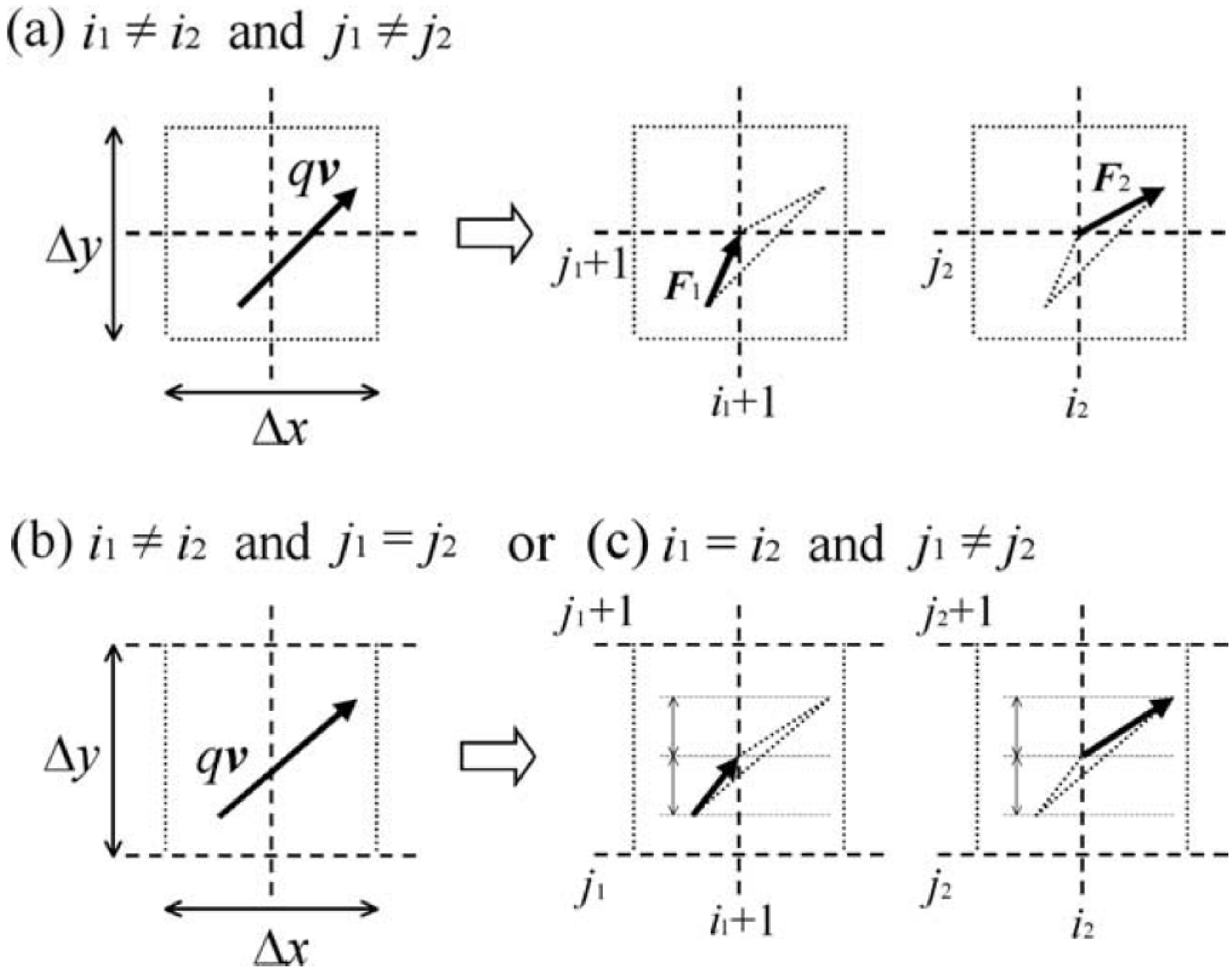
$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2} \qquad K_x = \frac{\sin(k_x \Delta x/2)}{\Delta x/2} \qquad K_y = \frac{\sin(k_y \Delta y/2)}{\Delta y/2}$$

$$\Omega^2 = c^2 (K_x^2 + K_y^2) \qquad \sqrt{2} c \Delta t < \Delta x$$

Comment on Zigzag Method

The Zigzag method for computation of the current density is slightly faster than the electrostatic correction method with charge density computation. It can enhance the electromagnetic fluctuations because of the non-physical trajectories of particles assumed to simplify the computation of the current.

Zigzag Method (current2)

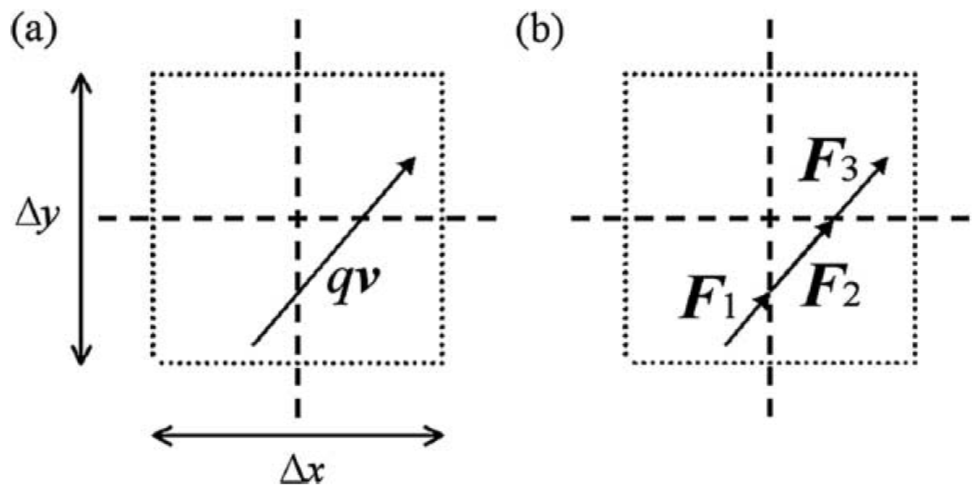


[*Umeda et al., CPC,* 2003]

Charge Conservation Method

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{J} = 0$$

$$\frac{J_x^{t+\Delta t/2}(i+\frac{1}{2}, j) - J_x^{t+\Delta t/2}(i-\frac{1}{2}, j)}{\Delta x} + \frac{J_y^{t+\frac{\Delta t}{2}}(i, j+\frac{1}{2}) - J_y^{t+\Delta t/2}(i, j-\frac{1}{2})}{\Delta y}$$
$$= \frac{\rho^t(i, j) - \rho^{t+\Delta t}(i, j)}{\Delta t}.$$



[*Villasenor & Buneman, CPC*, 1992]

KEMPO2

References

H. Matsumoto and Y. Omura, Particle Simulations of Electromagnetic Waves and its Applications to Space Plasmas, Computer Simulations of Space Plasmas, edited by H. Matsumoto and T. Sato, Terra Pub. and Reidel Co., pages 43-102, 1984.

T. Umeda, Y. Omura, T. Tominaga, and H. Matsumoto, A new charge conservation method in electromagnetic particle-in-cell simulations, Computer Physics Communications, Vol.156, No.1, pp.73-85, 2003.