Energetic electron tail production from binary encounters of discrete electrons and ions in a sub-Dreicer electric field

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Experiment Observation Provides Motivation

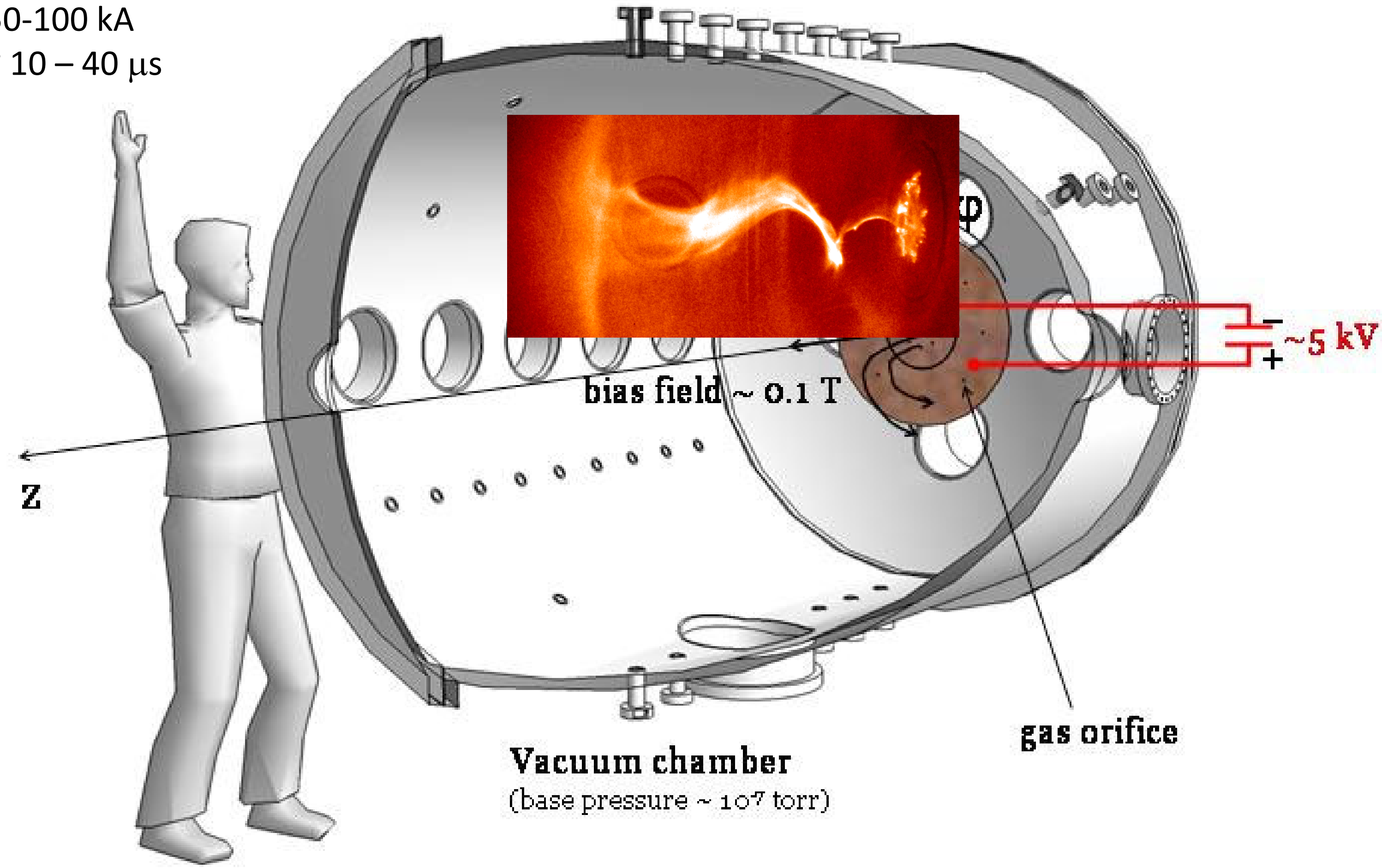
Experimental plasma jet is cold and highly collisional:

T = 2 eV and $n = 10^{22}$ m^{-3} $\rightarrow$ 1 μm electron mean free path

Yet, unstable jet emits 6 keV X-ray burst, implying 6 keV electrons!

Experimental jet

Current ~ 50-100 kA
Duration ~ 10 – 40 μs



Observed experimental sequence:

Jet kinks at a critical length (q=1)

↓

Kink lateral acceleration

↓

Effective gravity

↓

Rayleigh-Taylor instability

↓

Kinetic instability

↓

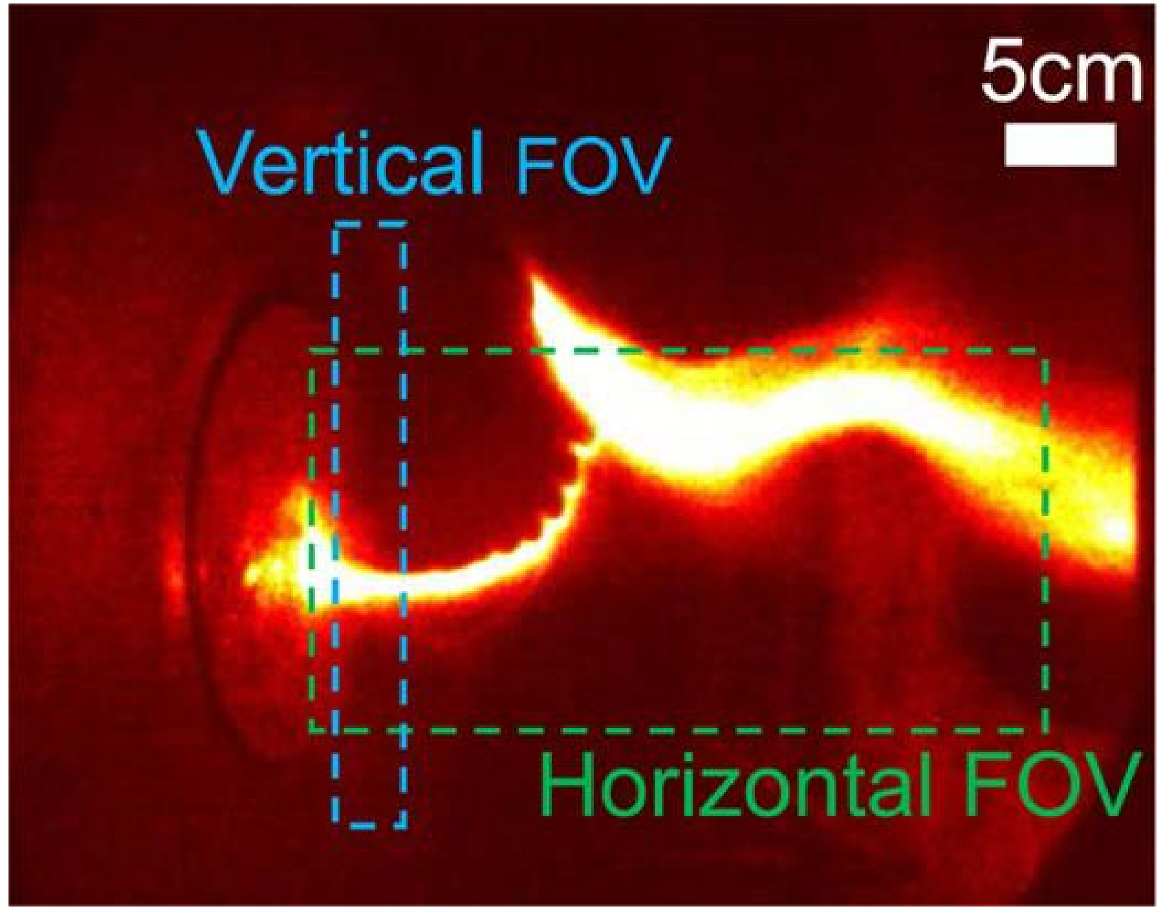
Large electric field

↓

Energetic electrons

↓

Scintillators observe 1 μs X-ray burst



Zhou/Pree/Bellan RSI 94, 013504 (2023)

Model for energetic electron tail

Compute Rutherford scattering of 5000 ions & 5000 electrons

No assumption of Ohm's law or differential cross-section

Follow each Rutherford encounter

Intersperse electron-electron and electron-ion encounters

Allow both grazing and large angle deflections

Three situations considered:

1. Electric field <u>off</u>, establish 3D Maxwellians

2. Electric field <u>on</u>, all electrons run away

3. Electric field on, <u>enable line radiation</u>, get energetic tail as in experiment

Problem with Ohm's law

Suppose Ohm's law is true: Then **E** = η**J** where **E** is constant and $\eta \sim T_e^{-3/2}$

Then dissipated power is $\eta J^2 = E^2/\eta$

Dissipated power increases T_e

$$\frac{d}{dt}\left(\frac{3}{2}n\kappa T_e\right) = \frac{E^2}{\eta} \quad \text{where} \quad \eta = \eta_0 (T_0/T_e)^{3/2}$$

Solution of this equation is

$$\frac{T_e(t)}{T_{e,0}} = \frac{1}{\left(1 - \frac{E^2/\eta_0}{3n\kappa T_{e0}} t\right)^2}$$

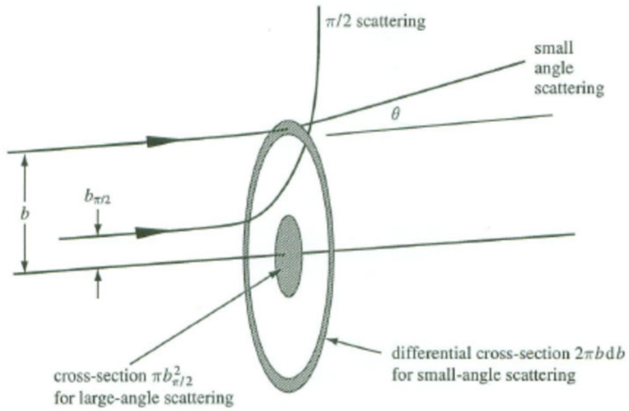
$$\rightarrow \infty \text{ when } t = \frac{3n\kappa T_{e0}}{E^2/\eta_0}$$

For Caltech experiment parameters, solution predicts T_e quadruples in 240 ps, becomes infinite in 480 ps

Infinite temperature not observed.

This implies it is incorrect to assume **E** = η**J** with constant **E** and $\eta \sim T_e^{-3/2}$.

Problems with differential cross-section concept



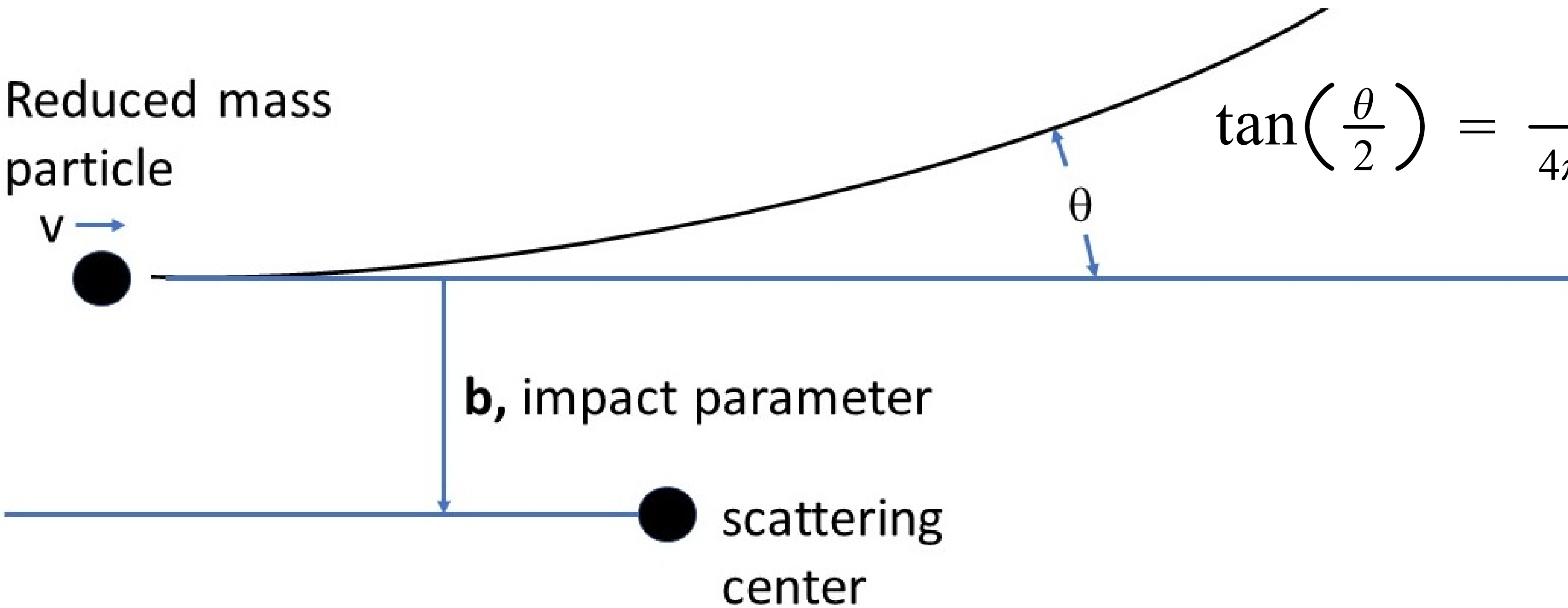
1. Differential cross-section concept assumes that
   an electron collides with a sequence of ions and <u>then</u> with a sequence of electrons
   - reality is that e-e and e-i collisions are <u>interspersed</u>
   - e-e and e-i collisions are qualitatively different
     - e-e collisions make electrons Maxwellian in electron center of mass frame
     - e-i collisions make electrons isotropic in ion frame

2. Differential cross-section concept assumes that an electron is not deflected after a collision
   when summing effect of many collisions

New model:

Do not make traditional assumptions, go back to first principles

Rutherford scattering encounter

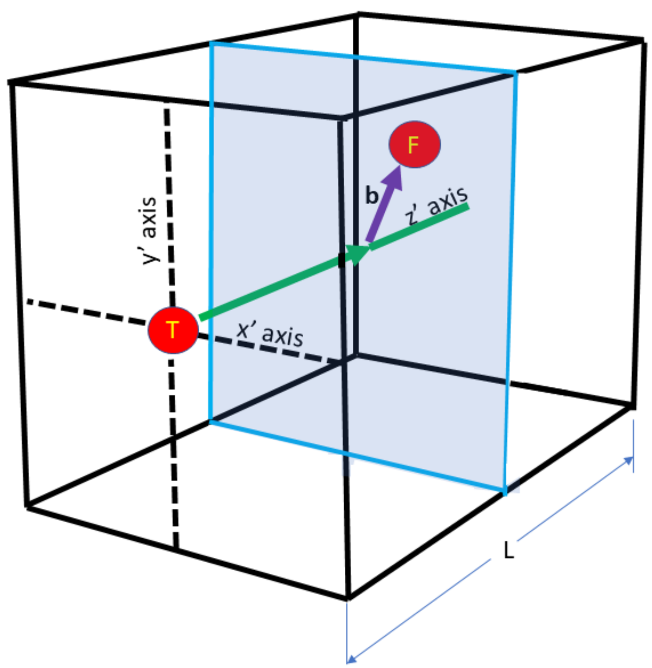


$$\tan\left(\frac{\theta}{2}\right) = \frac{q_T q_F}{4\pi\varepsilon_0 b \mu v^2}$$

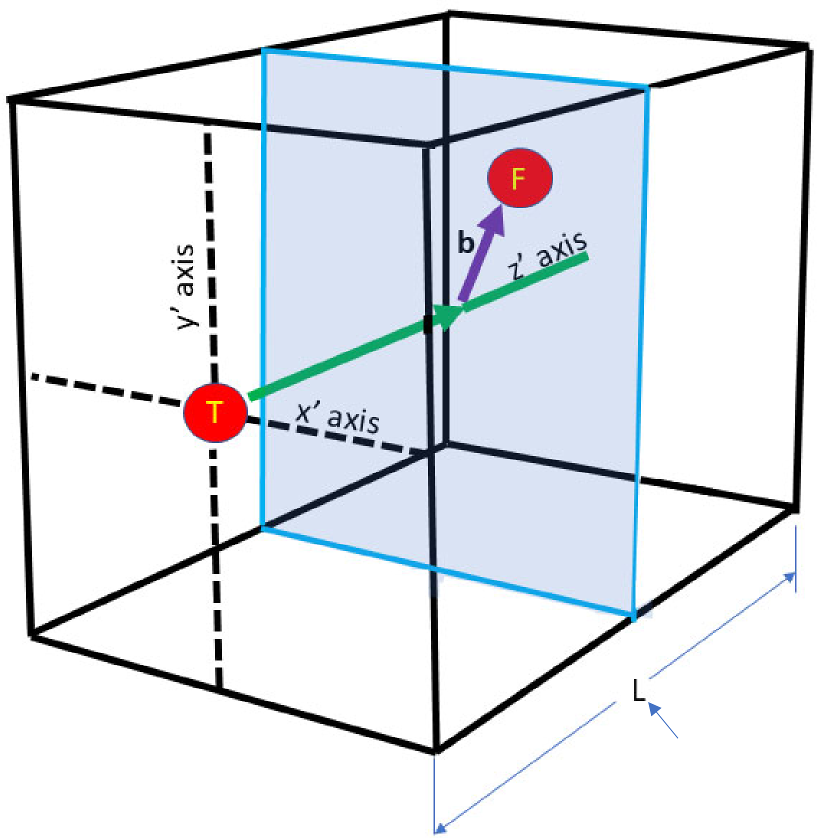
Interparticle spacing distance

$L = n^{-1/3}$

so <u>one</u> field particle in a cube having side L, $nL^3=1$



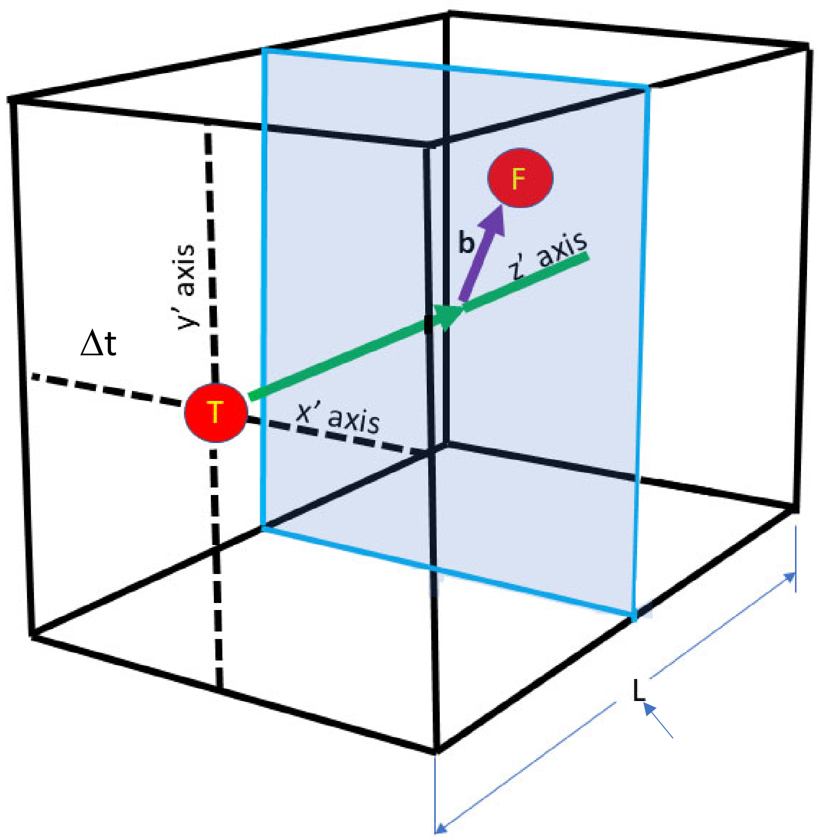
Scattering cube for each encounter



Field particle (electron or ion) is somewhere in blue square

Blue square is somewhere along z' axis (green line)

Scattering cube for each encounter



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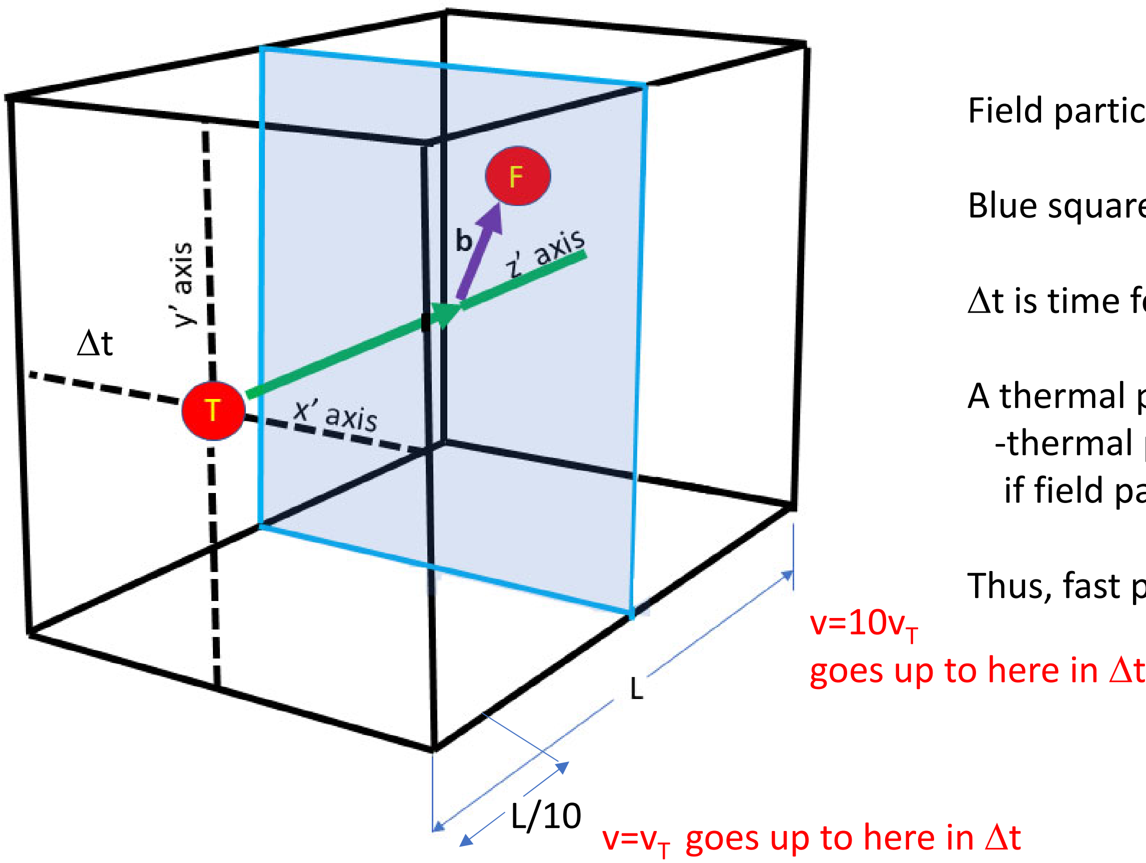
Blue square is somewhere along z' axis (green line)

Δt is time for fast particle (10 x thermal) to traverse cube.

A thermal particle traverses only L/10 in Δt
-thermal particle will not encounter a field particle
if field particle is not in first tenth of cube

Thus, fast particles have more encounters in time Δt

Scattering cube for each encounter



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Simplified example of calculation sequence:

Count all possible ways 4 electrons and 4 ions can collide

$$
\begin{bmatrix}
* & & & & & & & \\
0: e1-e0 & * & & & & & & \\
1: e2-e0 & 7: e2-e1 & * & & & & & \\
2: e3-e0 & 8: e3-e1 & 13: e3-e2 & * & & & & \\
3: \mathbf{i0-e0} & 9: \mathbf{i0-e1} & 14: \mathbf{i0-e2} & 18: \mathbf{i0-e3} & * & & & \\
4: \mathbf{i1-e0} & 10: \mathbf{i1-e1} & 15: \mathbf{i1-e2} & 19: \mathbf{i1-e3} & 22: i1-i0 & * & & \\
5: \mathbf{i2-e0} & 11: \mathbf{i2-e1} & 16: \mathbf{i2-e2} & 20: \mathbf{i2-e3} & 23: i2-i0 & 25: i2-i1 & * & \\
6: \mathbf{i3-e0} & 12: \mathbf{i3-e1} & 17: \mathbf{i3-e2} & 21: \mathbf{i3-e3} & 24: i3-i0 & 26: i3-i1 & 27: i3-i2 & *
\end{bmatrix}
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\end{bmatrix}
$$

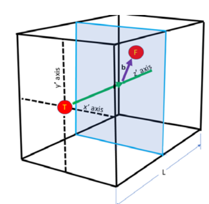
Map matrix to linear vector

$$
\left[\, 0: e1-e0,\ 1: e2-e0,\ 2: e3-e0,\ 3: \mathbf{i0-e0},\ \ldots\ 25: i2-i1,\ 26: i3-i1,\ 27: i3-i2 \,\right]
$$

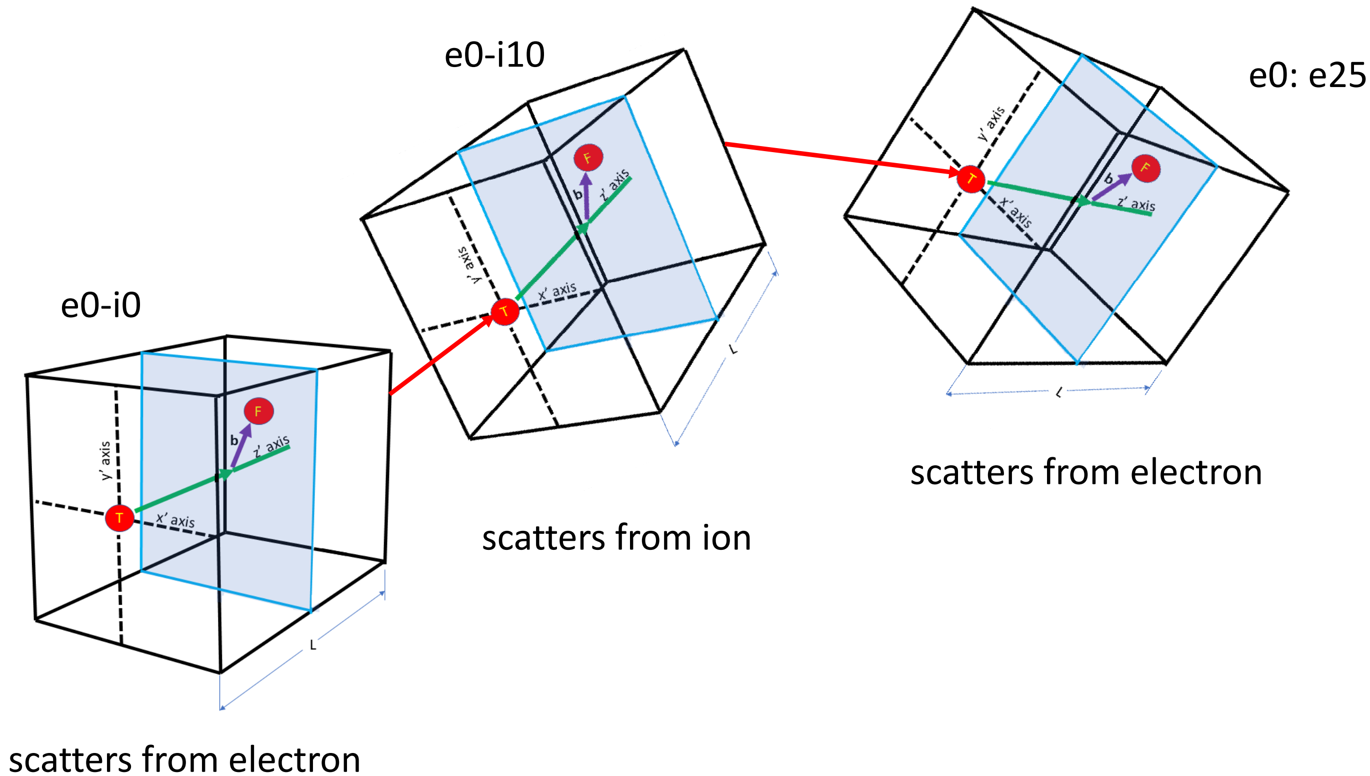
Randomly shuffle vector

$$
\left[\, 3: \mathbf{i0-e0},\ 24: i3-i0,\ 4: \mathbf{i1-e0},\ 3: \mathbf{i0-e0},\ \ldots\ 1: e2-e0,\ 26: i3-i1,\ 0: e1-e0 \,\right]
$$

Calculate Rutherford deflection for each element using scattering cube



Successive scattering cubes for electron ‘e0’



Velocity distribution movies

5000 electrons & 5000 ions all colliding with each other (10,000 particles)

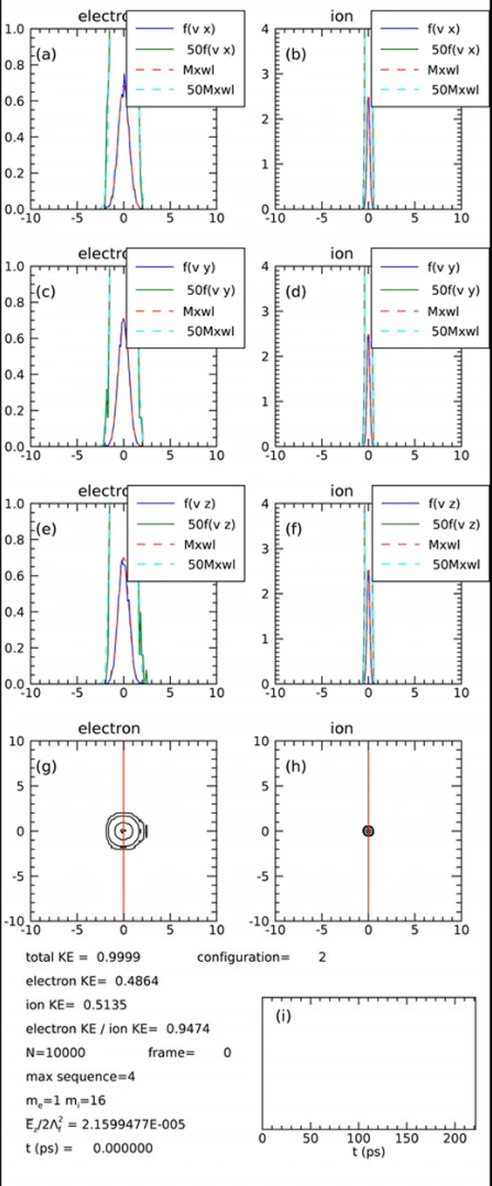
e-e, e-i, i-i Rutherford encounters calculated in center of mass frame, then converted to lab frame

N=10,000, matrix off-diagonal is $(N^2-N)/2$ = 49,995,000 elements

49,995,000 elements in sequence vector

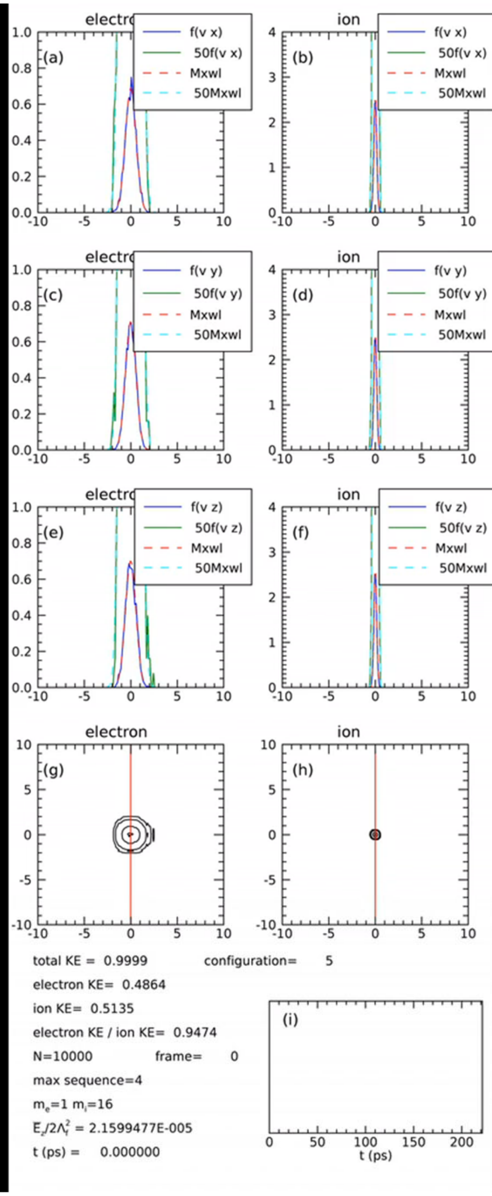
$$\left[\, 3:\mathbf{i0}-\mathbf{e0},\ 24:i3-i0,\ 4:\mathbf{i1}-\mathbf{e0},\ 3:\mathbf{i0}-\mathbf{e0},\ \ldots.\ 1:e2-e0,\ 26:i3-i1,\ 0:e1-e0 \,\right]$$

Electric field <u>on</u>, start with Maxwellian,
No line radiation
All electrons run away, heat up



Electric field on, start with Maxwellian,
Line radiation enabled,
Energy radiated for certain small impact parameters

Energetic tail forms -survivor electrons that never had small impact parameter and so never lost energy from line radiation but accelerated in electric field



Line radiation **OFF**

All electrons run away!

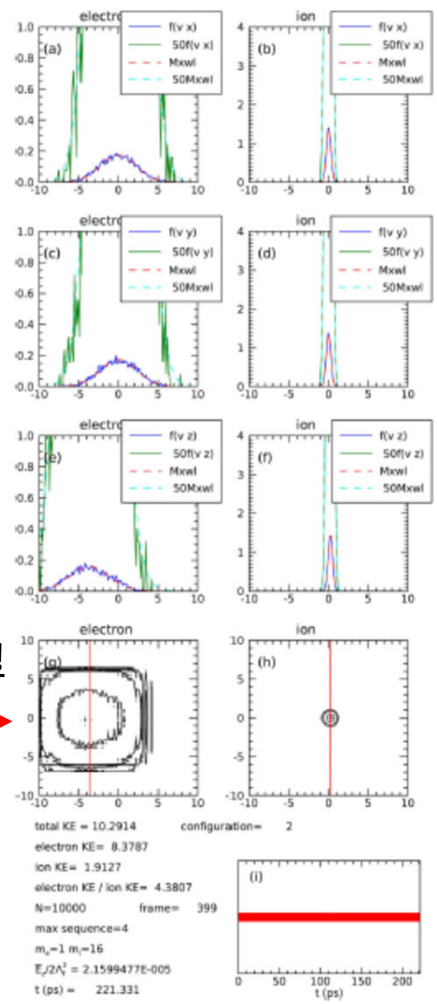


FIG. 5. Configuration No. 2 (still figure is the last figure of animation). $\bar{E}_z/2\Lambda_t^2 = 2.16 \times 10^{-5}$ and no radiation. (a)–(f) show as blue lines the components of numerically computed velocity distribution function f. Also shown (red dashed lines) is the Maxwellian g defined by Eq. (73) having the same average kinetic energy and same mean velocity. Also shown (dark green and dashed light green) are plots of 50f and 50g. Contours in (g) and (h) are at 0.001, 0.01, 0.25, and 0.9. Red vertical line in (g) and (h) denotes mean z velocity, i.e., u_{ez}. (i) Clock for animation running time. Multimedia available online.

Line radiation **ON**

Energetic tail!

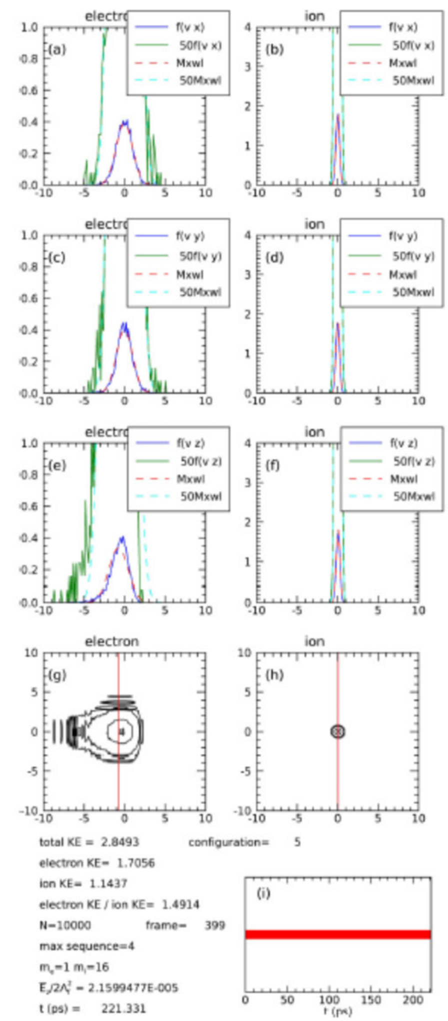


FIG. 9. Configuration No. 5. The same as Fig. 7 except $\delta_{ant} = 0.2$ (still figure is the last figure of animation). Multimedia available online.

Interpretation

1. Cumulative effect of grazing collisions is **not** a true cross-section

   In a true cross-section, a particle either collides **or it does not**

   For grazing collisions, all electrons behave the same way, so there are **no** electrons that do not collide

2. If **no** energy loss mechanism, then **all** electrons heat up and run away in of order 200 ps

Consistent with explosive heating $$\frac{T_e(t)}{T_{e,\,0}} = \frac{1}{\left(1 - \frac{E^2/\eta_0}{3n\kappa T_{e0}}\, t\right)^2}$$

.

$$\to \infty \text{ when } t = \frac{3n\kappa T_{e0}}{E^2/\eta_0}$$

3. Unlike grazing collisions, effect of line radiation **is** a true cross-section.

Electron either **does or does not** cause line radiation (quantum effect).

Line radiation comes from the occasional **small impact parameter collisions** discarded in classical model.

After each line-radiation mean free path, $1 - 1/e$ of the electrons have lost energy (caused photon emission).

Consequently $1/e$ of the electrons have **not** lost energy.

These $1/e$ of the electrons **gain energy** in E field.

After N mean free paths, $(1/e)^N$ of electrons have **never lost energy** from line radiation.

These electrons behave as if they were in the situation with the electric field on and no line radiation

These electrons are called `survivor' electrons.

Survivor electrons constitute an energetic tail.

Survivor electrons ultimately collide and produce X-rays.

Details in:

P. M. Bellan, Physics of Plasmas (2023)
doi: 10.1063/5.0167004

Electric field off,
Initial state: counterstreaming beams

Collisions form 3D Maxellians

Electrons thermalize first, then ions

Then electrons and ion temperatures equilibrate