Test particle simulation for electrons accelerated by kinetic Alfvén waves and precipitating into the ionosphere

ISSS-15 + IPELS-16

08.08.2024 17:40–18:00

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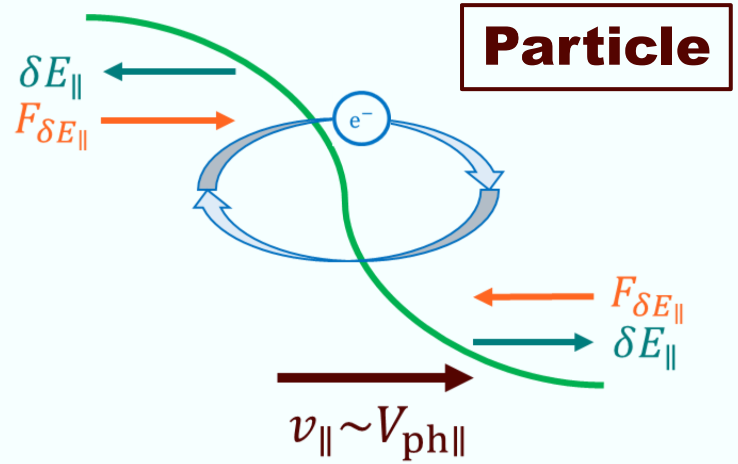
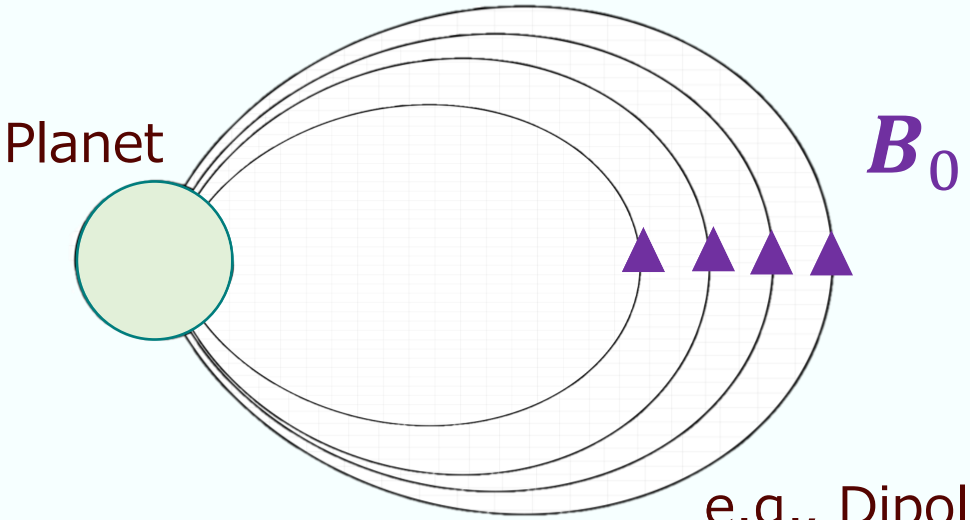
Short talk – particle trapping (Landau resonance)

A wave propagates in a uniform field.



Particles are trapped by the wave.



e.g., Dipole magnetic field

A wave propagates in a non-uniform field.

How do particles move by the wave?

Theory:
2nd-order resonance theory

Method: test particle simulation

1. Introduction

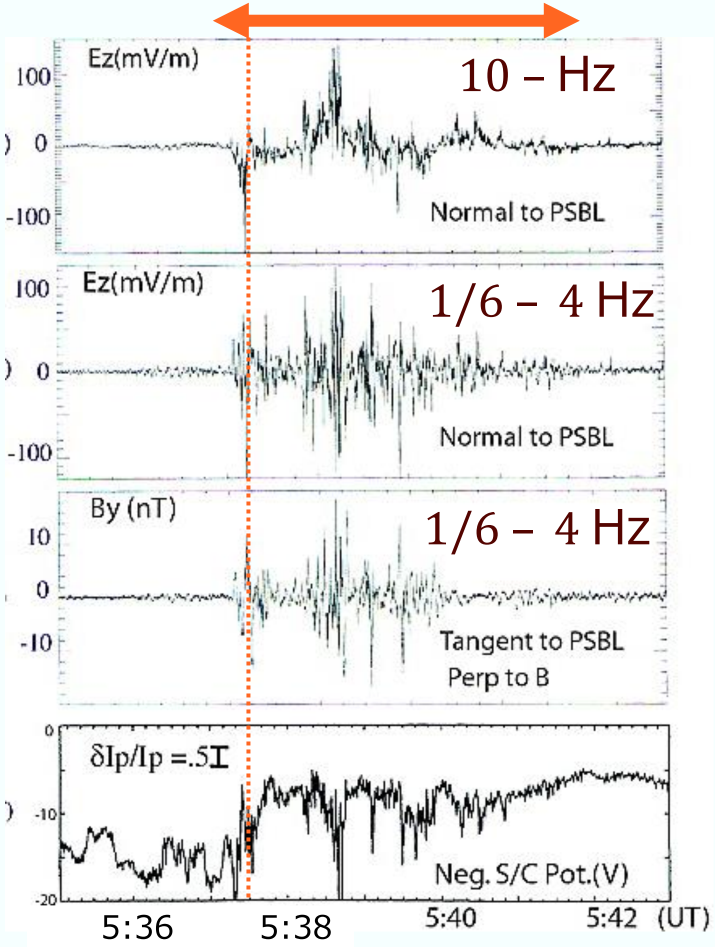
1.1 Introduction: Kinetic Alfvén waves

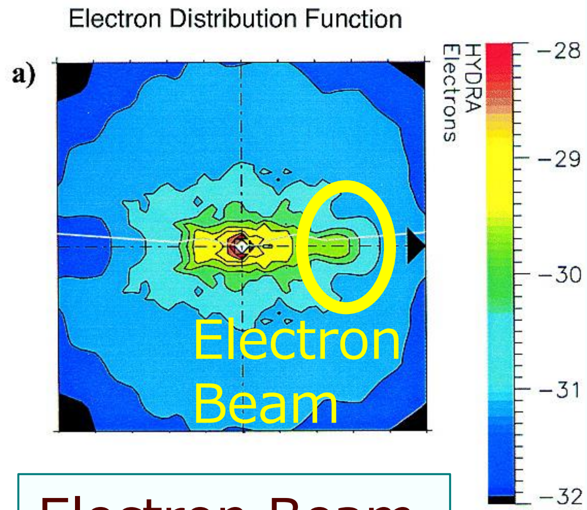
Observation

$R = 4\text{–}6\,\mathrm{R_E}, L = 7.5\sim9$

Kinetic Alfvén waves (KAWs)

Polar spacecraft





Electron Beam

Parallel energy:
$\sim1\text{–}2\,\mathrm{keV}$

Perpendicular energy:
$25\text{–}50\,\mathrm{eV}$

[Wygant et al., 2002]

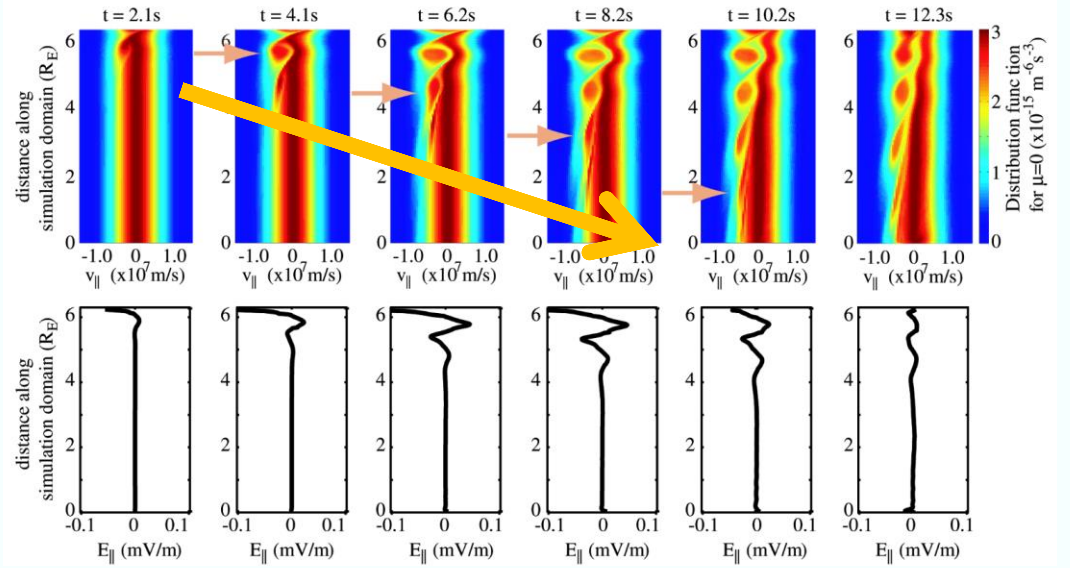
KAWs are observed at the plasma sheet boundary layer.

$\delta E_\parallel$ carried by KAWs can accelerate electrons to precipitate into the ionosphere.

Simulation

DK-1D simulation

Variation of distribution function $(\mu = 0)$



Electrons $(\mu = 0)$ are trapped by the KAW and transported to the ionosphere.

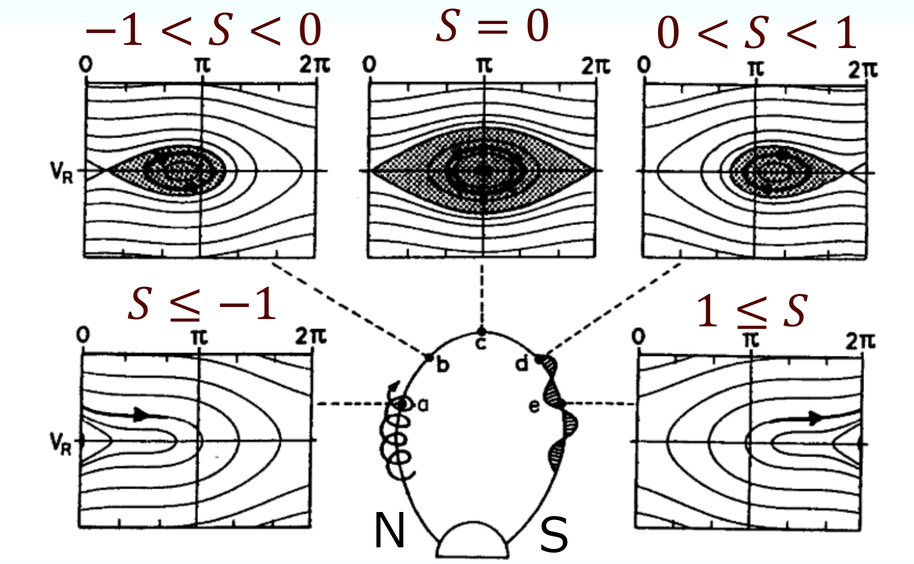
Electrons with large μ are reflected before reaching the ionosphere due to the mirror force and do not contribute to auroral brightening.

[Watt and Rankin, 2009]

Question

Under what conditions are electrons accelerated and precipitated into the ionosphere?
Under what conditions are electrons greatly accelerated?

2nd-order resonance theory



[Omura et al., 1991]

Second-order nonlinear ordinary differential equation of the wave phase as seen from the electron

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = k_{\parallel}\left(v_{\parallel} - V_{\mathrm{ph}\parallel}\right) \equiv \theta, \qquad \frac{\mathrm{d}^2\psi}{\mathrm{d}t^2} = \frac{\mathrm{d}\theta}{\mathrm{d}t} = -\omega_{\mathrm{tr}}^2(\sin\psi + S)$$

[Artemyev et al., 2017; Tobita et al., 2018]

The value of S depends on the background magnetic field gradient, and the region of phase trapping varies with the position on the magnetic field line.

Purpose

Using the second-order resonance theory, we clarify the characteristics of electrons precipitating into the ionosphere from a detailed investigation.

2. Calculation Methods

2.1 Calculation Methods: Equations for 2nd-order resonance theory

Kinetic Alfvén waves (KAWs)

Dispersion Relation (ERMHD)

$$\omega = k_\perp \rho_\mathrm{i} k_\parallel v_\mathrm{A} \sqrt{\frac{1+\tau}{\beta_\mathrm{i}(1+\tau)+2\tau}}$$

$(\tau \coloneqq T_\mathrm{i}/T_\mathrm{e})$

[Schekochihin et al., 2009]

Assumption $k_\perp \rho_\mathrm{i} = 2\pi$

Wave phase $\psi = \int_0^z k_\parallel \,\mathrm{d}z' - \omega t + \psi_0$

Scalar potential $\varphi = \varphi_0 \cos\psi$

Electric field

$$\delta E_\parallel = k_\parallel \varphi_0 \left(2 + \frac{1}{\tau}\right) \sin\psi = k_\parallel \Phi_\mathrm{E} \sin\psi$$

Equations of Motion

$$\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = -\frac{\mu}{m_\mathrm{e}}\frac{\mathrm{d}B_0}{\mathrm{d}z} - \frac{e}{m_\mathrm{e}}\delta E_\parallel$$

Mirror force vs. $\delta E_\parallel$

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = 0$$

μ conservation

Material derivative of wave phase ψ

1st-order $\dfrac{\mathrm{d}\psi}{\mathrm{d}t} = k_\parallel (v_\parallel - V_{\mathrm{ph}\parallel}) \equiv \theta$

2nd-order $\dfrac{\mathrm{d}^2\psi}{\mathrm{d}t^2} = \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\omega_\mathrm{t}^2(\sin\psi + S)$

Pendulum equation

Wave phase speed: $V_{\mathrm{ph}\parallel} \coloneqq \dfrac{\omega}{k_\parallel}$

Trapping frequency: $\omega_\mathrm{t} \coloneqq k_\parallel \sqrt{\dfrac{e\Phi_\mathrm{E}}{m_\mathrm{e}}}$

Inhomogeneity factor:

$$S \coloneqq \frac{K}{e\Phi_\mathrm{E}}(1 + \Gamma\cos^2\alpha)\delta_1$$

Pitch angle coefficient: $\Gamma \coloneqq 1 + \dfrac{2\beta_\mathrm{i}(1+\tau)}{\beta_\mathrm{i}(1+\tau)+2\tau} \sim 1$

Magnetic field gradient scale:

$$\delta_1 \coloneqq \frac{1}{k_\parallel B_0}\frac{\mathrm{d}B_0}{\mathrm{d}z}$$

2.2 Calculation Methods: Phase-trapped/scattered states

Pendulum equations

$$\begin{cases} \dfrac{\mathrm{d}\psi}{\mathrm{d}t} = \theta = k_{\parallel}\left(v_{\parallel} - V_{\mathrm{ph}\parallel}\right) \\ \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\omega_{\mathrm{t}}^2\left(\sin\psi + S\right) \end{cases}$$

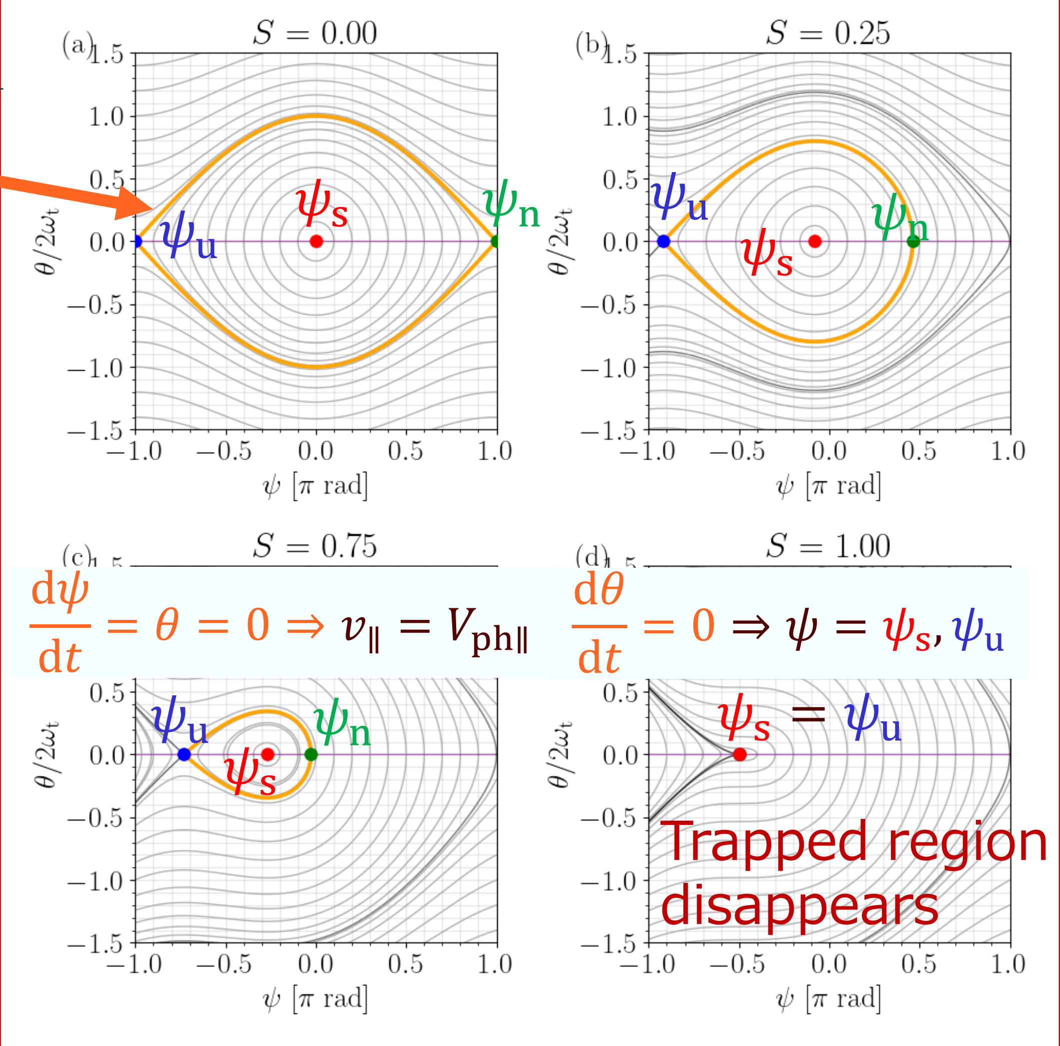
$\omega_{\mathrm{t}} = \mathrm{const.}$
$S = \mathrm{const.}$

$$\left(\frac{\theta}{2\omega_{\mathrm{t}}}\right)^2 - \frac{1}{2}\left(\cos\psi - S\psi\right) = \mathrm{const.}$$

Orange line: trapped–scattered boundary

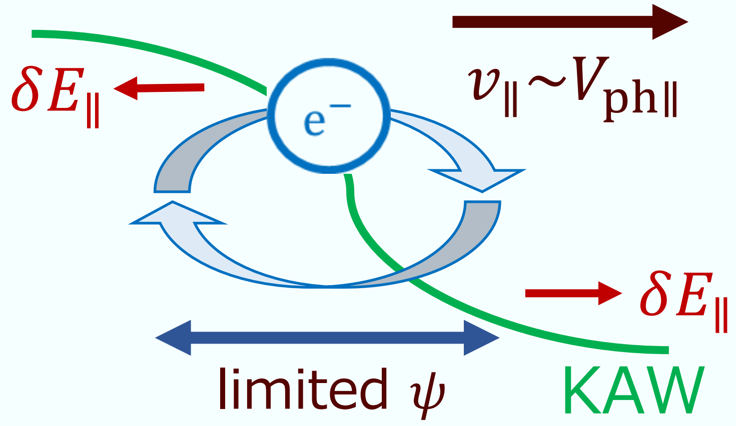
$$f(S,\psi,\theta,\omega_{\mathrm{t}}) := \left(\frac{\theta}{2\omega_{\mathrm{t}}}\right)^2 - \frac{1}{2}\left\{\cos\psi + \sqrt{1-S^2} - S(\psi + \pi - \arcsin S)\right\} = 0$$

and $\psi \in [\psi_{\mathrm{u}}, \psi_{\mathrm{n}}]$



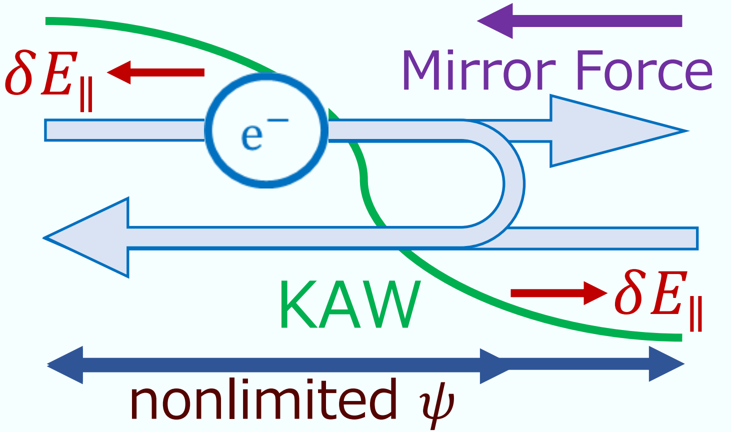
INSIDE

Phase-trapped state



OUTSIDE

Phase-scattered state

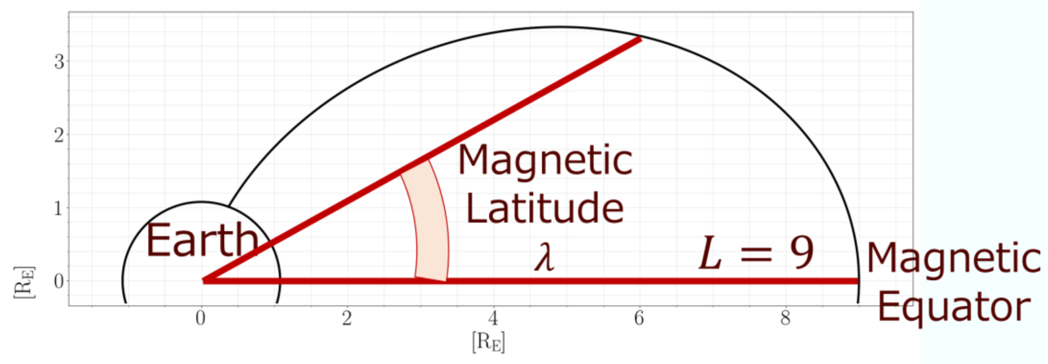


Calculation Settings

Calculation Field

Earth's $L = 9$ dipole magnetic field line

Background plasma: $n = 1\ \mathrm{cm}^{-3}, T_\mathrm{i} = 1\ \mathrm{keV}, T_\mathrm{e} = 100\ \mathrm{eV}$



KAW model

Perpendicular wavelength:

$$k_\perp \rho_\mathrm{i} = 2\pi$$

Wave frequency:

$$f_\mathrm{KAW} = \frac{\omega}{2\pi} = 0.15\ \mathrm{Hz}$$

$$\varphi = 2000\ \mathrm{V}\ \left(\because\ (k_\parallel \Phi_\mathrm{E})\Big|_{\lambda=0} \approx 1\,\mathrm{mV/m}\right)$$

[Chaston et al., 2012]

Particle Calculation Method

4th-order Runge–Kutta method

Time step: 10^{-3} s

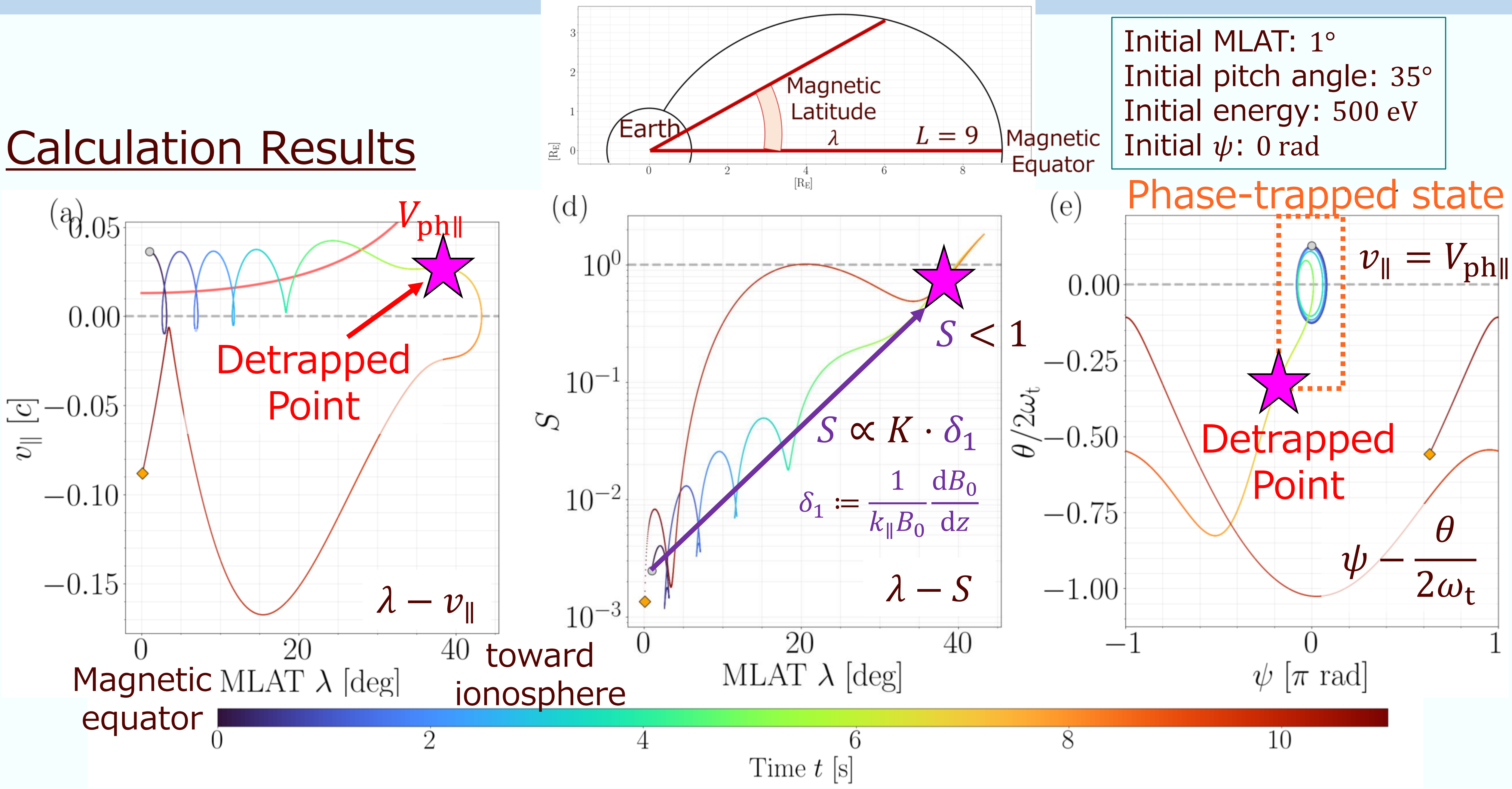
Equations:

$$\begin{cases} \dfrac{\mathrm{d}\psi}{\mathrm{d}t} = \theta \\ \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\omega_\mathrm{t}^2 (\sin\psi + S) \\ \dfrac{\mathrm{d}\lambda}{\mathrm{d}t} = \dfrac{(\theta + \omega)}{k_\parallel} \dfrac{1}{r_\mathrm{eq} \cos\lambda \sqrt{1 + 3\sin^2\lambda}} \end{cases}$$

2.4 Calculation Methods: Electron trajectory

Initial MLAT: 1°
Initial pitch angle: 35°
Initial energy: 500 eV
Initial ψ: 0 rad

Calculation Results



3. Results

3.1 Results: Detrapped point

Question

Under what conditions are electrons accelerated and precipitated into the ionosphere?
Under what conditions are electrons greatly accelerated?

Detrapped Point

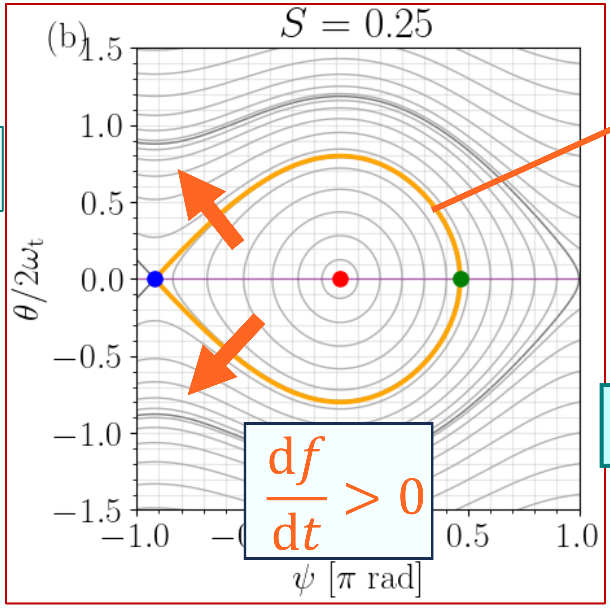
Phase-Scattered State

- $\theta > 0 \;\&\; v_{\parallel} > 0$
- Pass through $\theta = 0$
- $\theta \leq 0 \;\&\; v_{\parallel} > 0$
- Reach the ionosphere → **Auroral Precipitation**

$$\theta = k_{\parallel}(v_{\parallel} - V_{\mathrm{ph}\parallel})$$

$$S = \frac{K}{e\Phi_{\mathrm{E}}}(1 + \Gamma \cos^2 \alpha)\delta_1$$

$$\delta_1 = \frac{1}{k_{\parallel} B_0} \frac{\mathrm{d}B_0}{\mathrm{d}z}$$



Trapped–Scattered boundary
$f = 0$

$\frac{\mathrm{d}f}{\mathrm{d}t} > 0$

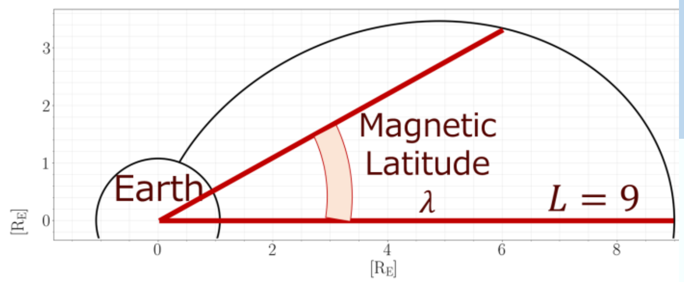
Detrapped point conditions

- $S \leq 1$
- $\psi \in [\psi_{\mathrm{u}}, \psi_{\mathrm{n}}]$
- $f(S, \psi, \theta, \lambda) = 0$
- $\frac{\mathrm{d}f}{\mathrm{d}t}(S, \psi, \theta, \lambda) > 0$

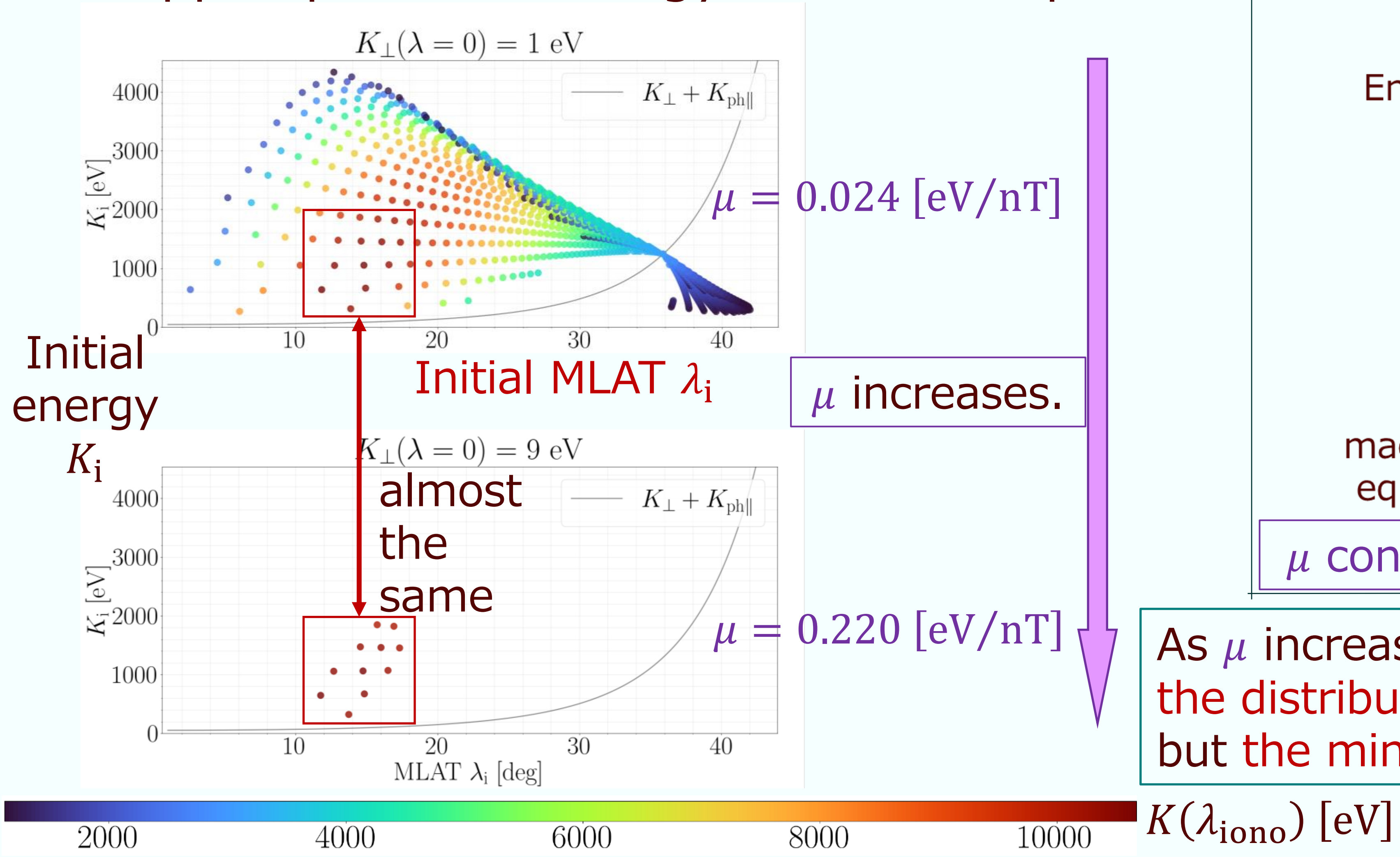
Detrapped point $(S_{\mathrm{i}}, \psi_{\mathrm{i}}, \mu, \theta_{\mathrm{i}}, \lambda_{\mathrm{i}})$

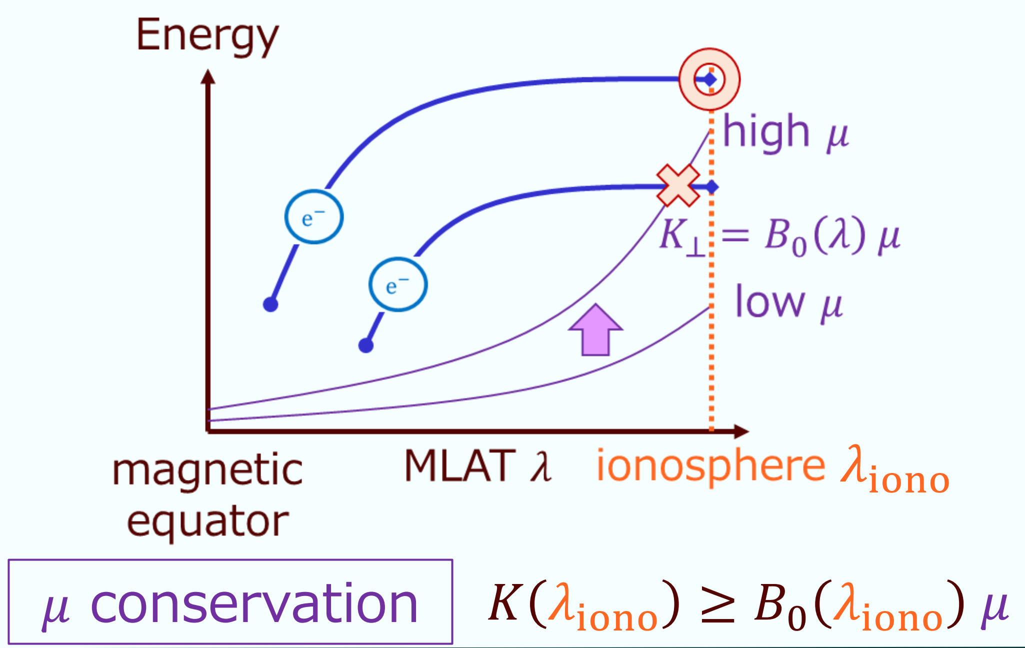
$S_{\mathrm{i}}, \mu, \mathrm{sgn}\,\theta_{\mathrm{i}}, \psi_{\mathrm{i}} \rightarrow$ determine λ_{i}

3.2.1 Results: Detrapped point dependency (μ)



Detrapped point vs. Energy at the ionosphere





As μ increases,
the distribution remains almost the same,
but the minimum energy becomes larger.

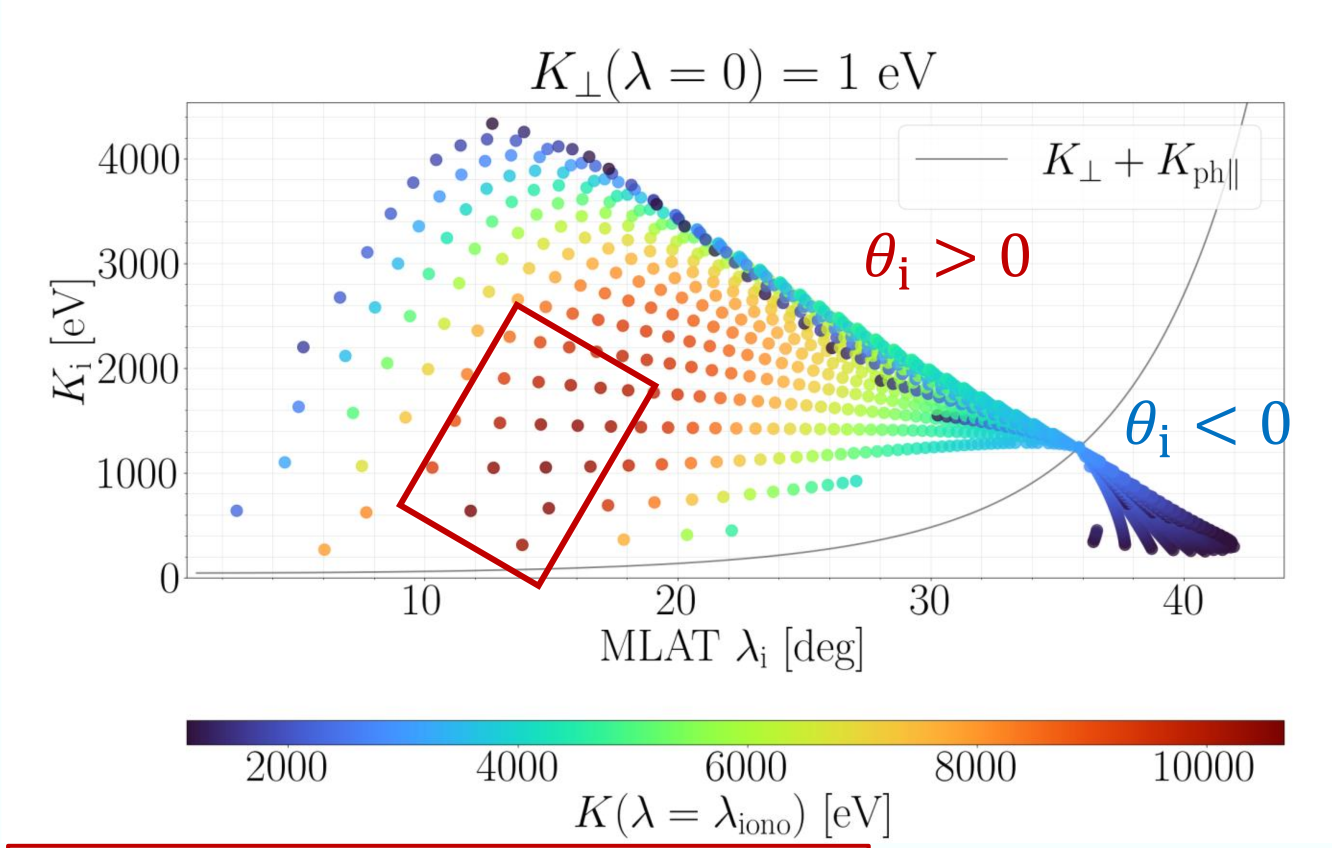
With a monochromatic KAW, $K(\lambda_{iono})$ varies depending on the detrapped point, and $K(\lambda_{iono})$ can be broadband.

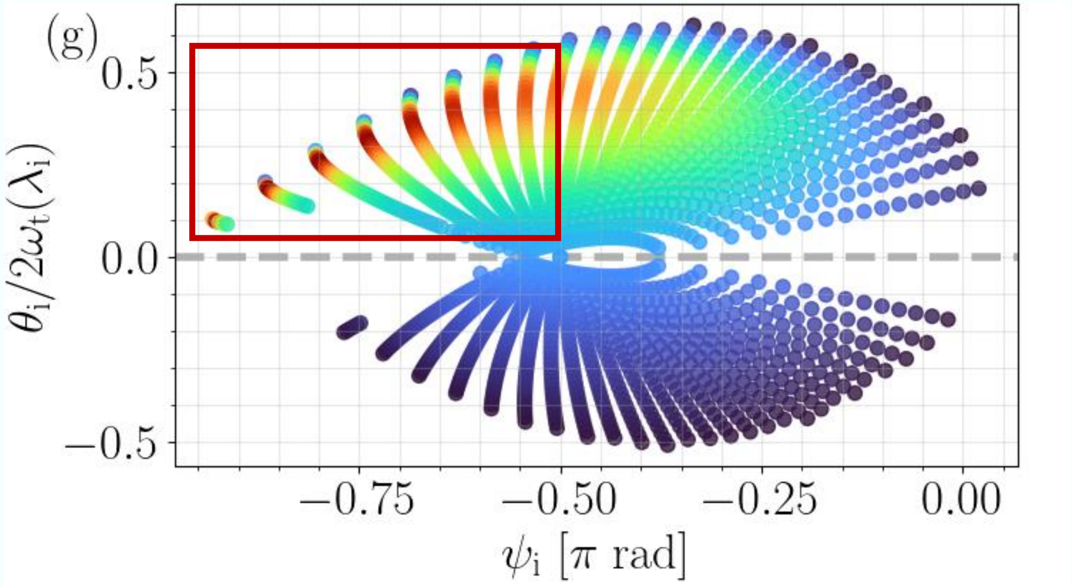
$\max K(\lambda_{iono}) \sim 10672$ eV

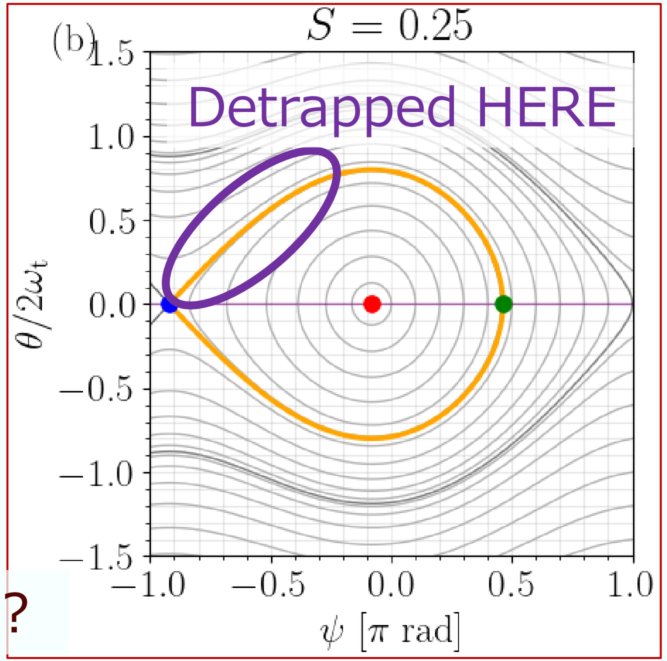
⇨ Necessary condition $\mu \lesssim 0.233$ [eV/nT]

3.2.2 Results: Detrapped point dependency $(\theta_\mathrm{i}, K_\mathrm{i}, S_\mathrm{i}, \lambda_\mathrm{i})$

We focus on the results of $\mu = 0.024$ [eV/nT].







Condition of $\max K(\lambda_\mathrm{iono})$

small S_i, $\psi_\mathrm{i} \sim \psi_\mathrm{ui}$, $K_\mathrm{i} \lesssim 2000$ eV, $\theta_\mathrm{i} > 0$, $\lambda_\mathrm{i} \in (10°, 20°)$

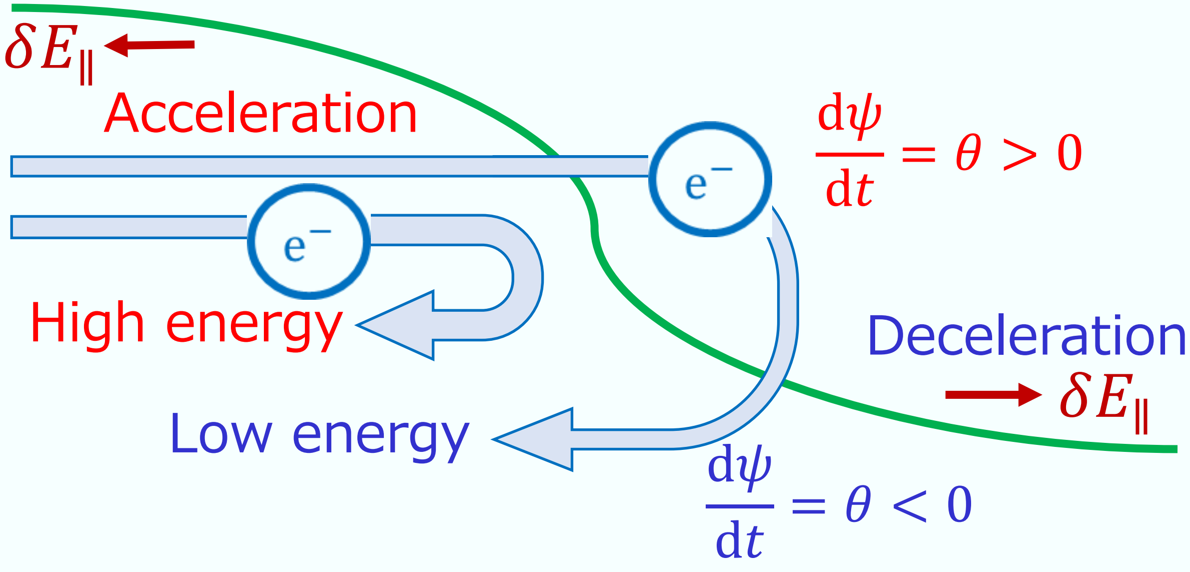
Why does $K(\lambda_\mathrm{iono})$ become large under the “just right” detrapped points?

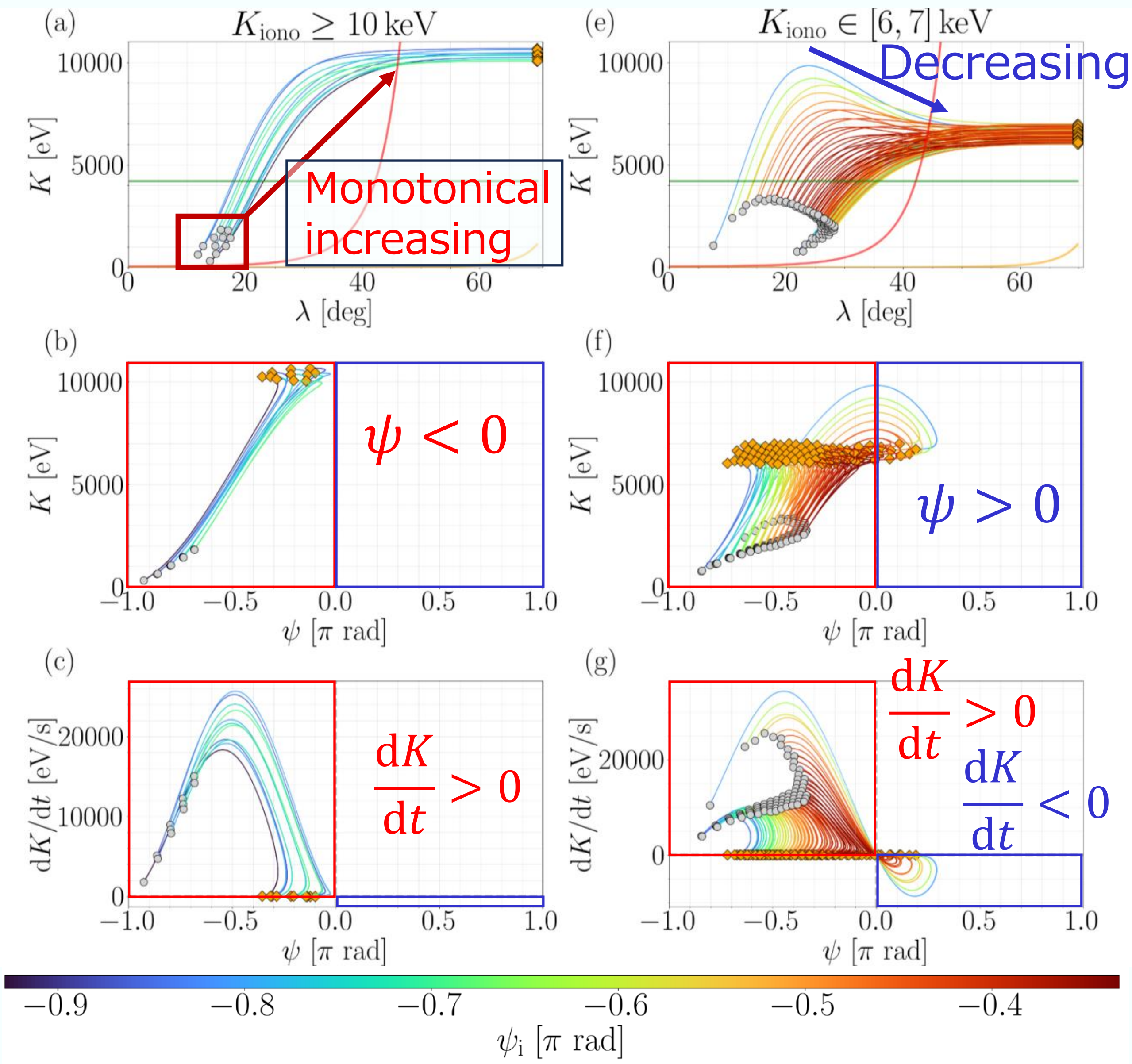
3.3 Results: Precipitating trajectories

Trajectories of electrons $\mu = 0.024$ [eV/nT]

$K(\lambda_{\mathrm{iono}}) \geq 10$ keV vs. $K(\lambda_{\mathrm{iono}}) \in [6,7]$ keV

$$\frac{\mathrm{d}K}{\mathrm{d}t} = v_\parallel \cdot (-e\delta E_\parallel) = -\sin\psi \cdot e\Phi_\mathrm{E}(k_\parallel v_\parallel)$$





When the electrons keep $\psi < 0$, $\frac{\mathrm{d}K}{\mathrm{d}t} > 0$, and $K(\lambda_{\mathrm{iono}})$ becomes maximum.

Question + Purpose

- Under what conditions are electrons accelerated and precipitated into the ionosphere?
- Under what conditions are electrons greatly accelerated?
- Using the second-order resonance theory, we clarify the characteristics of electrons precipitating into the ionosphere from a detailed investigation.

Approach

- We define the electron states using the second-order resonance theory.
- We investigate whether electrons at each detrapped point can reach the ionosphere using test particle simulations.

Results & Discussions

- $K(\lambda_{\mathrm{iono}})$ varies depending on the detrapped point.
- The maximum value of μ of electrons that can reach the ionosphere can be calculated from $K(\lambda_{\mathrm{iono}})$.
- A monochromatic KAW can result in a broadband energy spectrum of electrons.
- When the electrons keep $\psi < 0$, $\frac{\mathrm{d}K}{\mathrm{d}t} > 0$, and $K(\lambda_{\mathrm{iono}})$ becomes maximum.

References

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