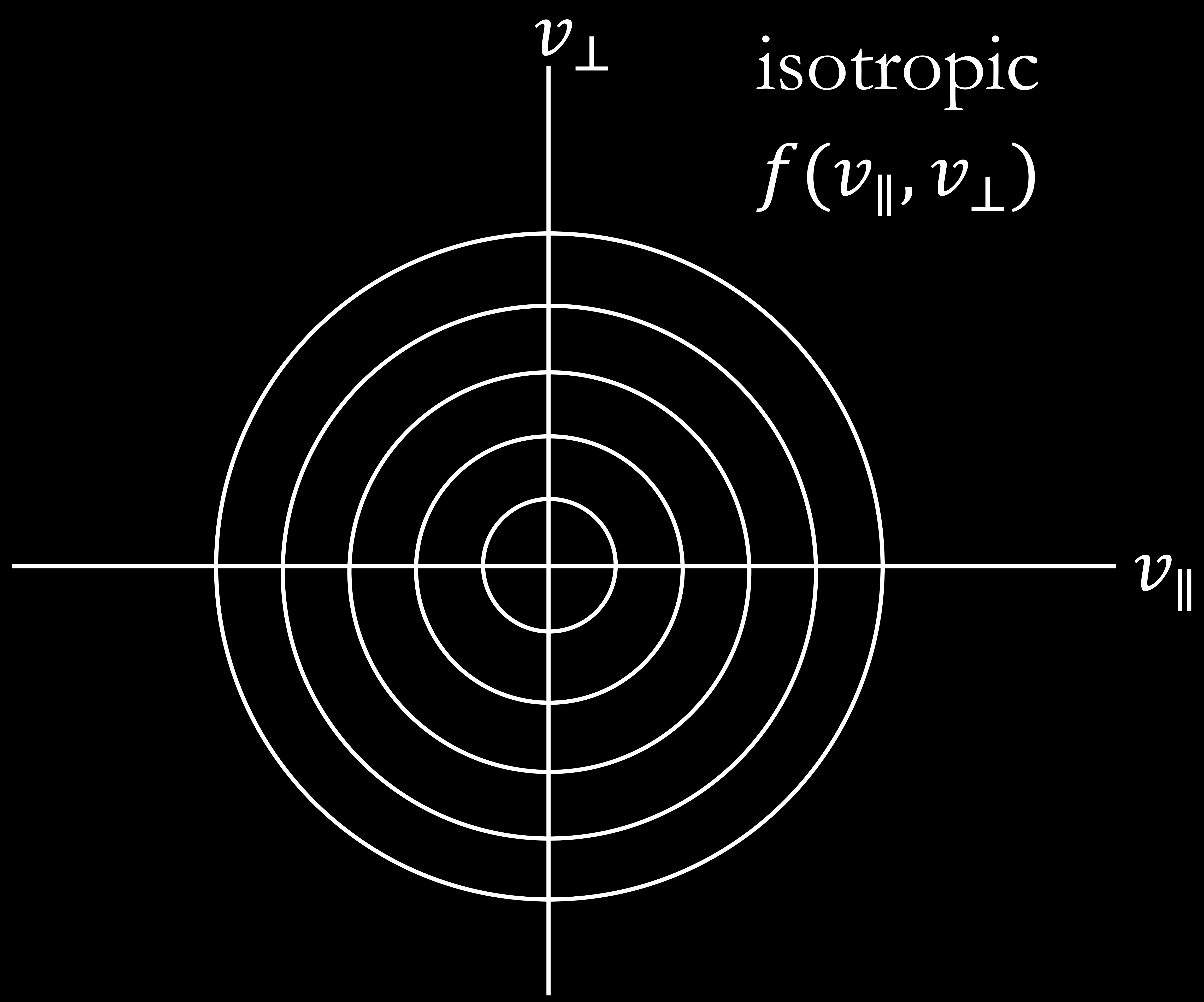
How non-equilibrium thermodynamics constrains magnetohydrodynamics in dilute astrophysical plasmas

Matthew Kunz
Princeton University

featuring work with J. Squire, A. Schekochihin, E. Quataert, Archie Bott, Muni Zhou, Stephen Majeski, Himawan Winarto, Lev Arzamasskiy, Denis St-Onge

with support from NSF CAREER and (past) NSF/DOE Partnership

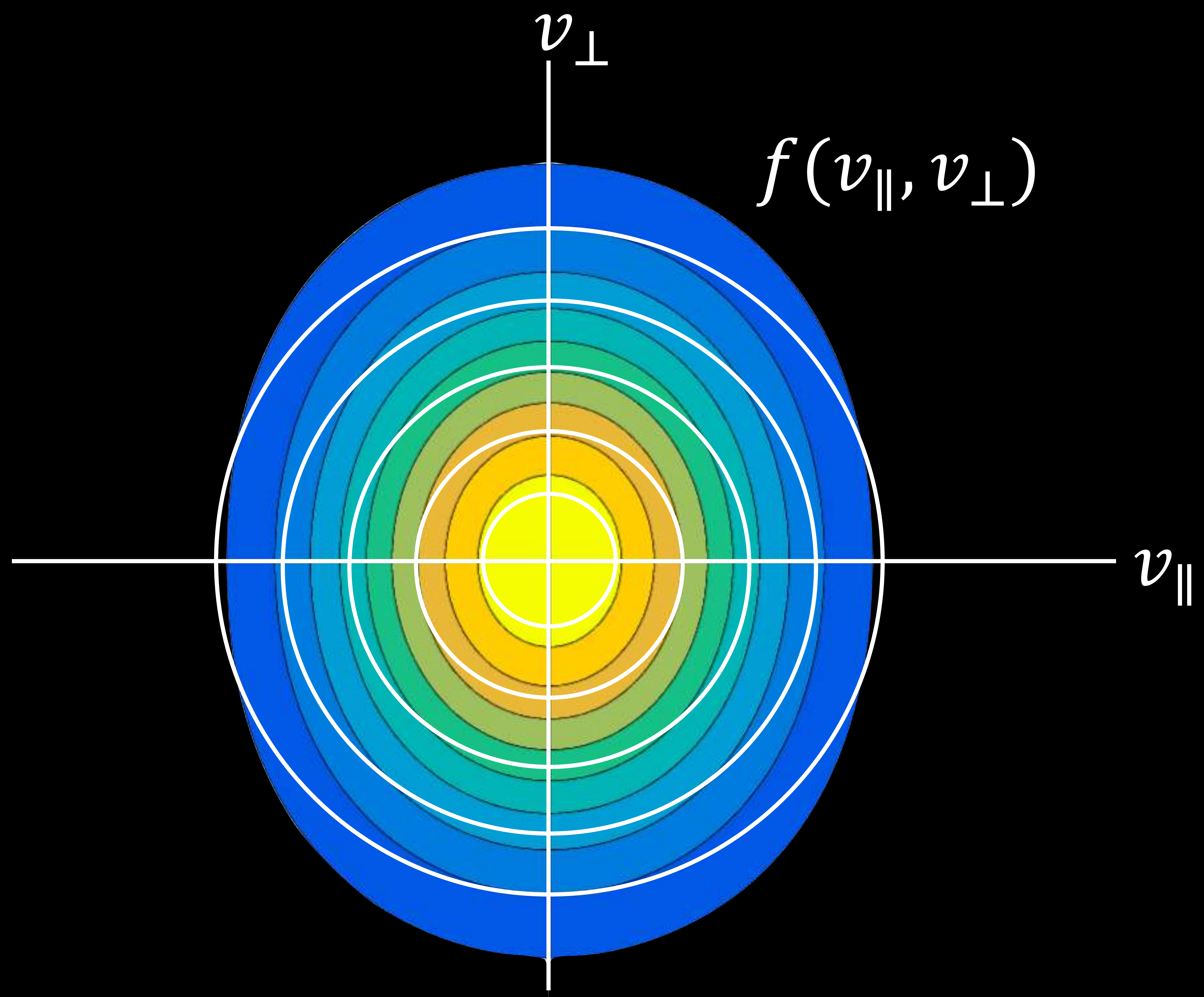
For a broad class of astrophysical and space plasmas,
departures from local thermodynamic equilibrium
are allowed by their low collisionalities,
are shaped by their magnetic fields, and
are large enough to be dynamically important.



isotropic

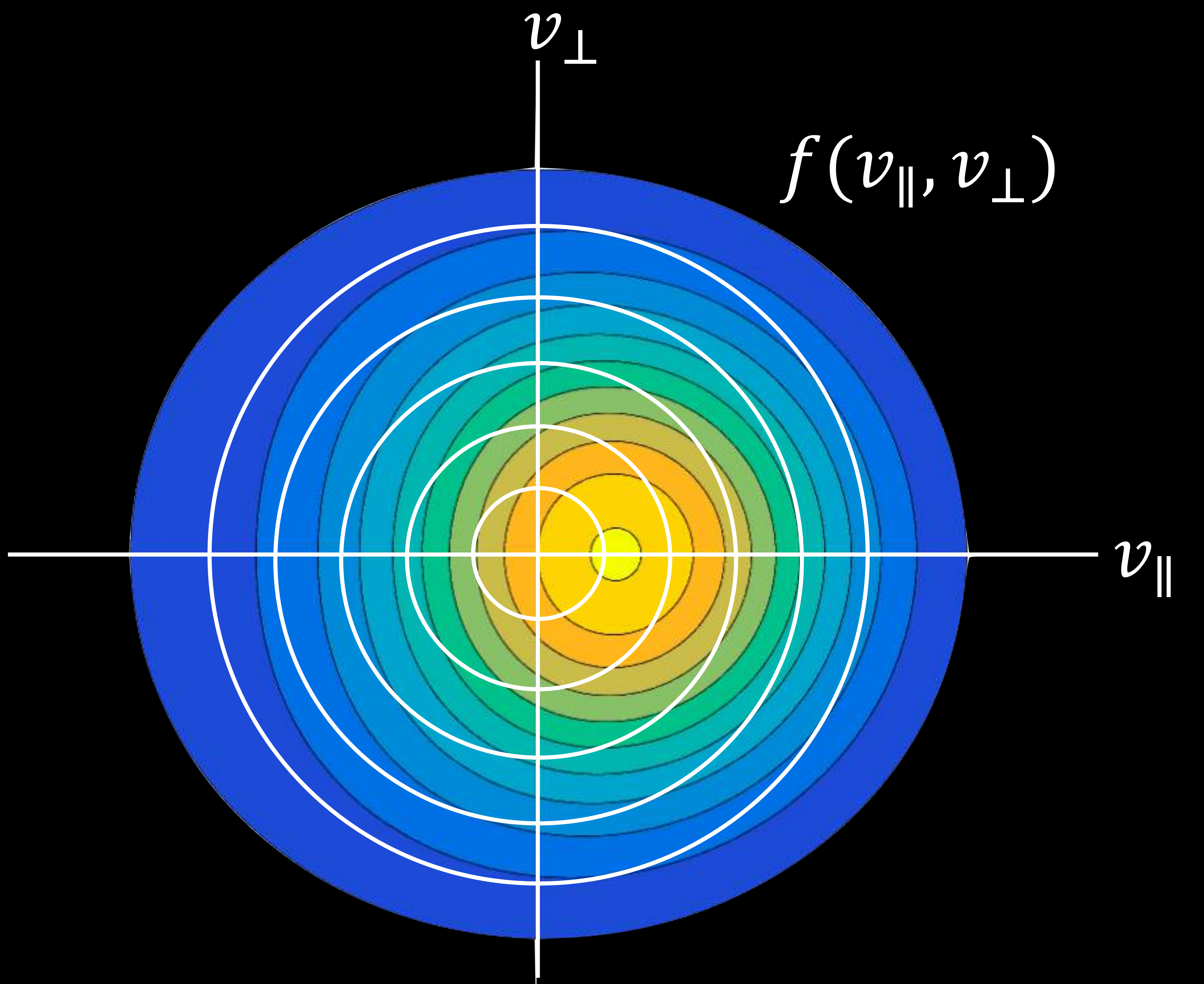
$f(v_\parallel, v_\perp)$

pressure anisotropy = viscosity

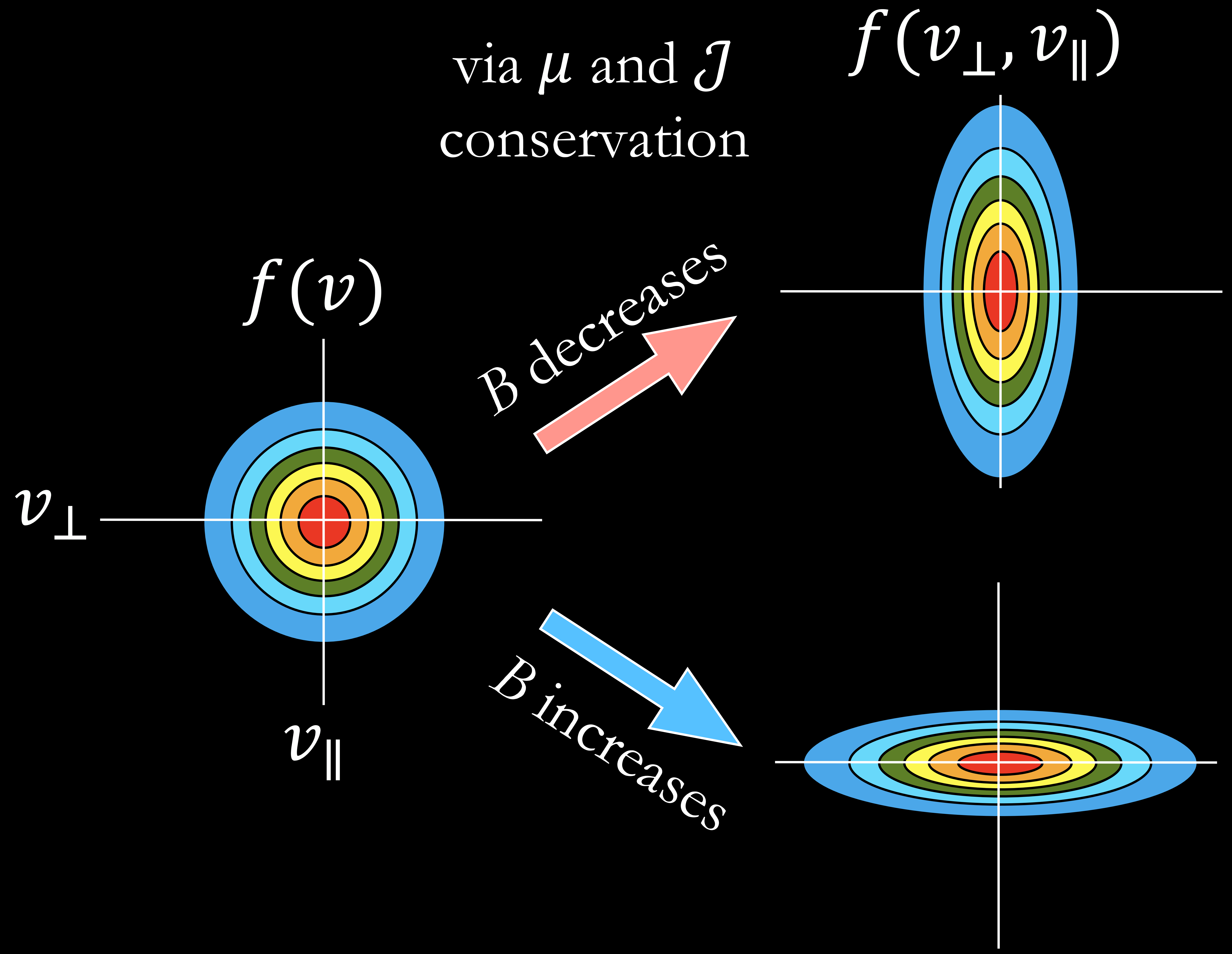


$$f_{\mathrm{M}}\left[1-\lambda_{\mathrm{mfp}}\left(\frac{2v_\parallel^2 - v_\perp^2}{v_{\mathrm{th}}^2}\right)\left(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}-\frac{\mathsf{I}}{3}\right):\boldsymbol{\nabla}\frac{\boldsymbol{u}}{v_{\mathrm{th}}}\right]$$

skewness = heat conduction



$$f_{\mathrm{M}}\left[1-\lambda_{\mathrm{mfp}}\frac{v_\parallel}{v_{\mathrm{th}}}\left(\frac{v^2}{v_{\mathrm{th}}^2}-\frac{5}{2}\right)\hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla}\ln T\right]$$



pressure anisotropy
can be driven **globally**
(e.g., expansion of solar wind,
compression during accretion,
differential rotation in disks)

or

can be generated ***in situ***
by **fluctuations**
(e.g., waves, turbulence,
contracting current sheets)

e.g., $p_\perp \neq p_\parallel$ in solar wind

at $r \sim 1$ au...

$n \sim 10\ \mathrm{cm}^{-3}$

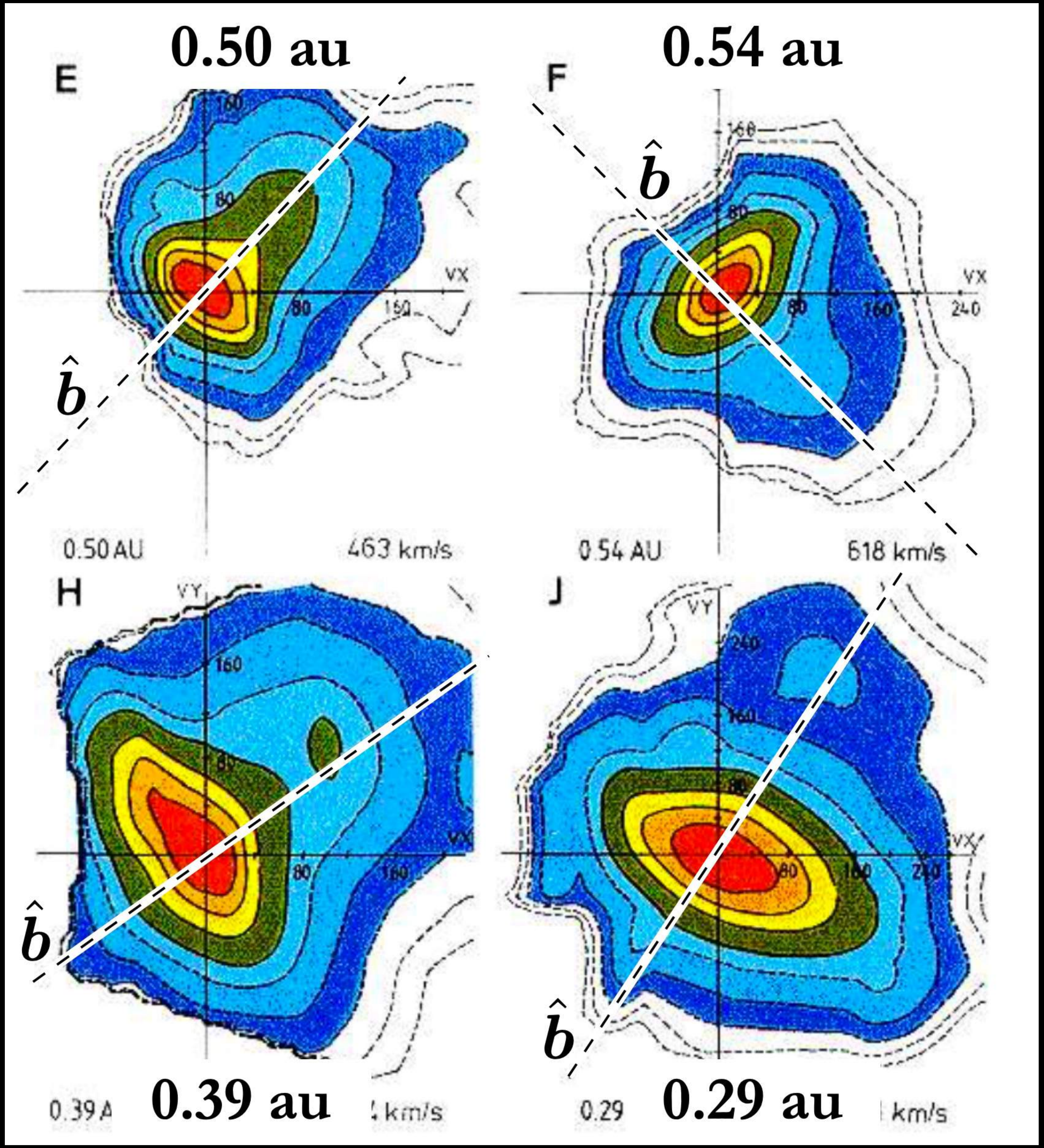
$T \sim 10$ eV

$B \sim 50\ \mu\mathrm{G}$

$\lambda_{\mathrm{mfp}} \sim 1$ au

$\rho_{\mathrm{i}} \sim 10^{-6}$ au

$\Omega_{\mathrm{i}} \sim 1\ \mathrm{s}^{-1}$



Marsch (2006)

e.g., $p_\perp \neq p_\parallel$ in solar wind

at $r \sim 1$ au…

$$n \sim 10 \text{ cm}^{-3}$$

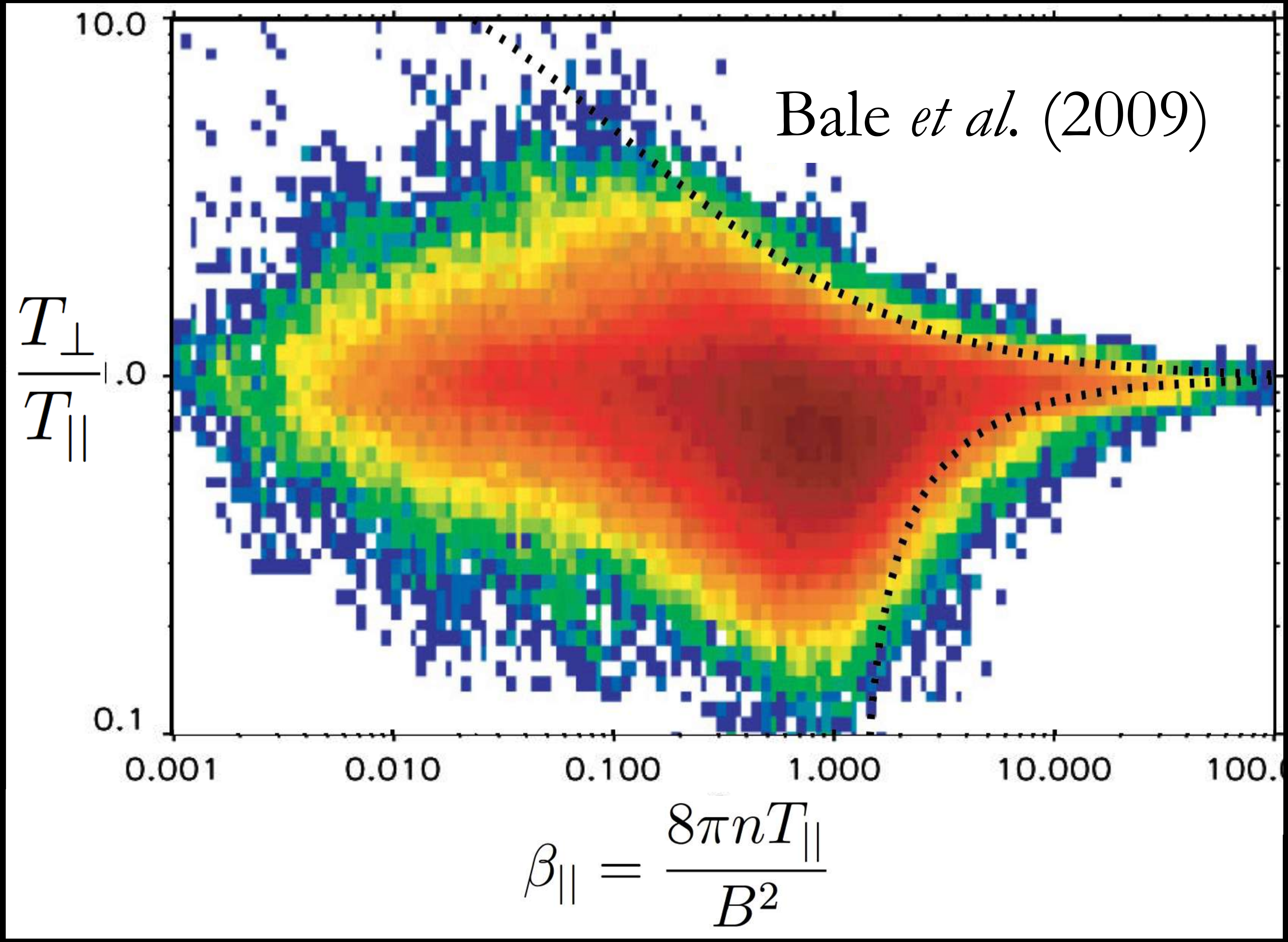
$$T \sim 10 \text{ eV}$$

$$B \sim 50 \ \mu\text{G}$$

$$\lambda_{\text{mfp}} \sim 1 \text{ au}$$

$$\rho_{\text{i}} \sim 10^{-6} \text{ au}$$

$$\Omega_{\text{i}} \sim 1 \text{ s}^{-1}$$



For a broad class of astrophysical and space plasmas,
departures from local thermodynamic equilibrium
are allowed by their low collisionalities,
are shaped by their magnetic fields, and
are large enough to be dynamically important.

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{p_\perp - p_\parallel}{p} \sim \frac{\mathrm{d}}{\mathrm{d}t}\ln\frac{B^3}{n^2} - \nu\frac{p_\perp - p_\parallel}{p}$$

adiabatic invariance

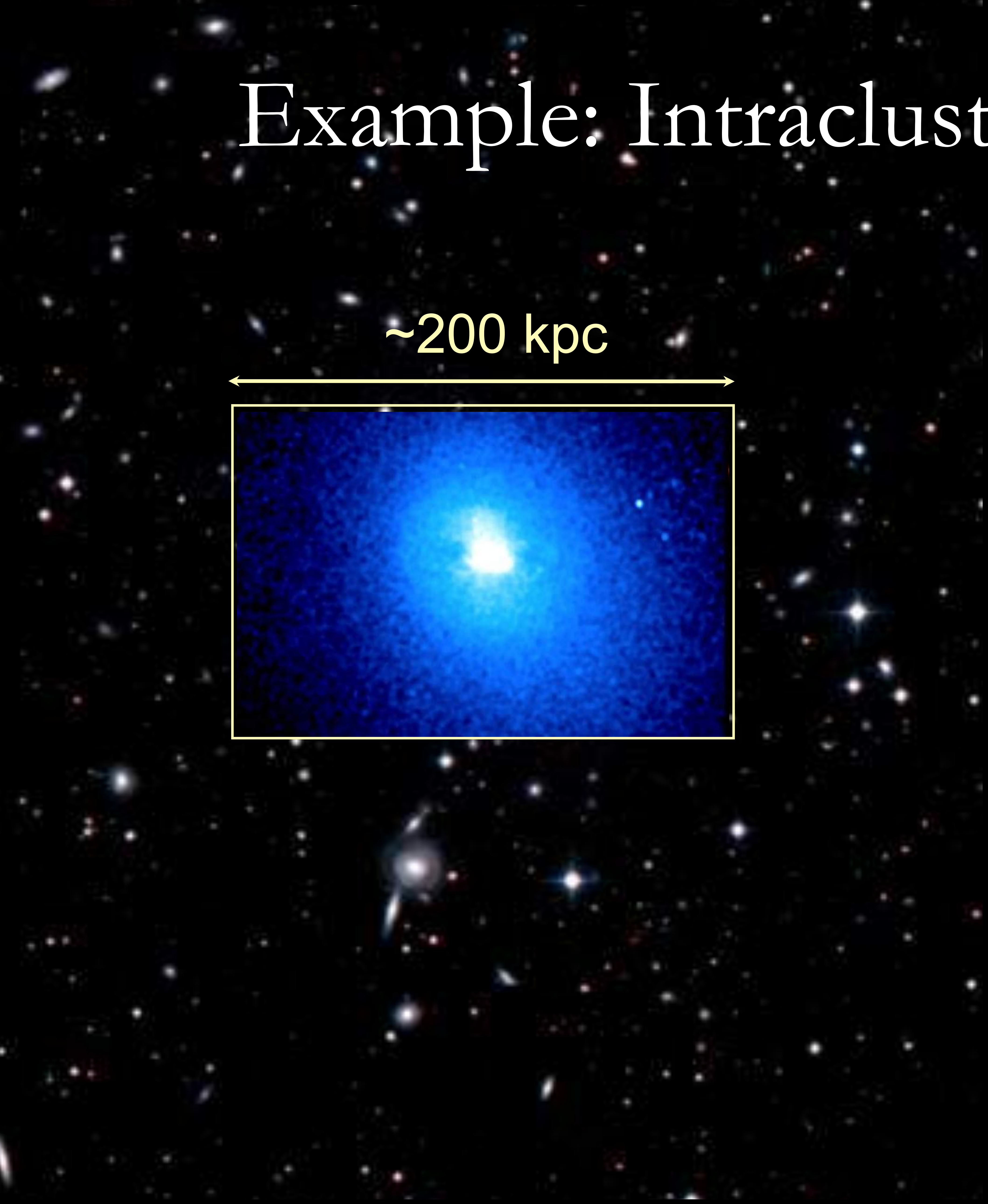
collisional regulation

$$mn\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\boldsymbol{\nabla}\left(p_\perp + \frac{B^2}{8\pi}\right) + \boldsymbol{\nabla}\cdot\left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\frac{B^2}{4\pi} + p_\perp - p_\parallel\right)\right]$$

competes with magnetic tension:

$$\Delta \doteq \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\beta}\,?$$

Example: Intracluster Medium of Galaxy Clusters



$\sim 10^{14-15}\ \mathrm{M}_\odot$ in ~ 1 Mpc

$\sim 10^{2-3}$ galaxies

ICM:

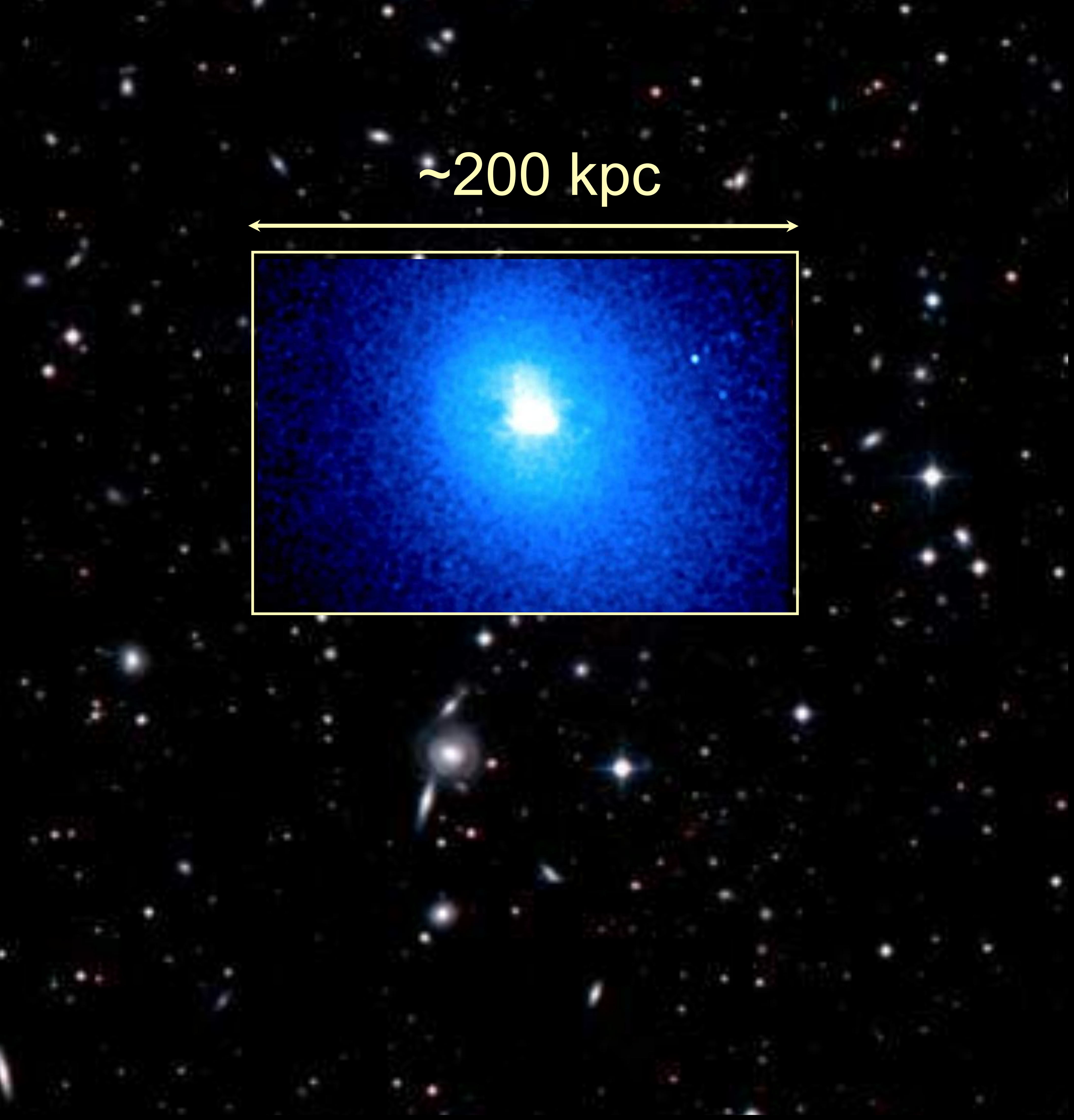
14% of thermal plasma

$T \sim 1 - 10$ keV

$n \sim 10^{-4} - 10^{-1}\ \mathrm{cm}^{-3}$

$B \sim 0.1 - 10\ \mu\mathrm{G}$

Example: Intracluster Medium of Galaxy Clusters



$\sim 10^{14-15}\ \mathrm{M}_\odot$ in ~ 1 Mpc

$\sim 10^{2-3}$ galaxies

ICM:

$\beta \sim 10^2 - 10^4$

$L \gtrsim 100$ kpc

$\lambda_{\mathrm{mfp}} \sim 1 - 10$ kpc

$\rho_{\mathrm{i}} \sim 1$ npc

(size of Jupiter)

$L \sim 100$ kpc

$\lambda_{\rm mfp} \sim$ few kpc

$\rho_{\rm i} \sim 1$ npc

$+$

$$\frac{U}{v_{\rm th,i}} \sim 0.1 - 0.3$$

Braginskii estimate of pressure anisotropy:

$$\frac{p_\perp - p_\parallel}{p} \sim \frac{u}{v_{\rm th,i}} \frac{\lambda_{\rm mfp,c}}{L} \sim 10^{-2}$$

compare to

$$\frac{1}{\beta} \sim 10^{-2}$$

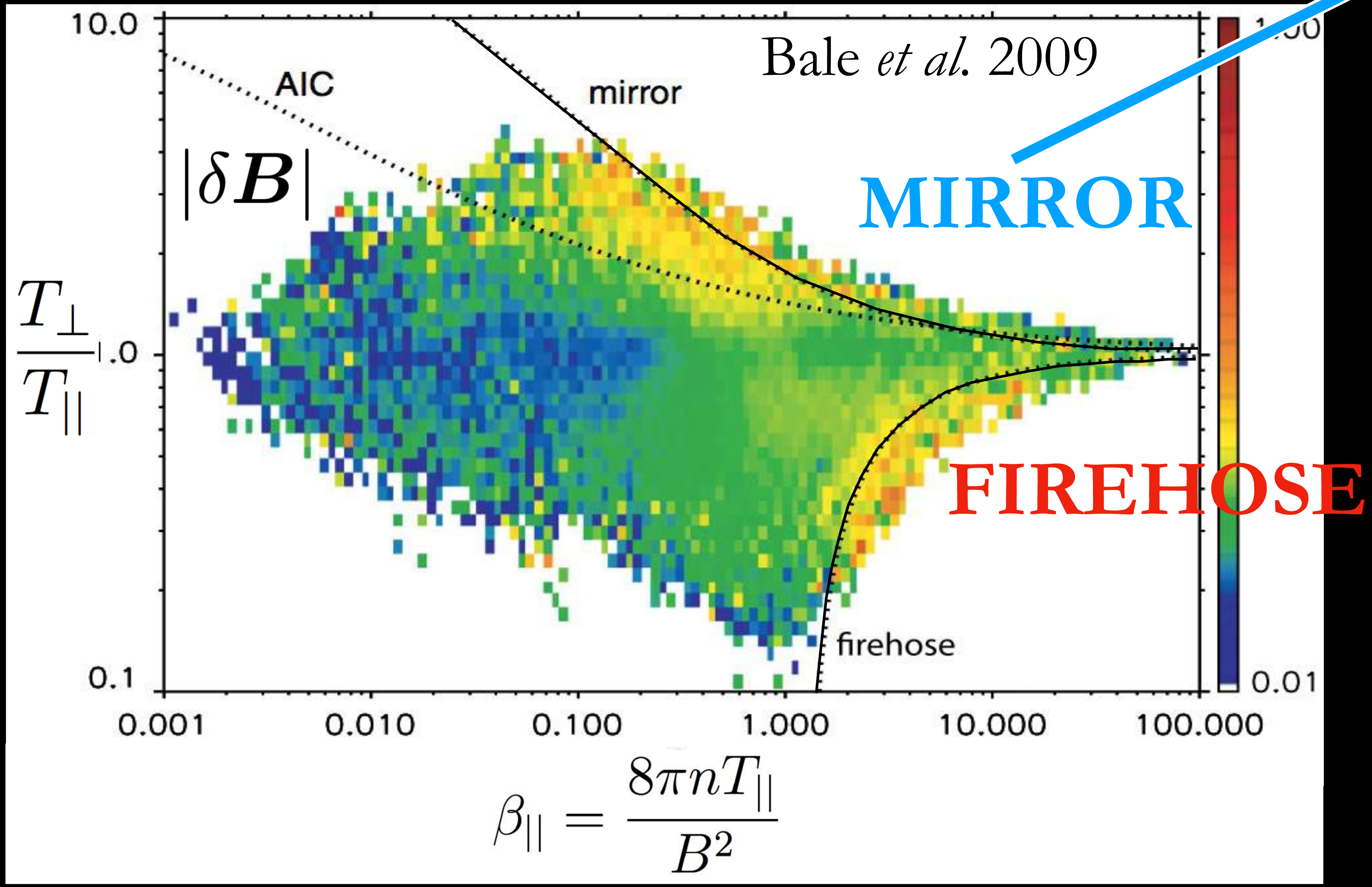
KE, ME, Δp all in approx. equipartition

(i) pressure anisotropy is ***dynamically important***

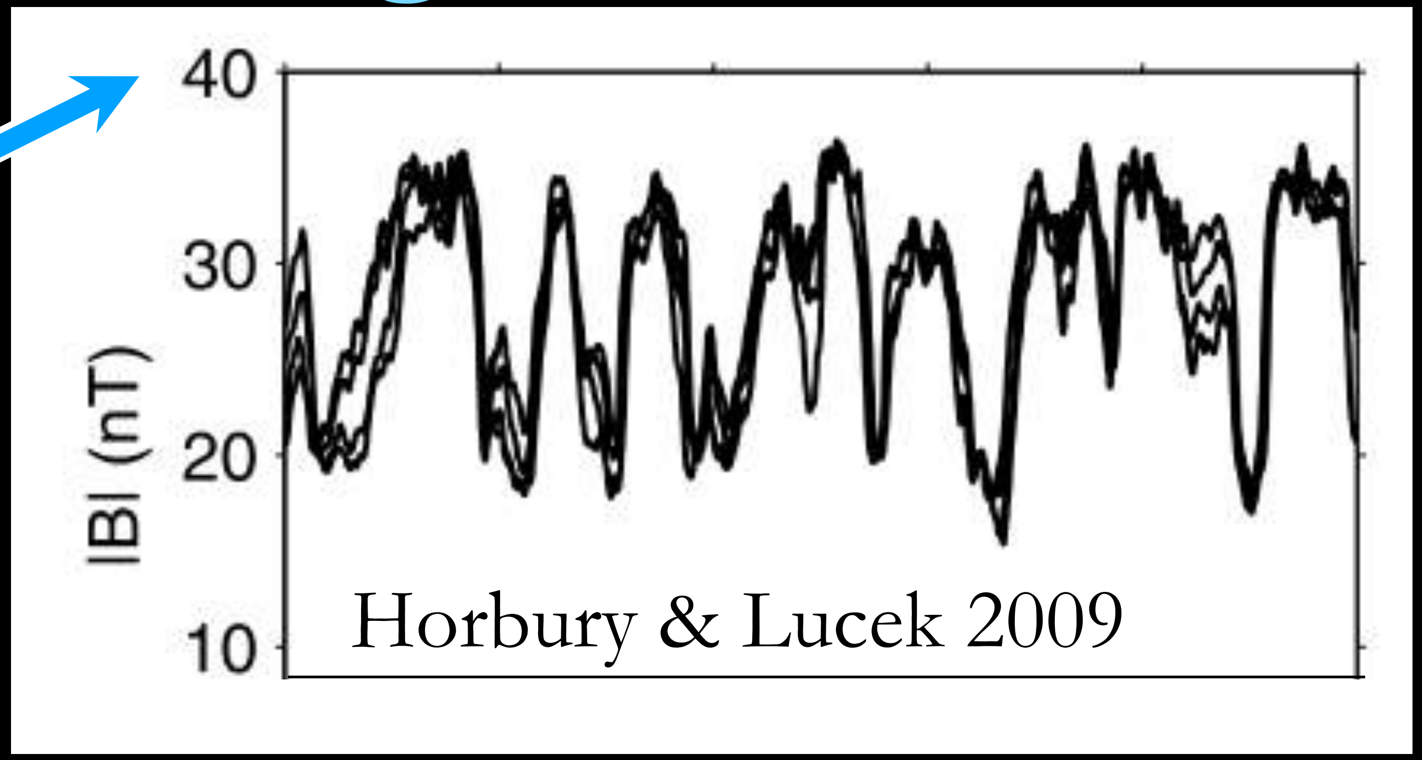
(ii) bulk ICM likely ***polluted with microscale B fluctuations***, which scatter ions and thereby provide an anomalous ***effective collisionality***

we've shown $\nu_{\rm eff} \sim \beta/\tau_{\rm dyn}$ many times now from tracked particles (e.g., Bott+ 2024)

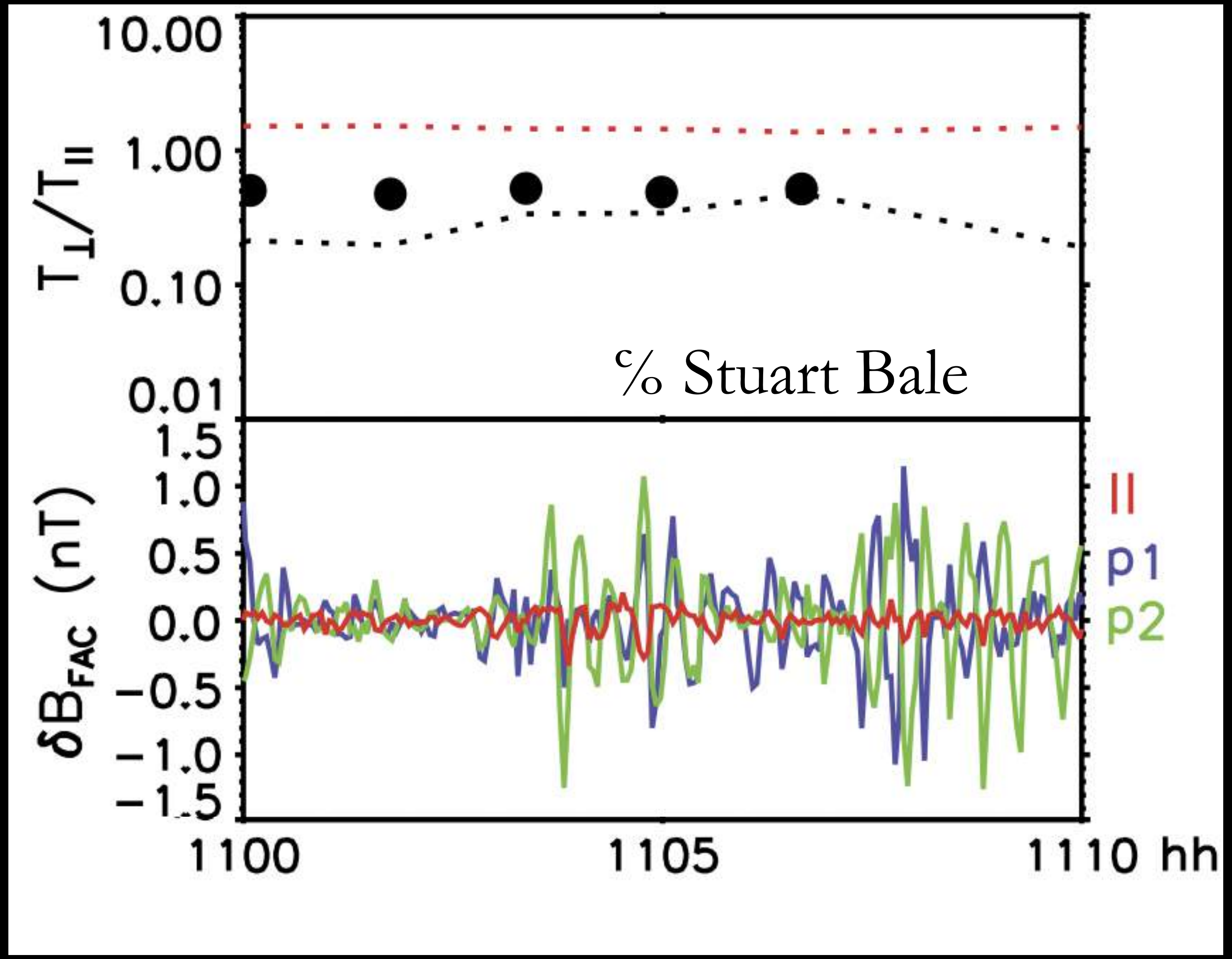
$p_\perp \neq p_\parallel$ regulated in solar wind



magnetic mirrors



firehose fluctuations



also Gary *et al.* 2001, Kasper *et al.* 2002, Hellinger *et al.* 2006, Matteini *et al.* 2007, Chen *et al.* 2016…

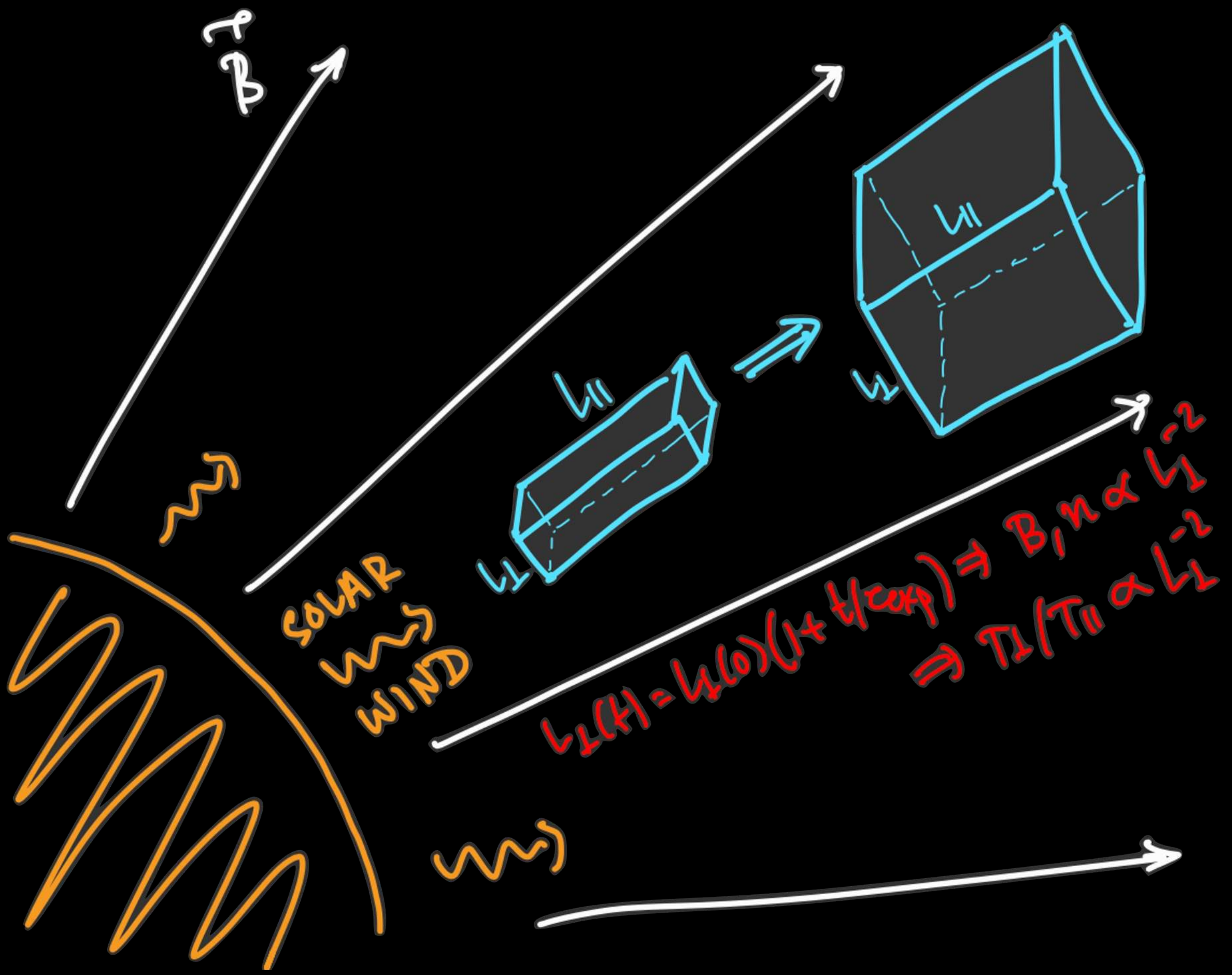
two ways to regulate departures from LTE (if Coulomb collisions aren't enough):

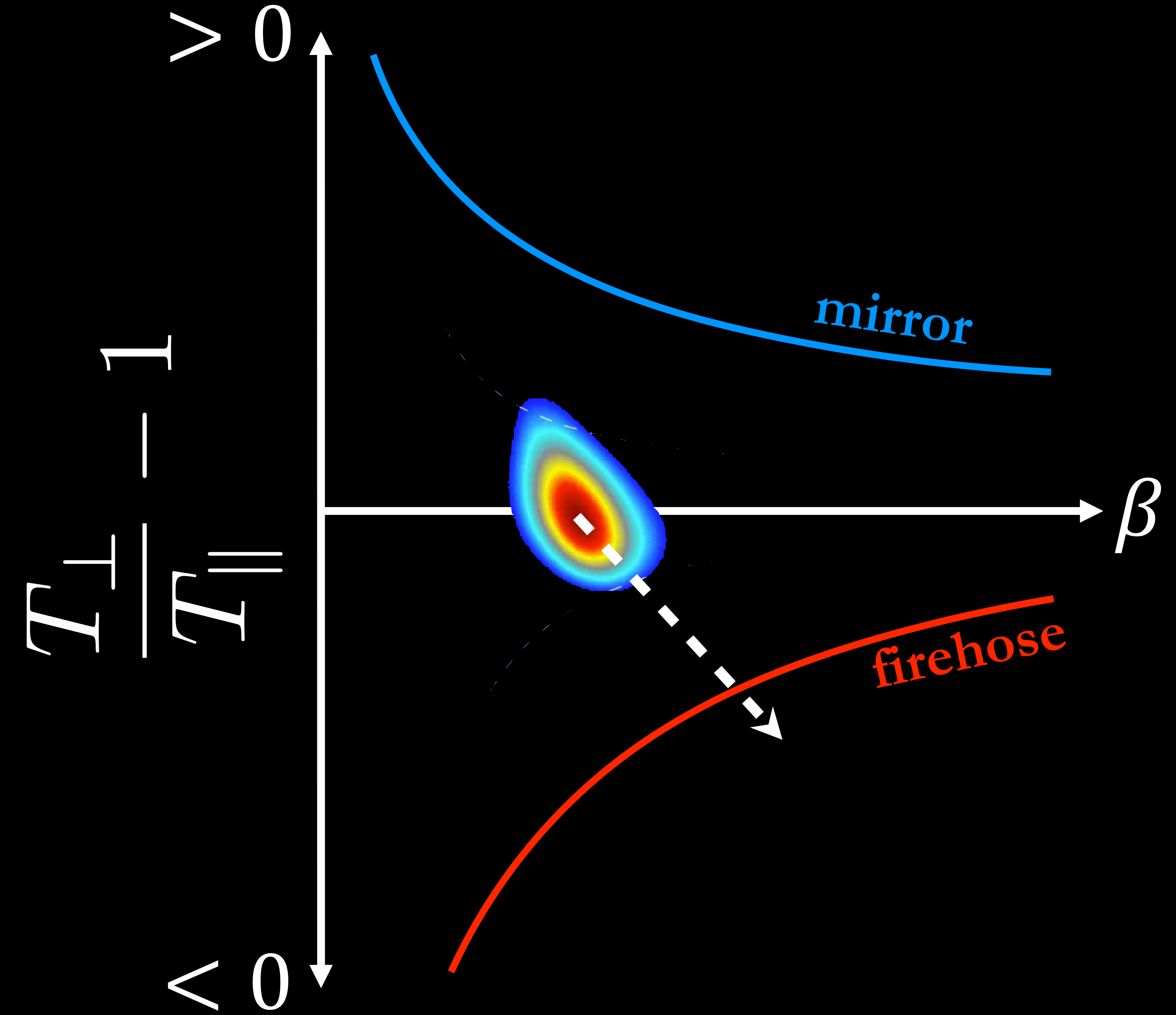
1. **particle scattering** off of Larmor-scale instabilities (e.g., firehose, mirror, whistler) at rate $\nu_{\rm eff} \sim \beta/\tau_{\rm dyn}$

dominant means of regulation in systems where departures from LTE are driven or imposed globally, i.e., are non-negotiable

this regulation directly links microscales and macroscales

Example: pressure anisotropy driven by global expansion





Bott, Arzamasskiy, Kunz, Quataert (2022, *ApJL*), using Pegasus++

many relevant studies on this problem (e.g. Matthaeus 1990; Grappin 1993; Velli 1997; Hellinger & Matsumoto 2001; Matteini *et al.* 2006; Hellinger & Travnıcek 2008; Matteini *et al.* 2013; Hellinger *et al.* 2015, 2017, 2019; Hellinger 2017; Innocenti 2019)

$L_\parallel / L_\perp(\tau_{\rm exp}) = 3$

$\beta_i(\tau_{\rm exp}) \approx 4$

$L_\parallel / L_\perp(0) = 6$

$\beta_i(0) \approx 2$

time

$256^2 \times 1536$ $\quad \tau_{\rm exp} = 10 L_\parallel / v_{\rm A}(0)$

Bott, Arzamasskiy, Kunz, Quataert (2022, *ApJL*) things to notice during expansion:

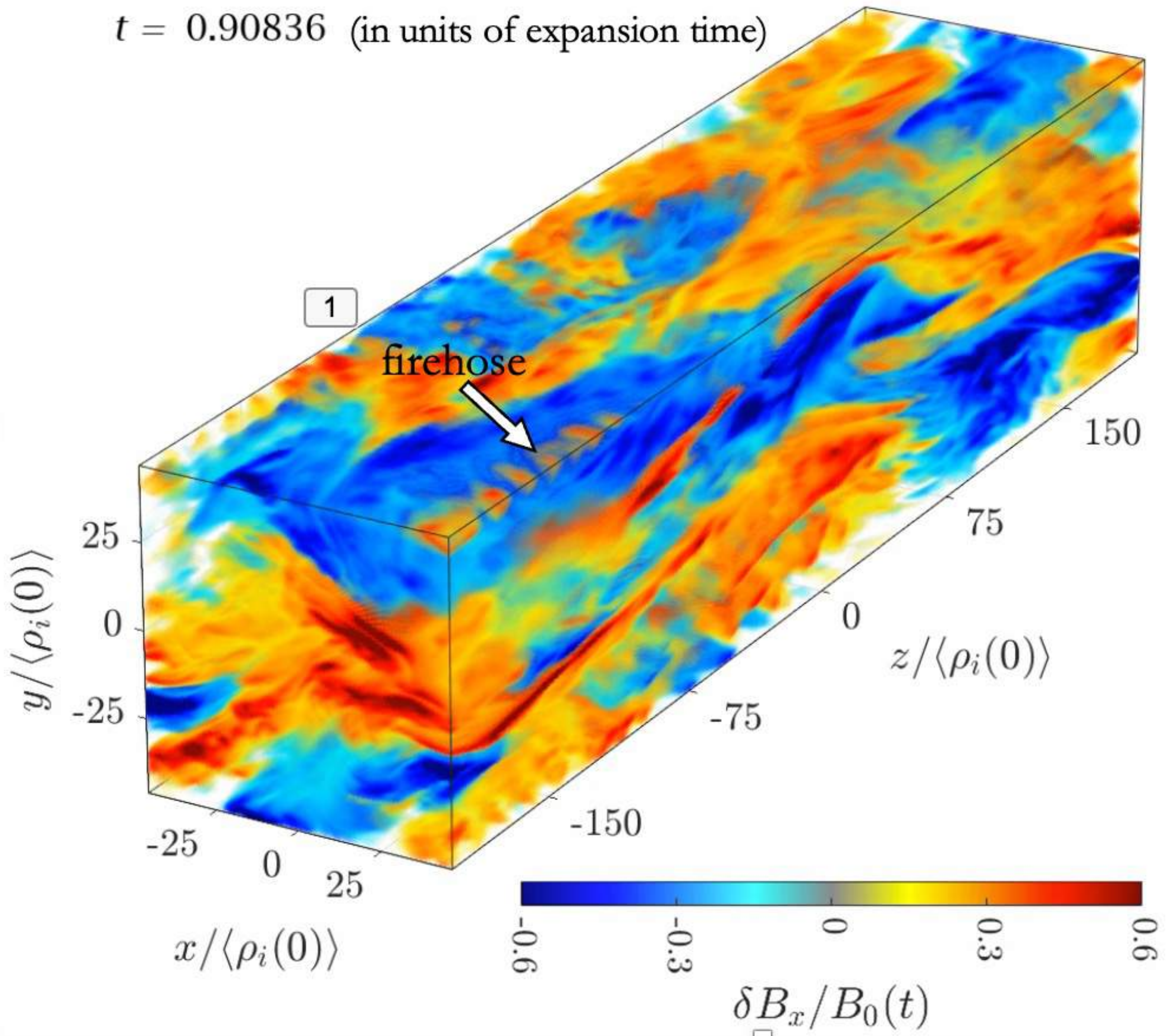
– fluctuations oscillate slower, as effective Alfvén speed drops

$$v_{\mathrm{A,eff}} = v_{\mathrm{A}} \sqrt{1 - \left(\frac{p_{\parallel} - p_{\perp}}{B^2/4\pi} \right)}$$

– $\delta B/B_0(t)$ gets larger, because it costs less energy to bend field

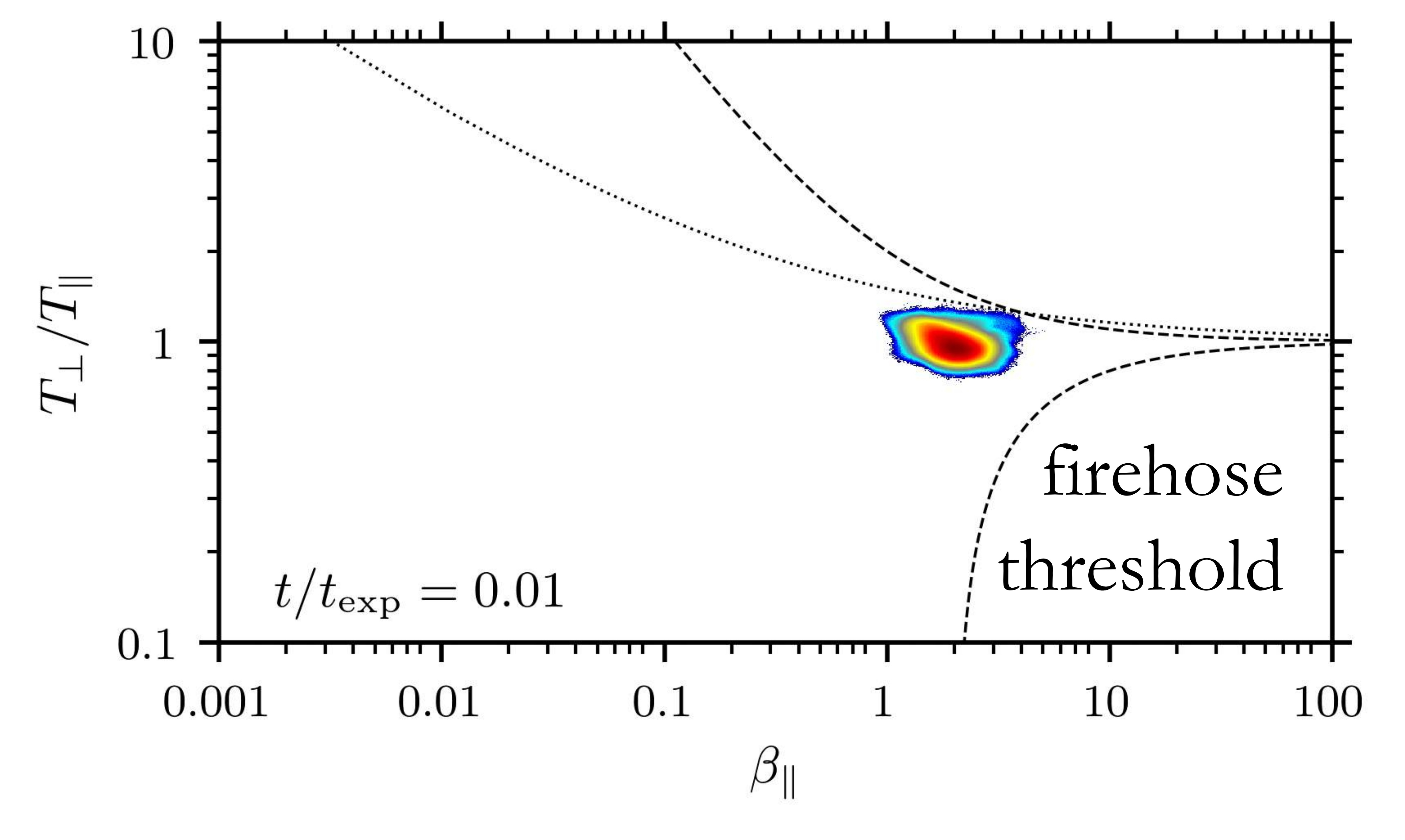
– burst of firehose fluctuations appears around $t \approx 0.9$

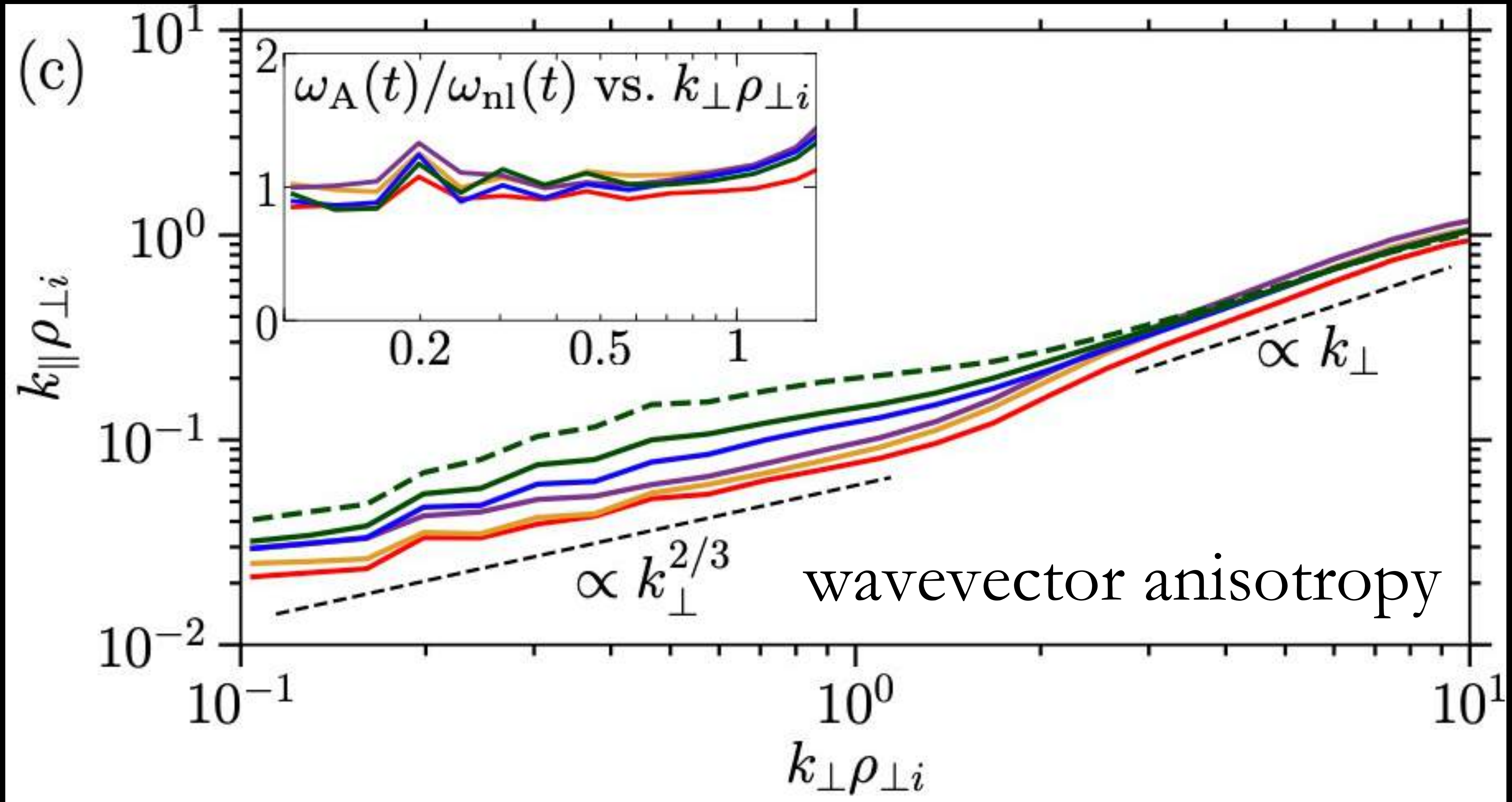
using Pegasus++



pressure anisotropy well regulated by kinetic firehose instability at $\Delta_\mathrm{i} \approx -1.4/\beta_{\mathrm{i}\|}$ via effective collisionality $\nu \sim \beta/\tau_{\mathrm{exp}}$

cuts effective tension of field lines in ~half

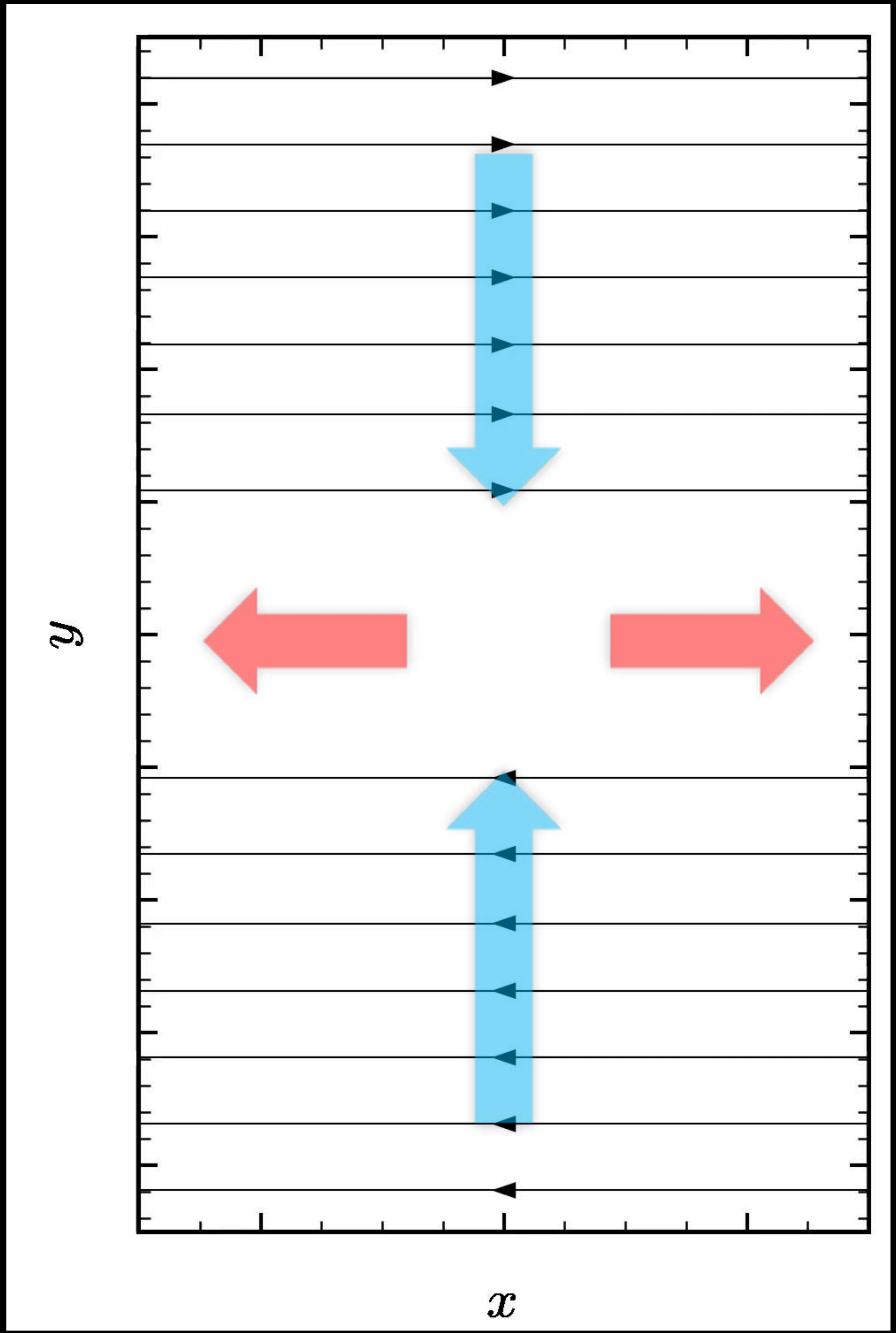




turbulence nonlinearly adapts to decreasing linear wave speed to remain in critical balance with $\omega_{\mathrm{lin}} \sim \omega_{\mathrm{nl}}$ throughout inertial range

microscale control of thermodynamics forces macroscale dynamics to adjust

Example: pressure anisotropy driven by forming current sheet



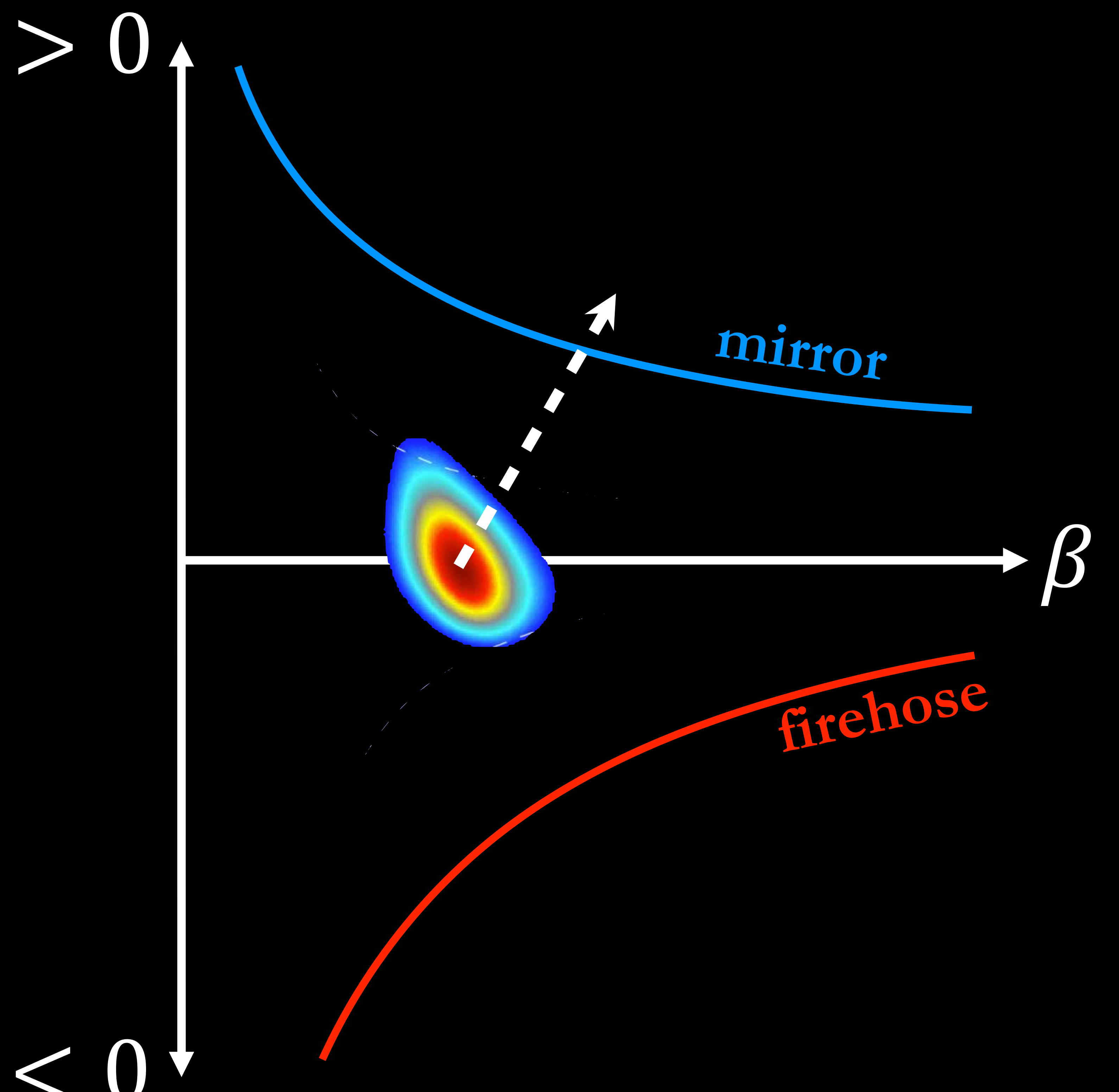
Consider a thinning current sheet in a collisionless, magnetized plasma

B will increase in inflowing fluid elements, driving $P_\perp > P_\parallel$

mirrors will rapidly grow and saturate above ion-Larmor scales,

changing $\Delta'(k)$

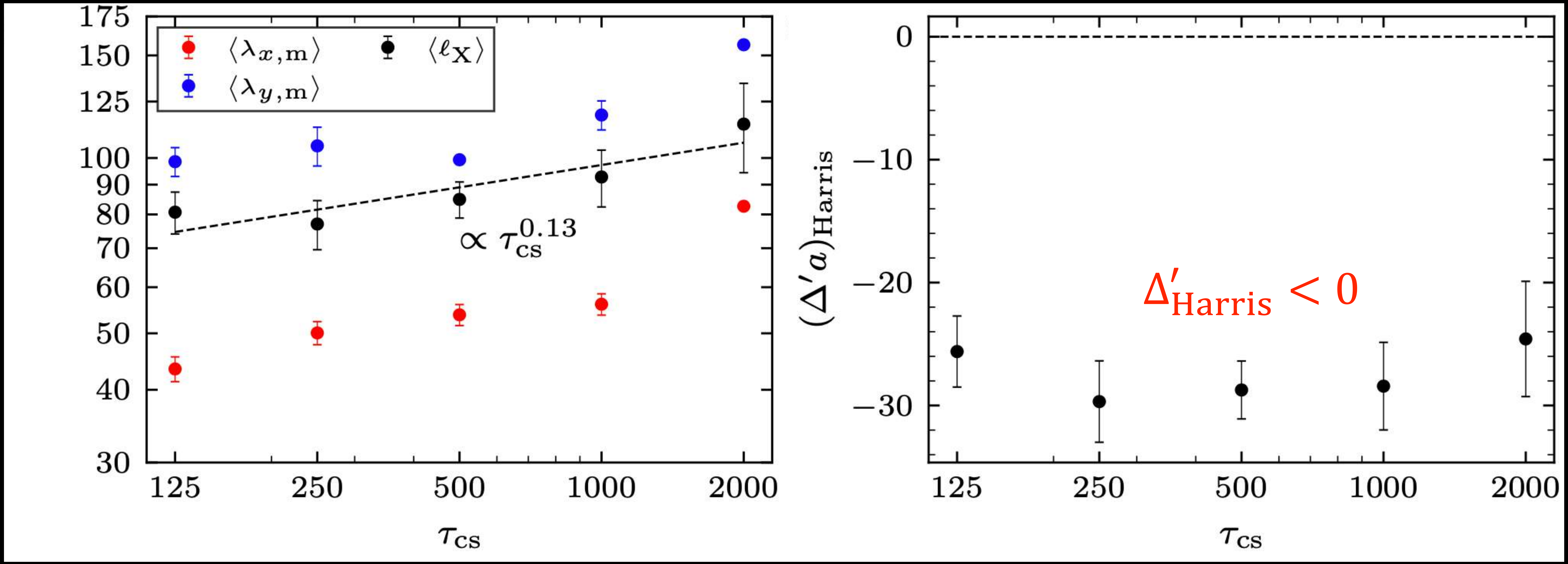
and triggering tearing



Alt & Kunz (2019)
Winarto & Kunz (2022)

triggering tearing in a forming current sheet with the mirror instability
(from Pegasus++ simulation; Winarto & Kunz 2022)

can compute characteristic X-point separation ℓ_X from simulation, and show that the **equivalent Harris-sheet profile would be stable to tearing**



provides a natural explanation for results from a recent laser–plasma experiment of driven reconnection with collisionless ions, which indicated an earlier onset of tearing having larger growth rates and significantly smaller scales than anticipated (W. Fox+, arXiv) two ways to regulate departures from LTE (if Coulomb collisions aren't enough):

1. **particle scattering** off of Larmor-scale instabilities (e.g., firehose, mirror, whistler) at rate $\nu_{\rm eff} \sim \beta/\tau_{\rm dyn}$

   dominant means of regulation in systems where departures from LTE are driven or imposed globally, i.e., are non-negotiable

2. **self-organization** of macroscale fluctuations, e.g., to avoid producing p. anisotropy in the first place

"Magneto-immutability" – self-organization to avoid Δp stress

(Squire *et al.* 2019; Kempski *et al.* 2019; Arzamasskiy, Kunz, Squire *et al.* 2022; Squire, Kunz *et al.* 2023; Majeski, Kunz & Squire 2024 ← develops theory)

recall the force equation:

$$mn\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\boldsymbol{\nabla}\left(p_\perp + \frac{B^2}{8\pi}\right) + \boldsymbol{\nabla}\cdot\left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\frac{B^2}{4\pi} + p_\perp - p_\parallel\right)\right]$$

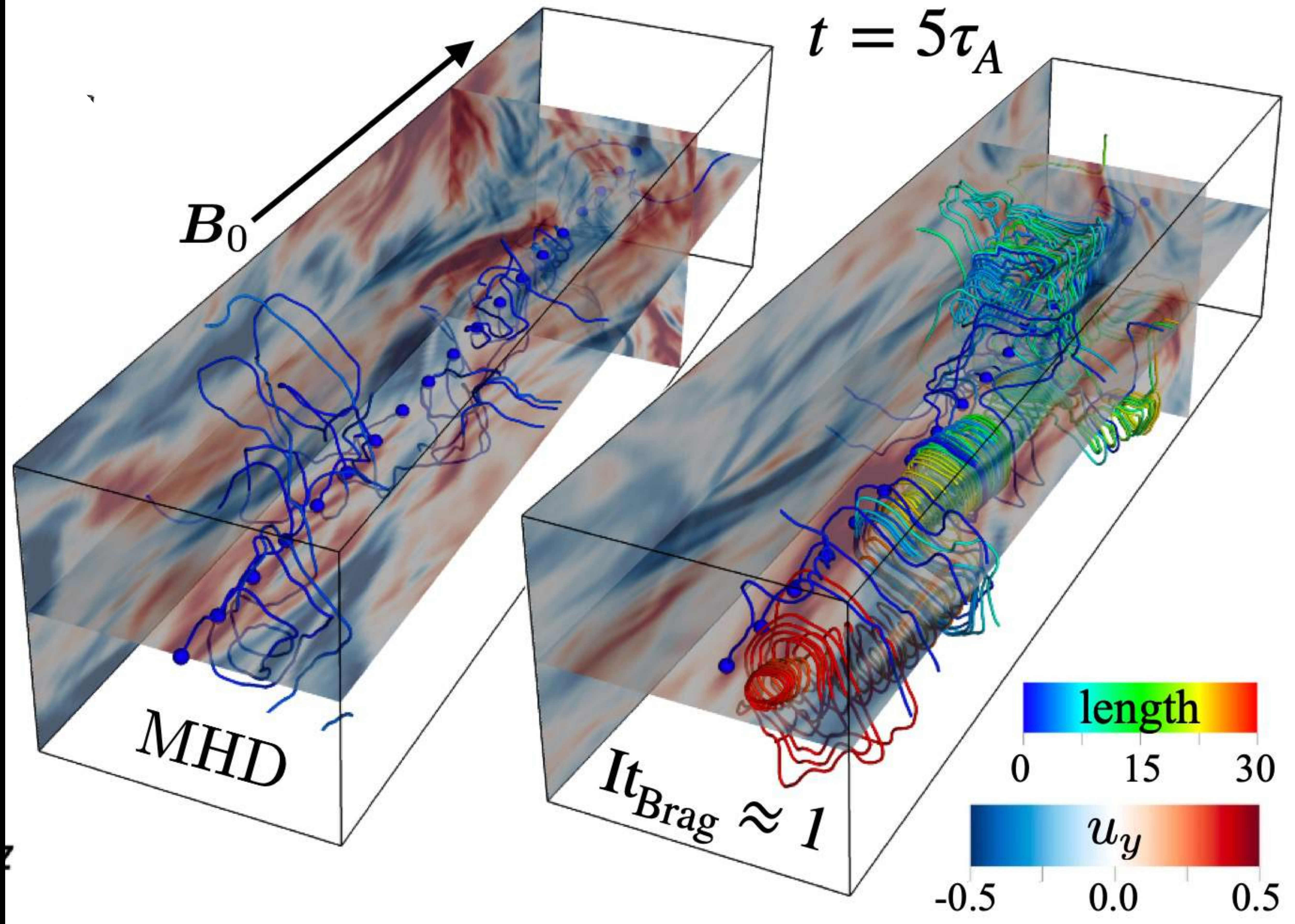
"Magneto-immutability" – self-organization to avoid Δp stress

(Squire *et al.* 2019; Kempski *et al.* 2019; Arzamasskiy, Kunz, Squire *et al.* 2022; Squire, Kunz *et al.* 2023; Majeski, Kunz & Squire 2024 ← develops theory)

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{p_\perp - p_\parallel}{p} \sim \frac{\mathrm{d}\ln B}{\mathrm{d}t} \sim \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\nabla\boldsymbol{u}$$

control Δp by adjusting this angle to minimize changes in B

velocity streamlines from Braginskii turb sim:

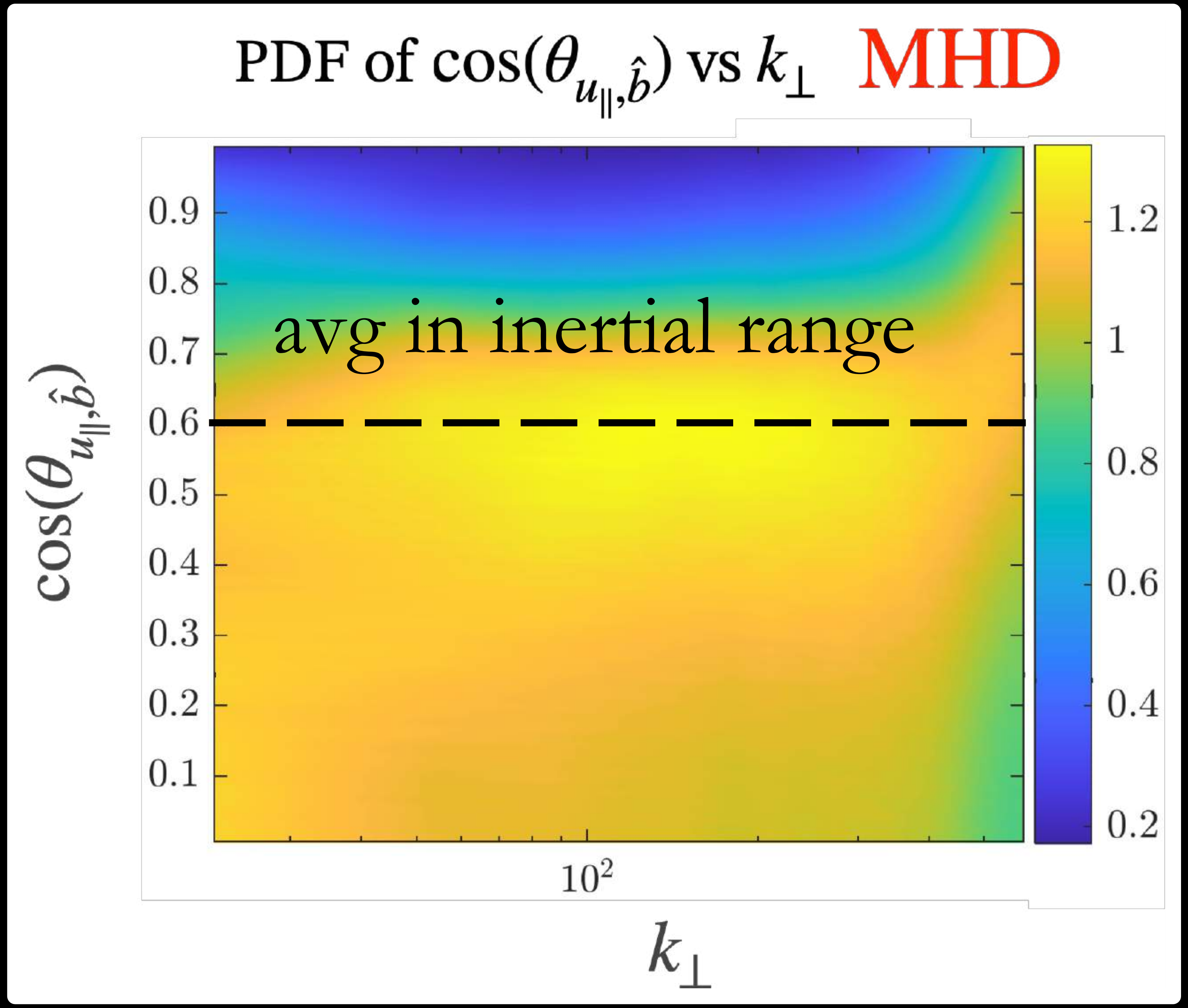


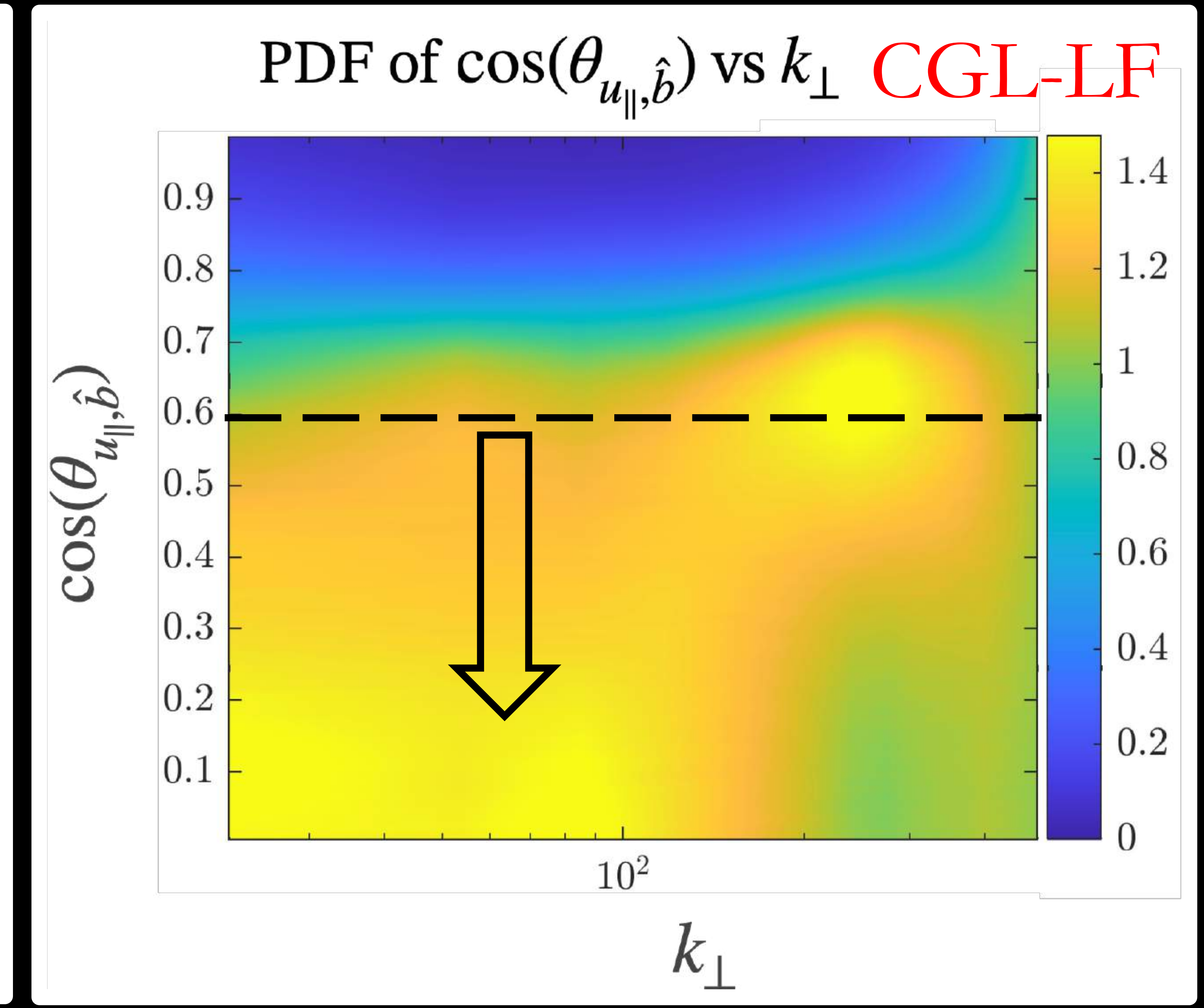
"Magneto-immutability" – self-organization to avoid Δp stress

(Squire *et al.* 2019; Kempski *et al.* 2019; Arzamasskiy, Kunz, Squire *et al.* 2022; Squire, Kunz *et al.* 2023; Majeski, Kunz & Squire 2024 ← develops theory)

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{p_\perp - p_\parallel}{p} \sim \frac{\mathrm{d}\ln B}{\mathrm{d}t} \sim \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\nabla\boldsymbol{u}$$

control Δp by adjusting this angle to minimize changes in B



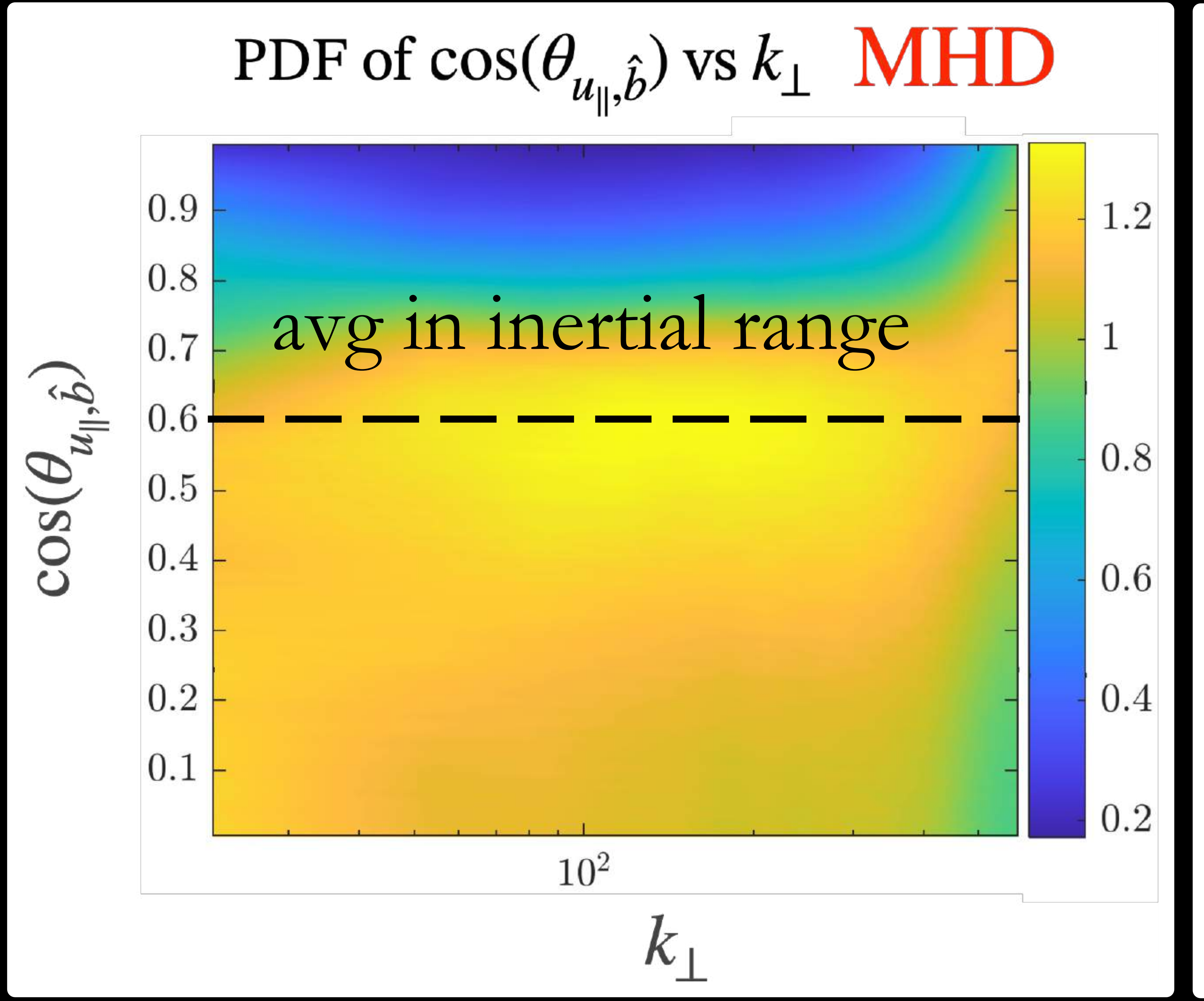


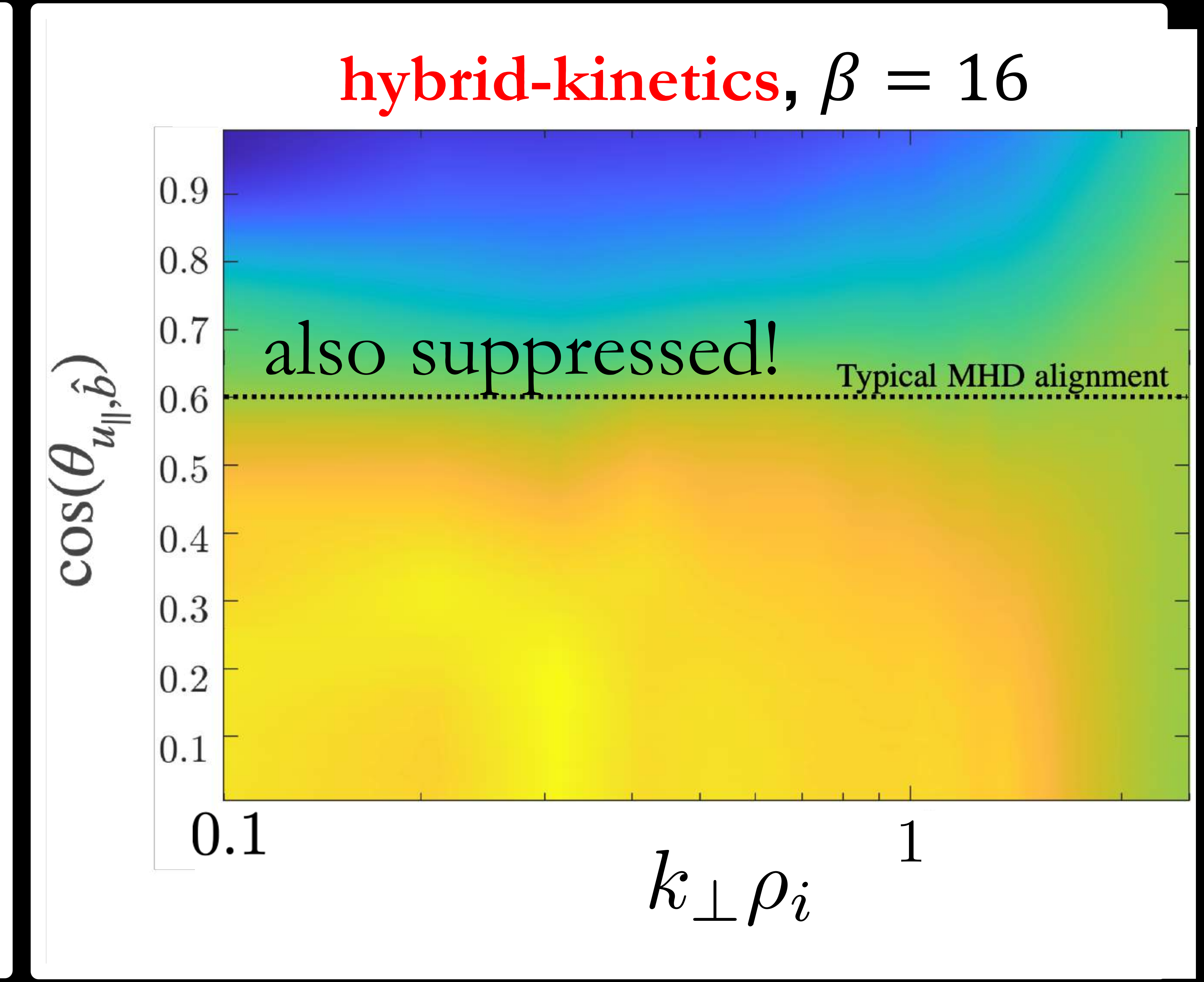
"Magneto-immutability" – self-organization to avoid Δp stress

(Squire *et al.* 2019; Kempski *et al.* 2019; Arzamasskiy, Kunz, Squire *et al.* 2022; Squire, Kunz *et al.* 2023; Majeski, Kunz & Squire 2024 ← develops theory)

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{p_\perp - p_\parallel}{p} \sim \frac{\mathrm{d}\ln B}{\mathrm{d}t} \sim \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\nabla\boldsymbol{u}$$

control Δp by adjusting this angle to minimize changes in B





- viscous damping ($\rightarrow$ heating) important near outer scale, damps ~1/2 of cascade energy
- the remainder self-organizes into a robust, nearly conservative GS95-like cascade to small scales

incompressibility:

large pressure force opposes any flow with $\boldsymbol{\nabla}\cdot\boldsymbol{u}\neq 0$ that attempts to change p.

p tied to ρ

$$\rho\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\boldsymbol{\nabla}p + \dots$$

$$\frac{\mathrm{d}\ln p}{\mathrm{d}t} = -\gamma\boldsymbol{\nabla}\cdot\boldsymbol{u}$$

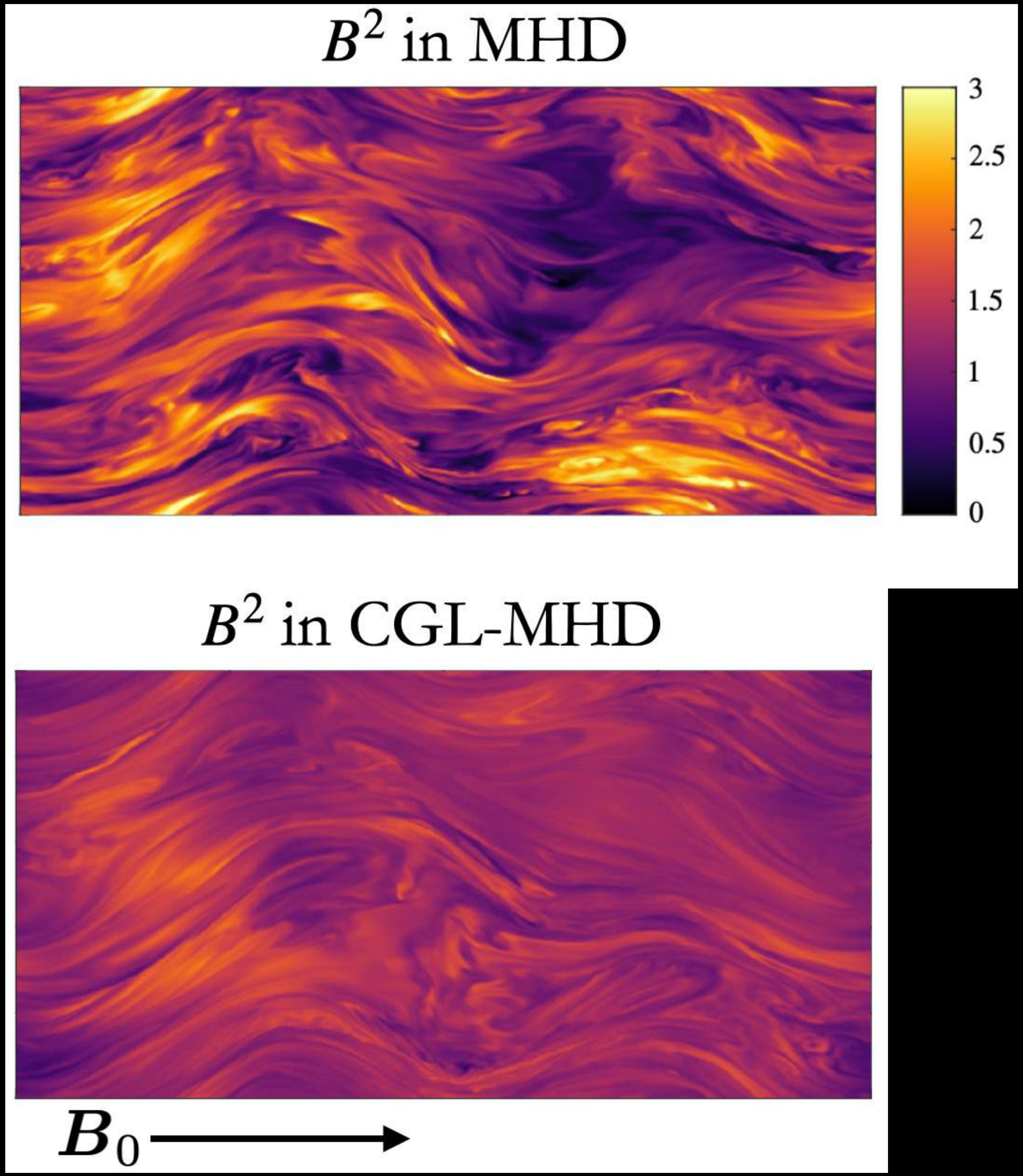
magneto-immutability:

large viscous force opposes any flow with $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\,\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u}\neq 0$ that attempts to change Δp.

Δp tied to B

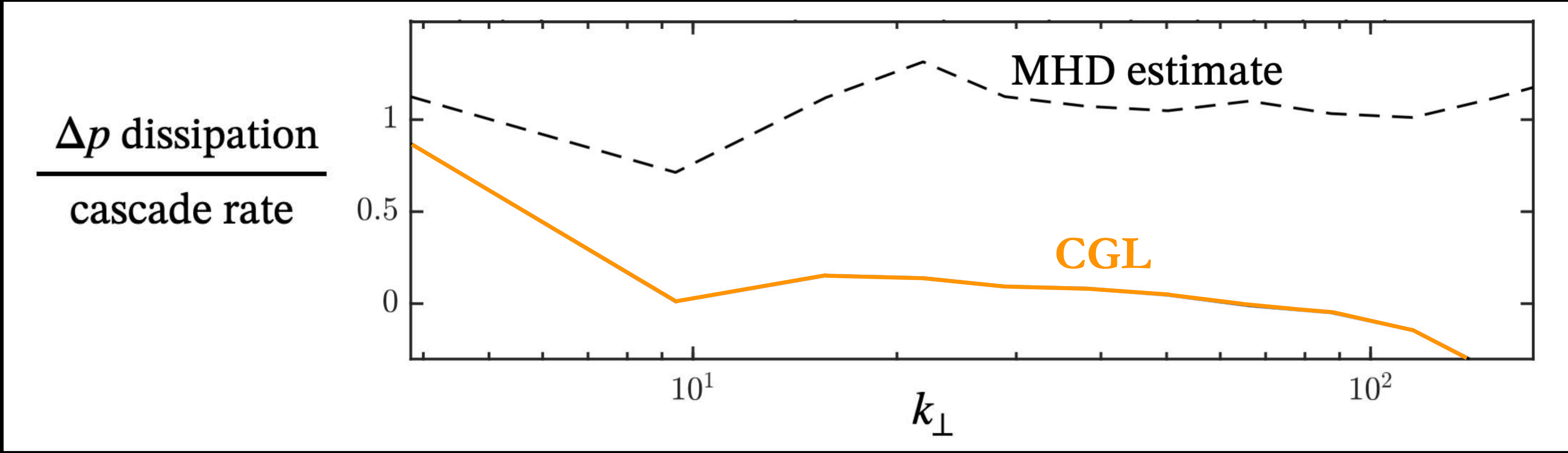
$$\rho\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \boldsymbol{\nabla}\cdot(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\Delta p) + \dots$$

$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} = (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} - \mathsf{I})\,\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u}$$



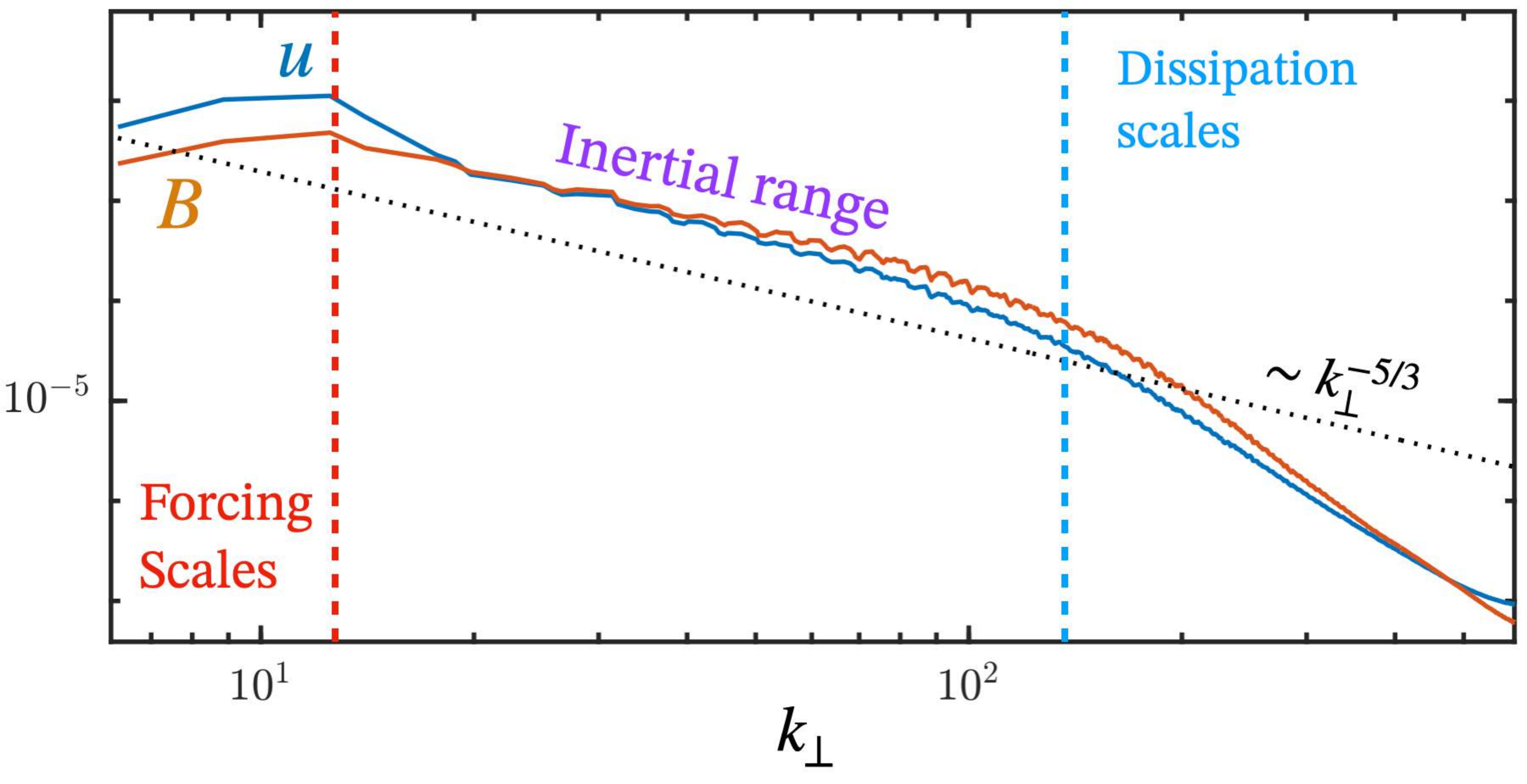
Use $\beta = 10$ MHD turbulence to estimate Δp effects as if they were active:

then turn those effects on and evolve to steady state:



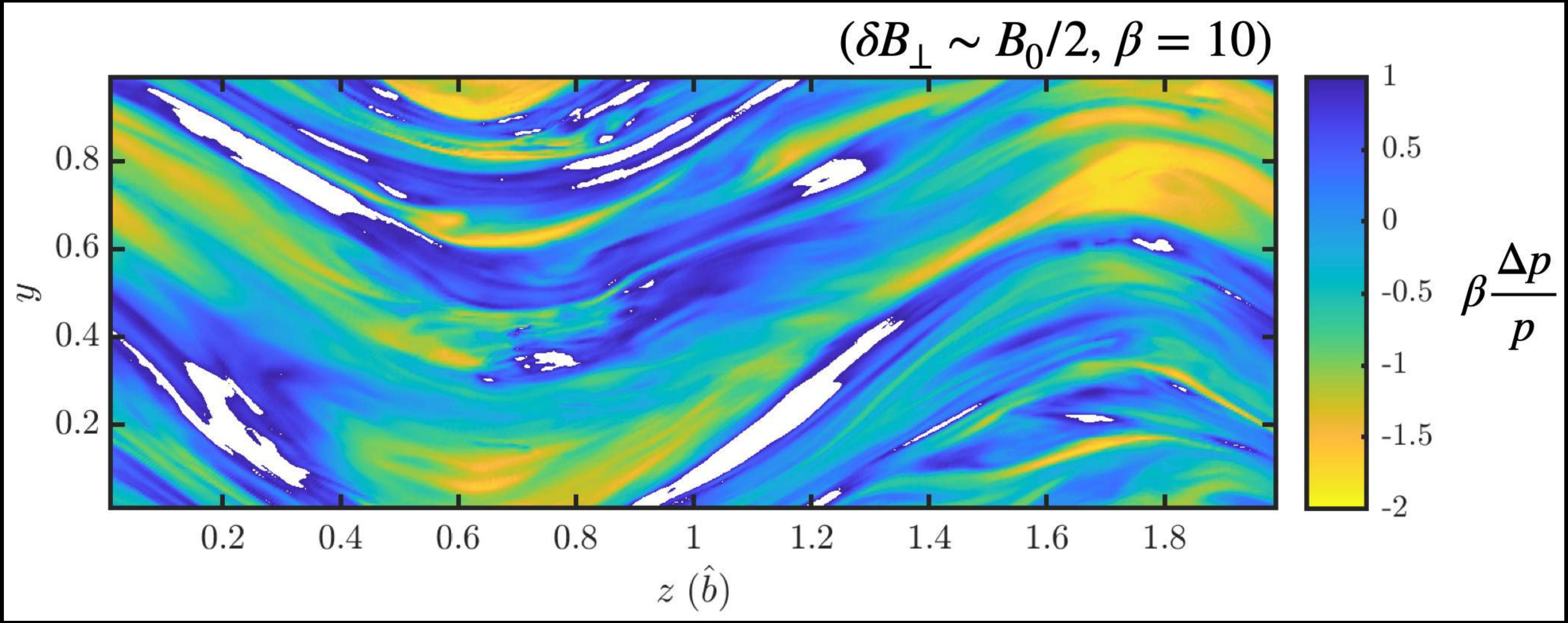
inertial-range turbulence self-organizes to avoid viscous dissipation!

the result is a robust, critically balanced cascade of Alfvénic fluctuations…



Majeski, Kunz & Squire 2024

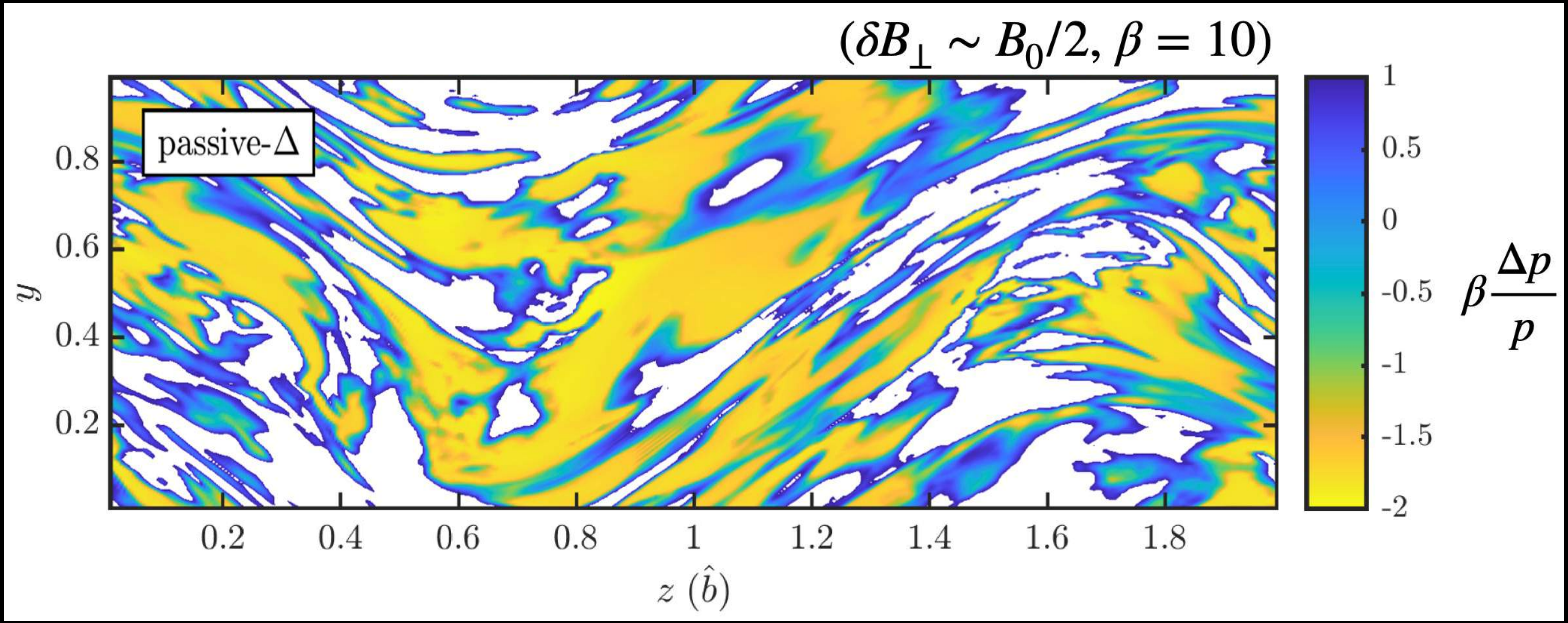
…with relatively little of the plasma (~10%) unstable to firehose/mirror



(white parts are kinetically unstable)

Majeski, Kunz & Squire 2024

…with relatively little of the plasma (~10%) unstable to firehose/mirror



(white parts are kinetically unstable)

Majeski, Kunz & Squire 2024

can obtain theory for magneto-immutable turbulence assuming:

$\beta \gg 1$, critical balance, approximate μ conservation

$$\epsilon \sim \frac{1}{\beta} \sim \frac{k_\|}{k_\perp} \sim \frac{u_\perp}{v_\mathrm{A}} \sim \frac{u_\|}{v_\mathrm{A}} \sim \frac{\delta B_\perp}{B_0} \sim \frac{\delta B_\|}{B_0} \sim \frac{\delta T_\perp}{T_0} \sim \frac{\delta T_\|}{T_0} \ll 1$$

$$\omega \sim k_\| v_\mathrm{A}$$

(see Majeski, Kunz & Squire 2024 for details) can obtain theory for magneto-immutable turbulence assuming:

$\beta \gg 1$, critical balance, approximate μ conservation

$$\nabla_\perp \cdot \boldsymbol{u}_\perp = \nabla_\perp \cdot \delta \boldsymbol{B}_\perp = 0,$$

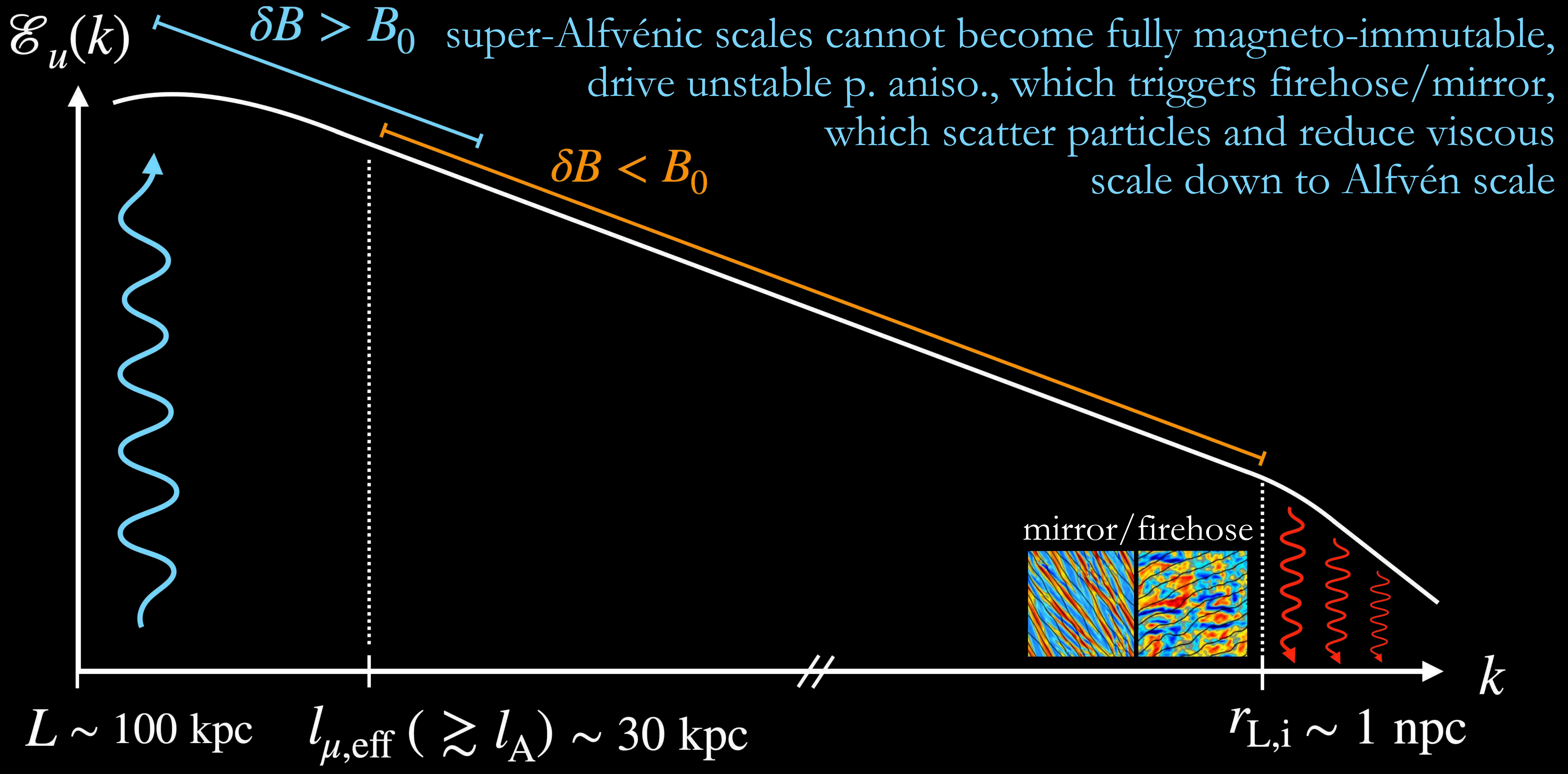
$$\frac{\mathrm{d}\delta \boldsymbol{B}_\perp}{\mathrm{d}t} = B_0 \hat{\boldsymbol{b}} \cdot \nabla_\perp \boldsymbol{u}_\perp,$$

$$\rho_0 \frac{\mathrm{d}\boldsymbol{u}_\perp}{\mathrm{d}t} = -\nabla_\perp P_{\mathrm{total}} + \left(1 + \frac{\beta}{2}\frac{\Delta p}{p_0}\right)\left(\frac{B_0 \hat{\boldsymbol{z}} + \delta \boldsymbol{B}_\perp}{4\pi}\right) \cdot \nabla \delta \boldsymbol{B}_\perp,$$

$$\boxed{\frac{\mathrm{d}\Delta p}{\mathrm{d}t} = \frac{\mathrm{d}\delta B_\parallel}{\mathrm{d}t} = \hat{\boldsymbol{b}} \cdot \nabla u_\parallel = \hat{\boldsymbol{b}} \cdot \nabla \Delta p = 0}$$

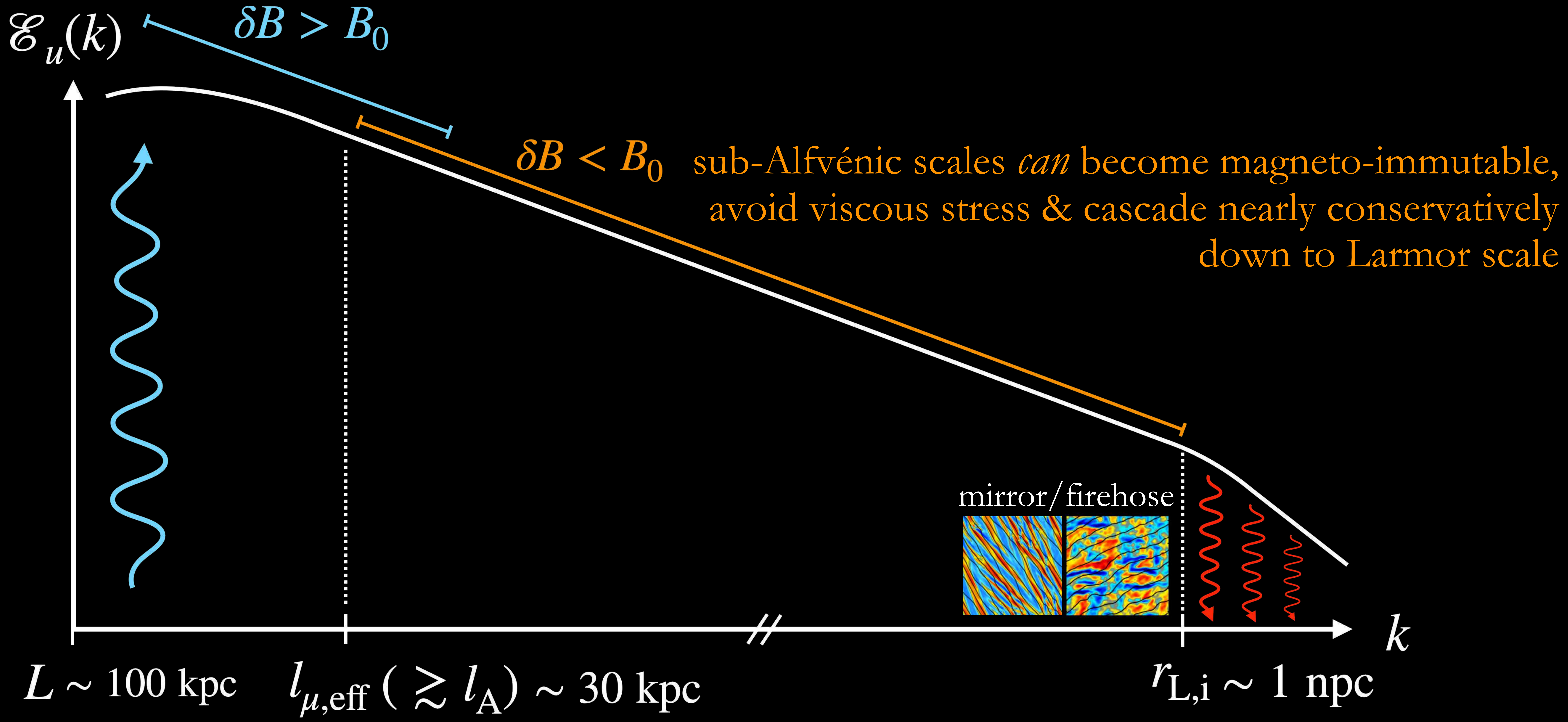
(see Majeski, Kunz & Squire 2024 for details)

motivates a theory of ICM turbulence:



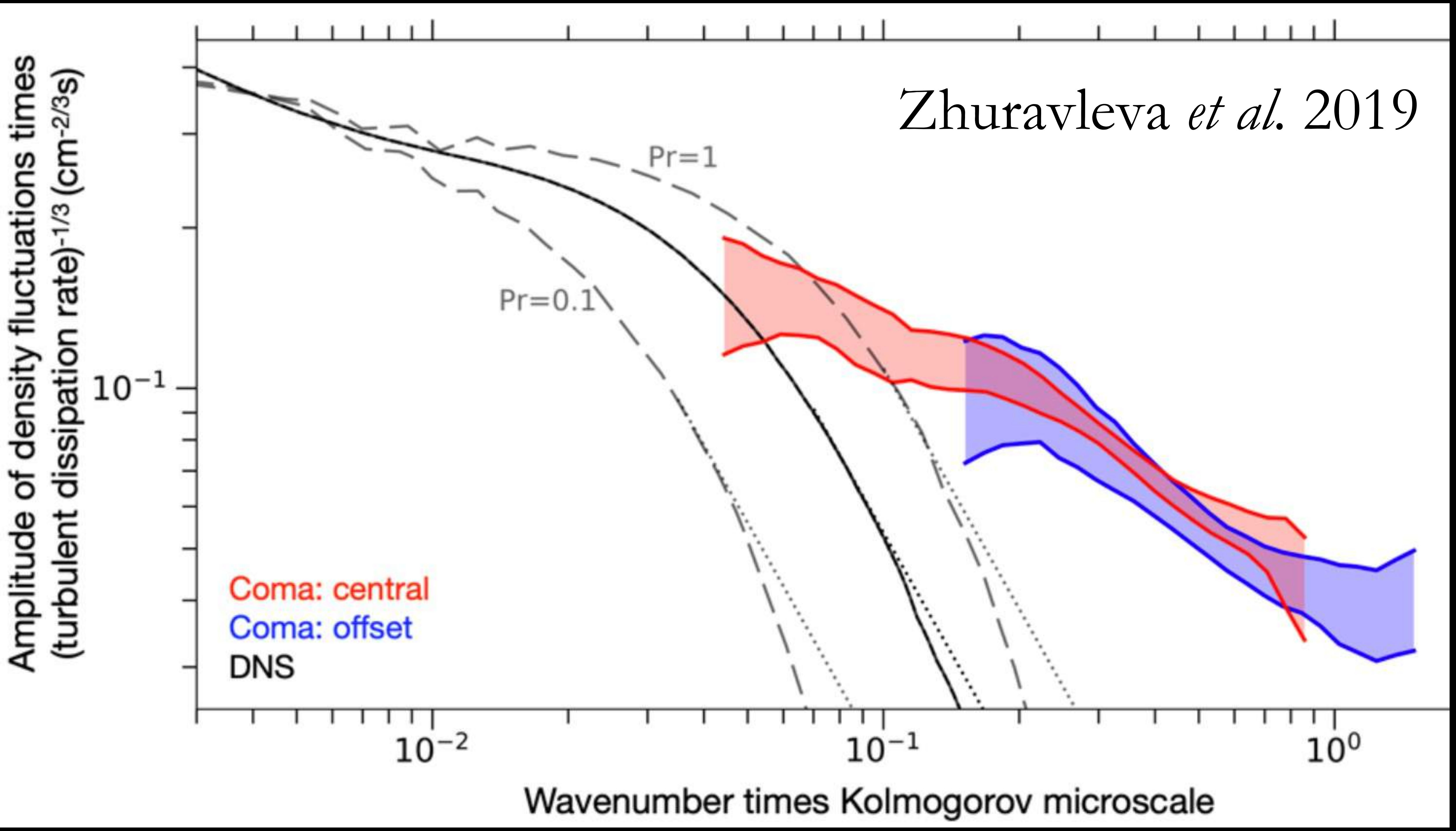
($l_{\mu,\mathrm{Coulomb}} \sim 300$ kpc; Coma)

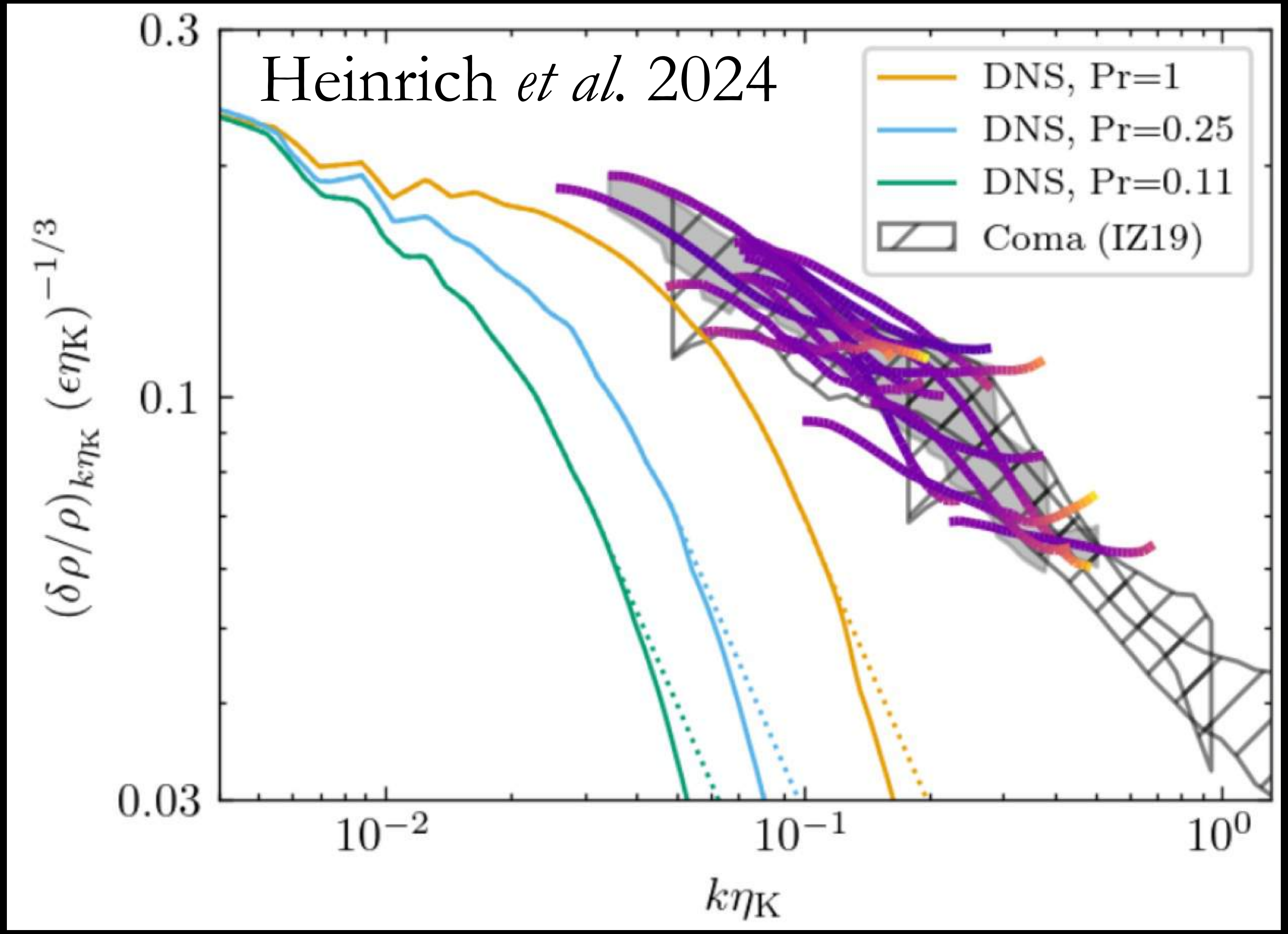
motivates a theory of ICM turbulence:



($l_{\mu,\mathrm{Coulomb}} \sim 300$ kpc; Coma)

Interestingly, X-ray observations of clusters can't seem to find viscous scale…





viscosity inferred from X-ray obs of density fluc's in Bremsstrahlung emission in 17 different clusters is significantly smaller than Spitzer (collisional) value

When kinetic instabilities regulate transport…

$$\nu_{\mathrm{eff},u} \sim \beta\, |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla}\boldsymbol{u}| \quad \to \quad \Pi_{\|} \sim \frac{B^2}{8\pi} \quad \text{instead of } \Pi_{\|} \propto T^{5/2}|\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla}\boldsymbol{u}|$$

$$\nu_{\mathrm{eff},T} \sim \beta\, |\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} \ln T| \quad \to \quad q_{\|} \sim T\frac{B^2}{8\pi} \quad \text{instead of } q_{\|} \propto T^{5/2}|\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} T|$$

thermodynamics depends on the magnetic-field strength

…but in some cases, system has freedom to self-organize in order to stay close to LTE (e.g., magneto-immutability)

in either situation, ***NOT MHD***!

some other adventures in high-beta astrophysical plasmas:

long-wavelength **ion-acoustic waves** having $\delta n/n \gtrsim \beta^{-1}$
trigger firehose/mirror, interrupting their Landau damping;
behave like Braginskii-MHD sound waves
(Kunz, Squire, Schekochihin & Quataert 2020, JPP)

long-wavelength **fast modes** having $\delta B/B \gtrsim 2\beta^{-1}$
trigger firehose/mirror, behave like Braginskii-MHD fast modes
(Majeski, Kunz & Squire 2023, JPP)

long-wavelength **non-propagating modes** having $\delta B/B \gtrsim 0.3$
trigger mirror, interrupting their transit-time damping;
behave like Braginskii-MHD entropy modes
(Majeski, Kunz & Squire 2023, JPP)

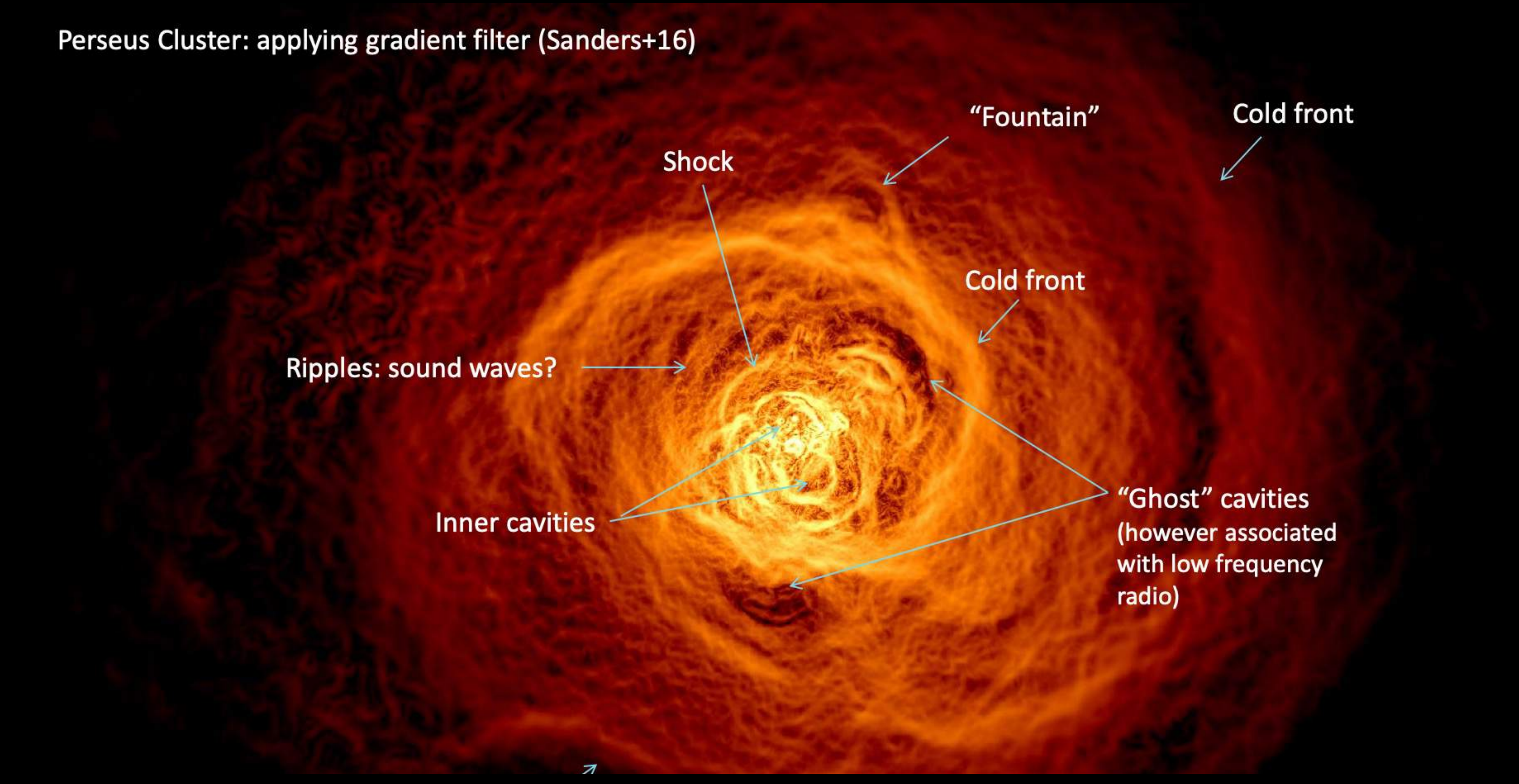
graphically…

resonant surfer demonstrating
Landau damping

not so much

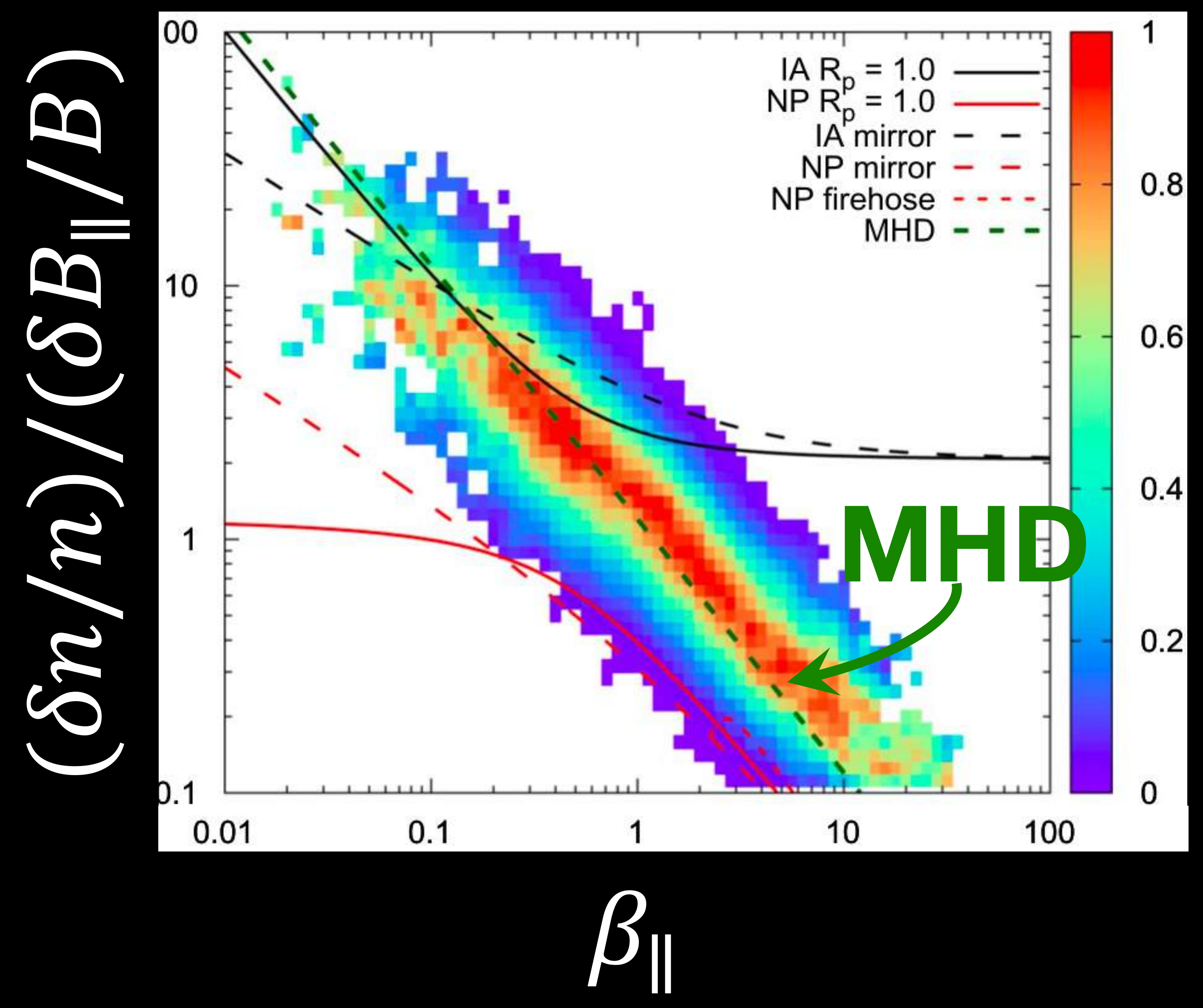
micro-instabilities impede maintainenance of Landau resonances,
halt otherwise strong collisionless damping and give fluid-like behavior

connection to
sound-wave propagation in ICM,
important for redistribution of
energy throughout ICM
(e.g., Fabian *et al.* 2003; Ruszkowski *et al.* 2004; Fabian *et al.* 2005, 2017; Zweibel *et al.* 2018)



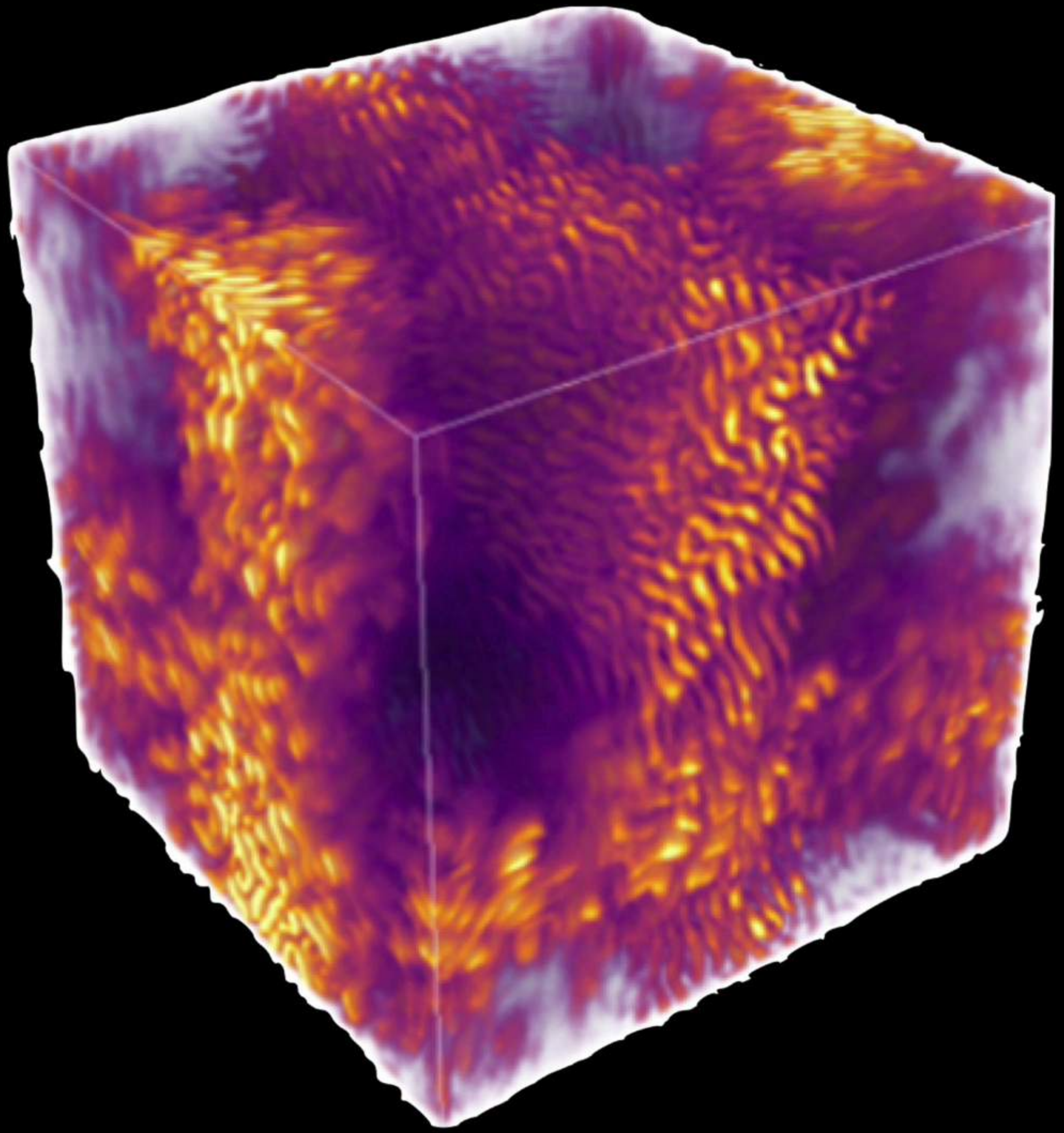
Fabian, Sanders *et al.* (B♭, 57 octaves below middle-C)

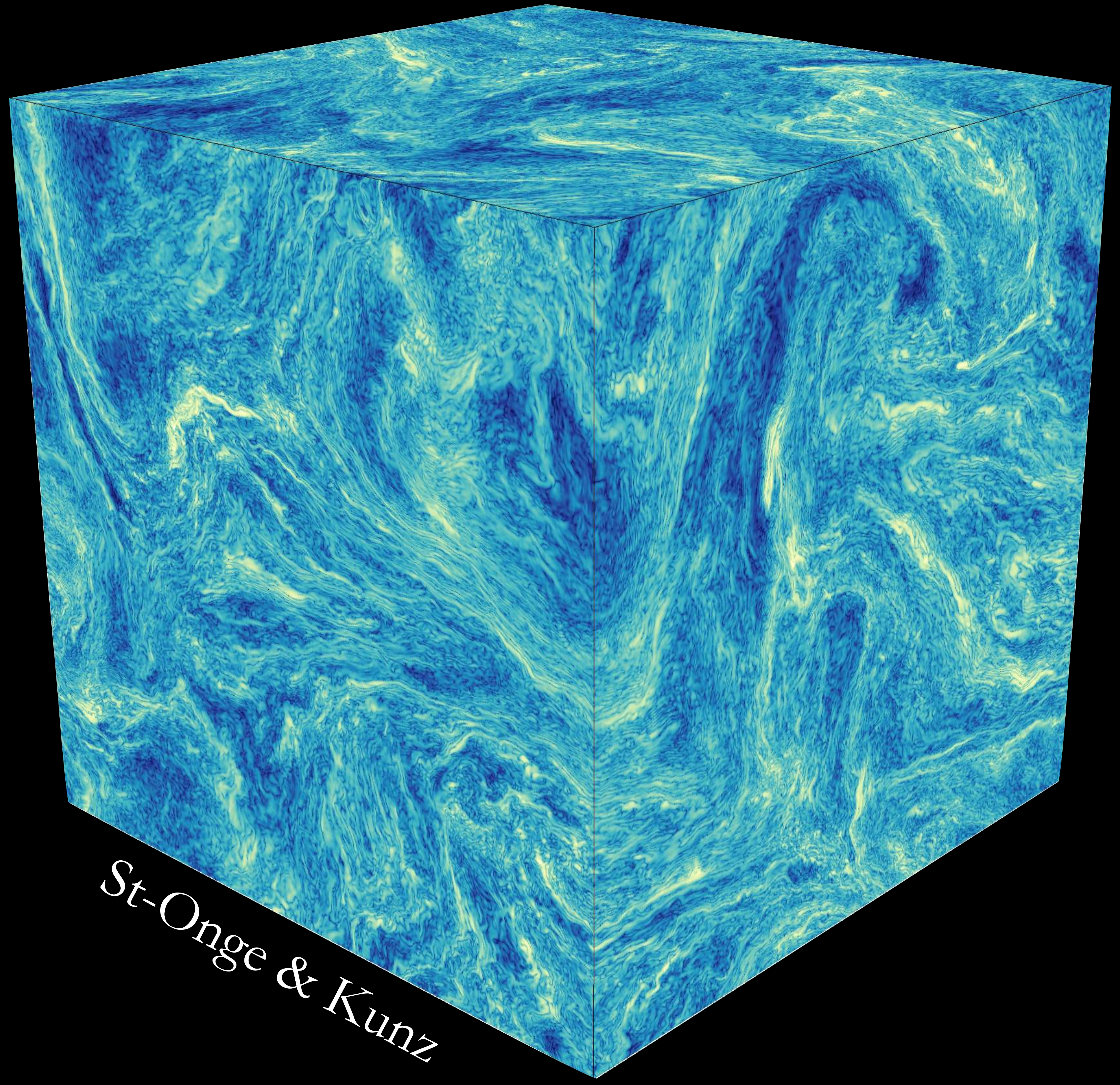
connection to
compressive fluctuations in SW,
which are more fluid-like than
kinetic in their polarizations
(e.g., Verscharen, Chen & Wicks 2017; Coburn, Chen & Squire 2022)



some other adventures in high-beta astrophysical plasmas:

Magnetogenesis in a collisionless plasma:
from Weibel instability to turbulent dynamo
(Zhou, Zhdankin, Kunz, Loureiro & Uzdensky, ApJ 2023)





CGL + anomalous collisionality:
Santos-Lima *et al.* (2014)

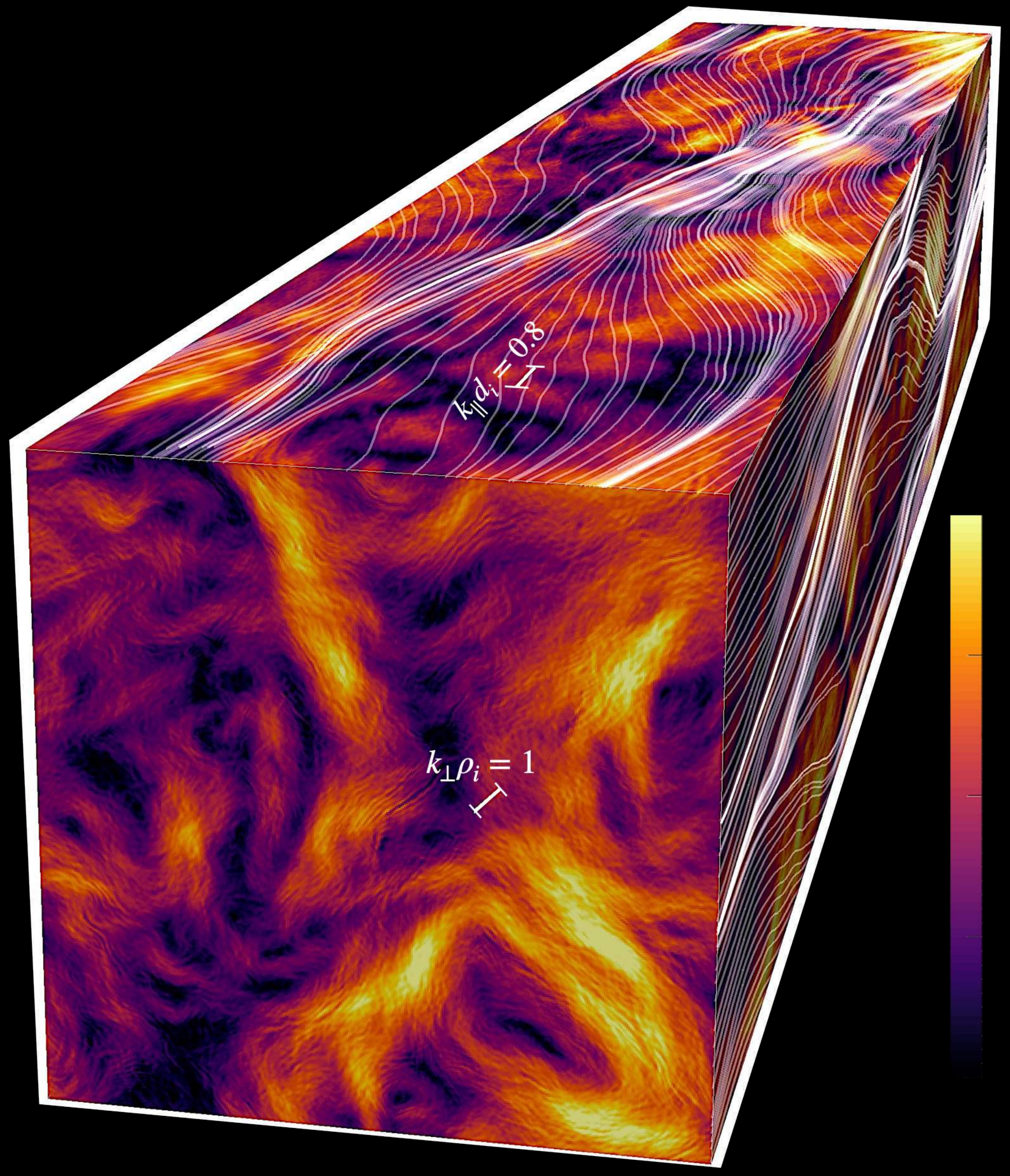
hybrid-kinetic:
Rincon *et al.* (2016), St-Onge & Kunz (2018), Achikanath Chirakkara *et al.* (2023)

Braginskii + semi-analytic modeling:
St-Onge *et al.* (2020)

kinetic pair plasma:
Sironi *et al.* (2023), **Zhou** *et al.* (2023)

“helicity barrier” in imbalanced turbulence:

how low-frequency turbulence can drive high-frequency heating of the solar wind



Squire, Meyrand, Kunz+ 2022, *Nat. Astro.*
Squire, Meyrand & Kunz 2023, *ApJL*

at low β only balanced portion of AW cascade can penetrate ion-Larmor scale (“helicity barrier”)
(Meyrand, Squire, Schekochihin & Dorland 2021)

inertial-range energy builds up, critical balance leads to higher frequencies

eventually excites ion-cyclotron waves, which heat ions perpendicularly to saturate the barrier, while KAW cascade heats the electrons

explains large number of SW observations

A few outstanding questions

- Plasma turbulence at high β is a frontier, with galaxy clusters as the premier cosmic laboratories (*XRISM*, AXIS, LEM, Athena, Lynx…) Can new insights on high-β plasma explain various observations of ICM? Can anything be done in the lab?
  - Turbulence cascades beyond the Coulomb viscous scale (Zhuravleva+ 2019; Heinrich+ 2024)
  - Self-sustaining sound-wave propagation to large distances in clusters to redistribute energy (see Fabian+ 2003; Zweibel+ 2018; Kunz+ 2020)
  - Thermally stable source of heating to suppress cooling flows? (Kunz+ 2011; Ley+ 2023)
  - Efficient confinement of sub-TeV cosmic rays? (Patrick Reichherzer+ 2024)
  - Clusters at $z \sim 0.7$ with inferred μG magnetic fields (Di Gennaro+ 2020) – fast plasma dynamo? (Schekochihin & Cowley 2006; St-Onge *et al.* 2020; Zhou *et al.* 2023; Sironi *et al.* 2023)
- More generically, when and how does a collisionless plasma behave as a fluid?