Bridging the Gap Between Fluid and Kinetic Simulations

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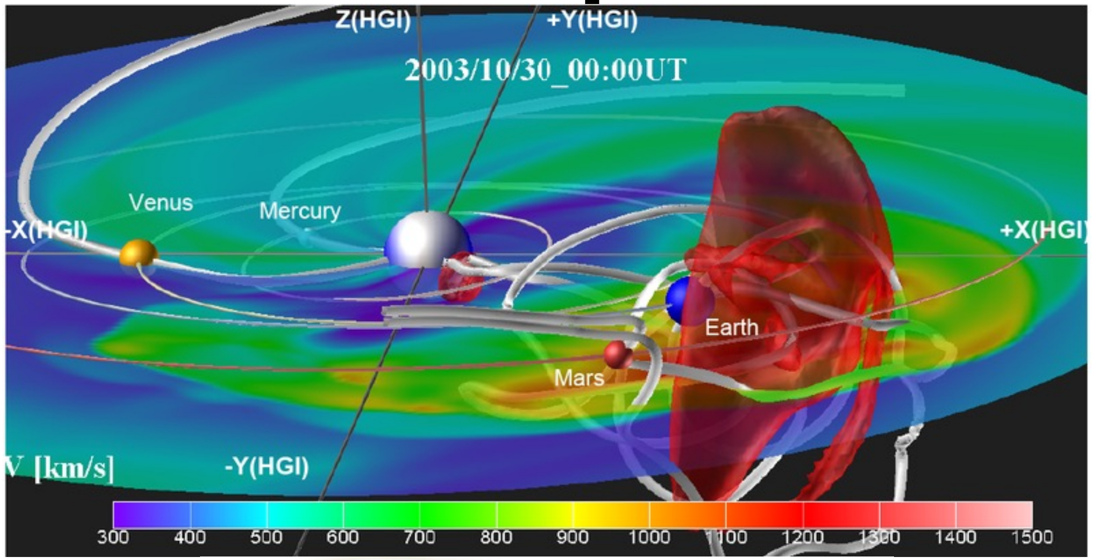
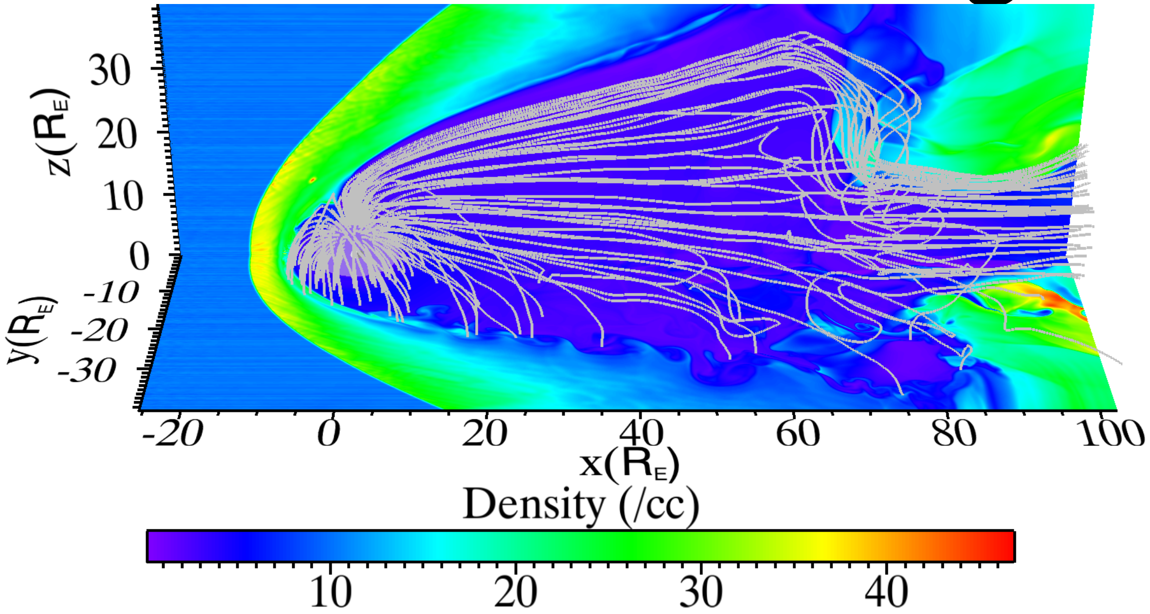
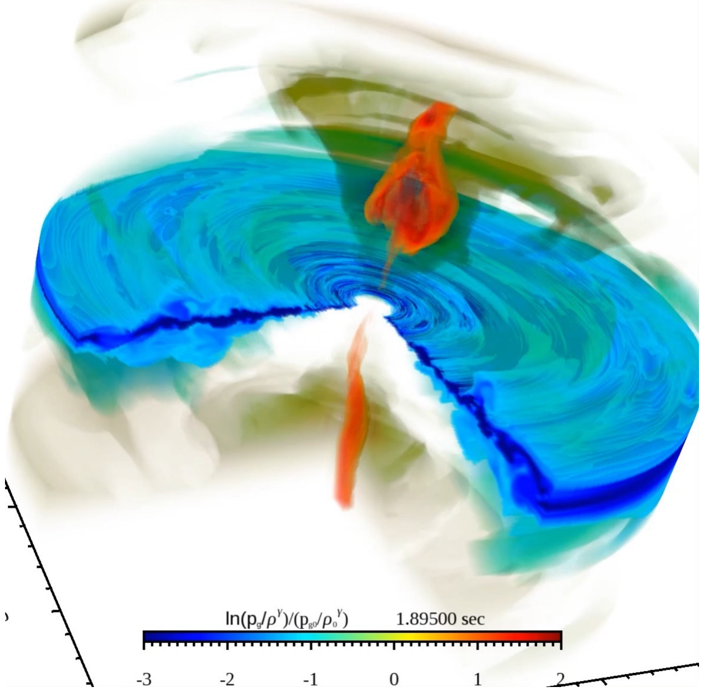
1. Jikei, T., Amano, T., 2021. A non-local fluid closure for modeling cyclotron resonance in collisionless magnetized plasmas. Physics of Plasmas 28, 042105. https://doi.org/10.1063/5.0045335
2. Jikei, T., Amano, T., 2022. Critical comparison of collisionless fluid models: Nonlinear simulations of parallel firehose instability. Physics of Plasmas 29, 022102. https://doi.org/10.1063/5.0077064

Macroscopic Collisionless Plasma Modeling



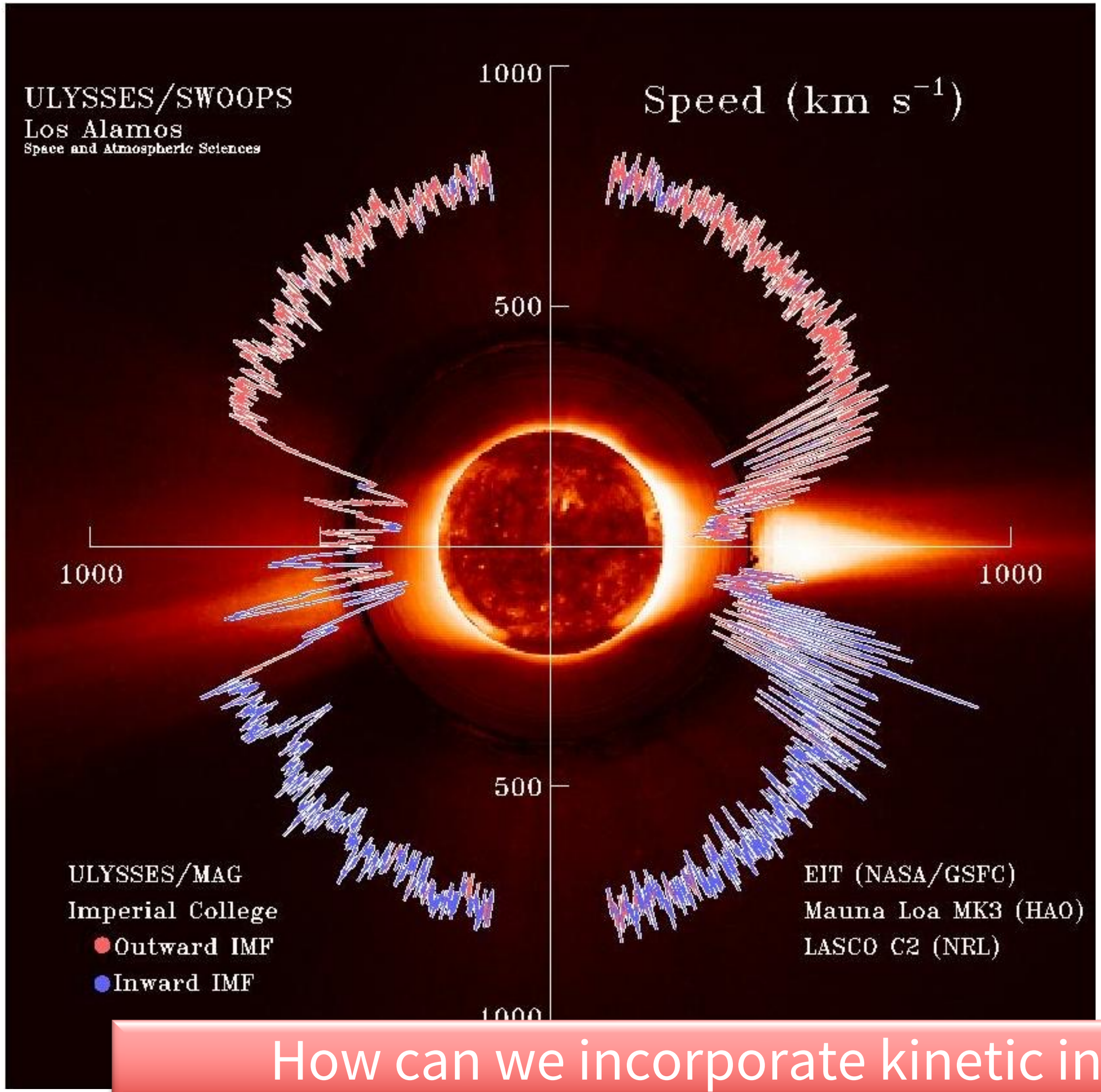


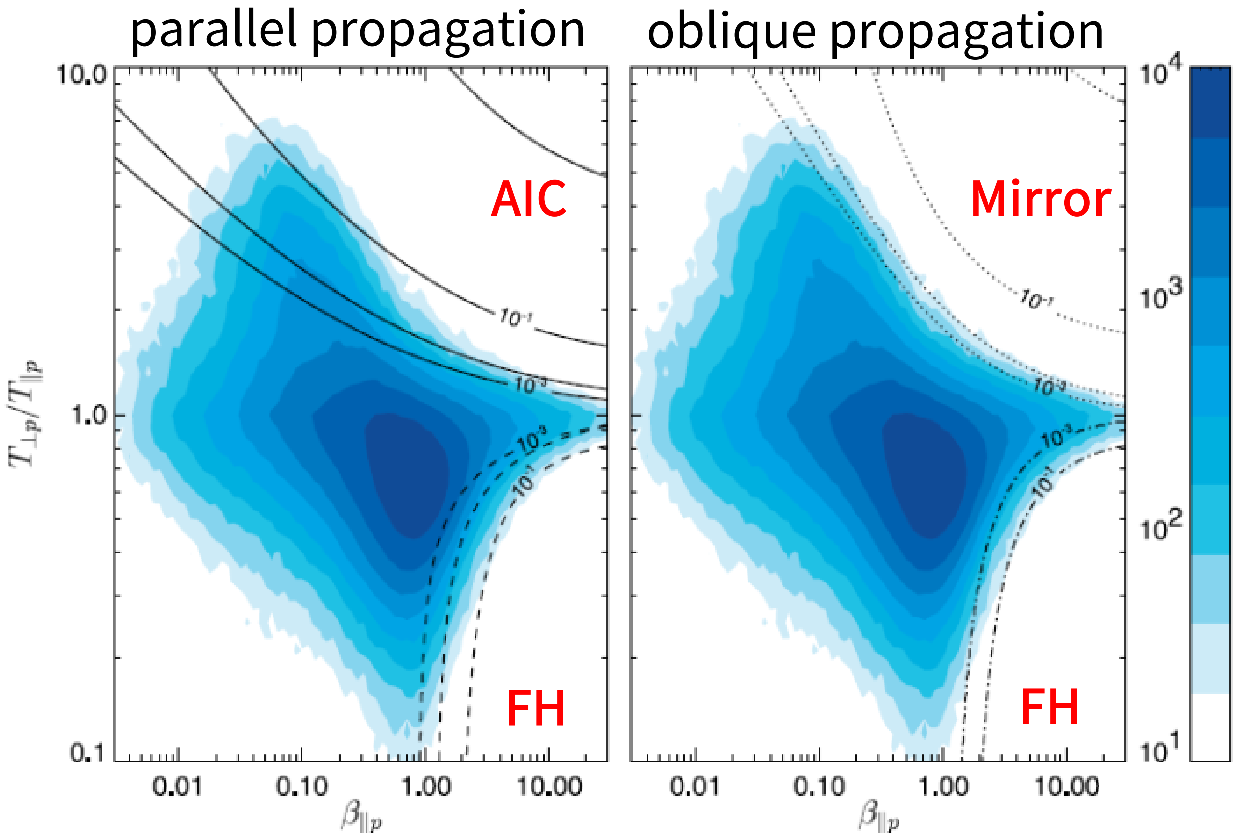


Issues in standard MHD

- Magnetic reconnection
- Temperature anisotropy
- Energetic particles (cosmic rays)
- Energy partition

"Anisotropic" Solar Wind





Fast solar wind (Hellinger+2007)

- Adiabatic expansion naturally induces anisotropy.
- There must be self-regulation to stabilize the system.

How can we incorporate kinetic instabilities in a macroscopic (fluid) model?

Chew-Goldberger-Low (CGL) Model

Anisotropic (yet gyrotropic) pressure tensor

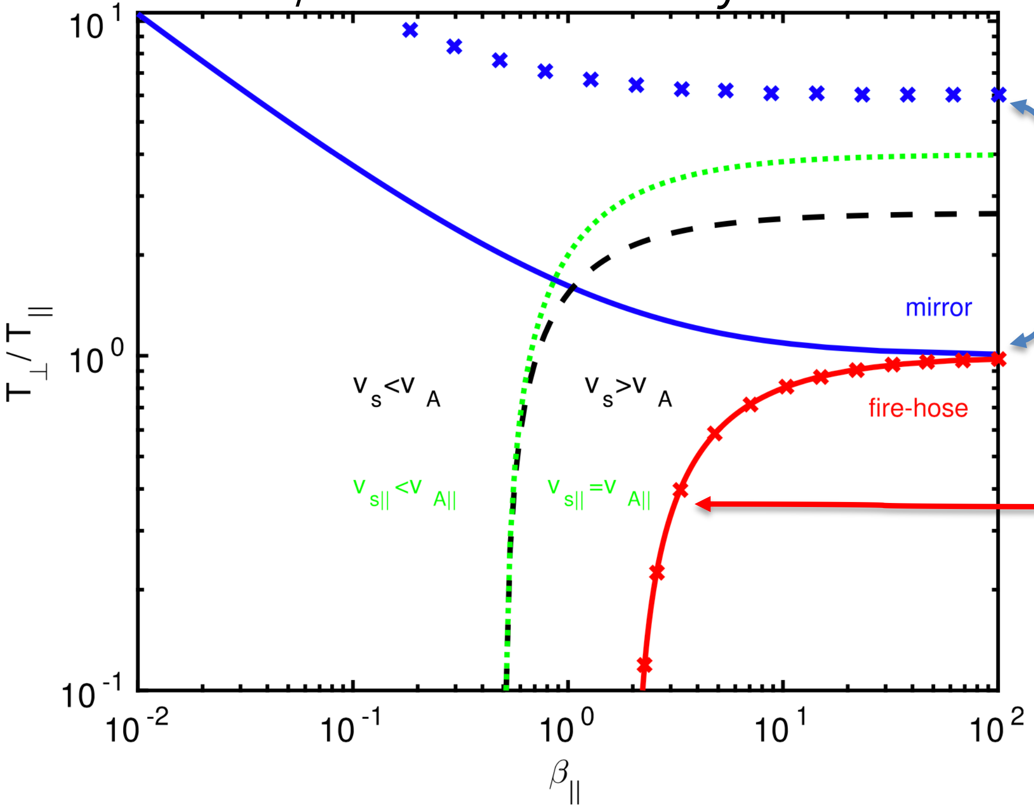
$$\boldsymbol{p} = p_\perp \boldsymbol{I} + (p_\parallel - p_\perp)\boldsymbol{bb}$$

CGL Equation of State

$$\text{first adiabatic invariant} \rightarrow \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = 0$$

$$\text{second adiabatic invariant} \rightarrow \frac{d}{dt}\left(\frac{B^2 p_\parallel}{\rho^3}\right) = 0$$

Mirror/Firehose Instability Threshold



discrepancy with the kinetic model for the mirror instability
(mirror instability is more sensitive to the parallel heat flux)

agrees with the kinetic model for the firehose instability

Hunana et al. 2019

Closure in Collisionless Plasmas

Vlasov equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{m}E\frac{\partial f}{\partial v} = 0$$

$$\int dv\, v^0 \rightarrow \frac{\partial}{\partial t}(n) + \frac{\partial}{\partial x}(nv) = 0$$

$$\int dv\, v^1 \rightarrow \frac{\partial}{\partial t}(nv) + \frac{\partial}{\partial x}(nv^2 + p) = \frac{nq}{m}E$$

$$\dots$$

$$\int dv\, v^k \rightarrow \frac{\partial}{\partial t}(M^k) + \boxed{\frac{\partial}{\partial x}(M^{k+1})} = k\frac{q}{m}EM^{k-1}$$

Naive moment expansion always involves the dependence on a higher order quantity!

“Standard” MHD

- ignores the heat flux completely (simple truncation)
- assumes instantaneous Maxwellianization (or local thermodynamic equilibrium) without concrete physical reasoning.

Phase Mixing

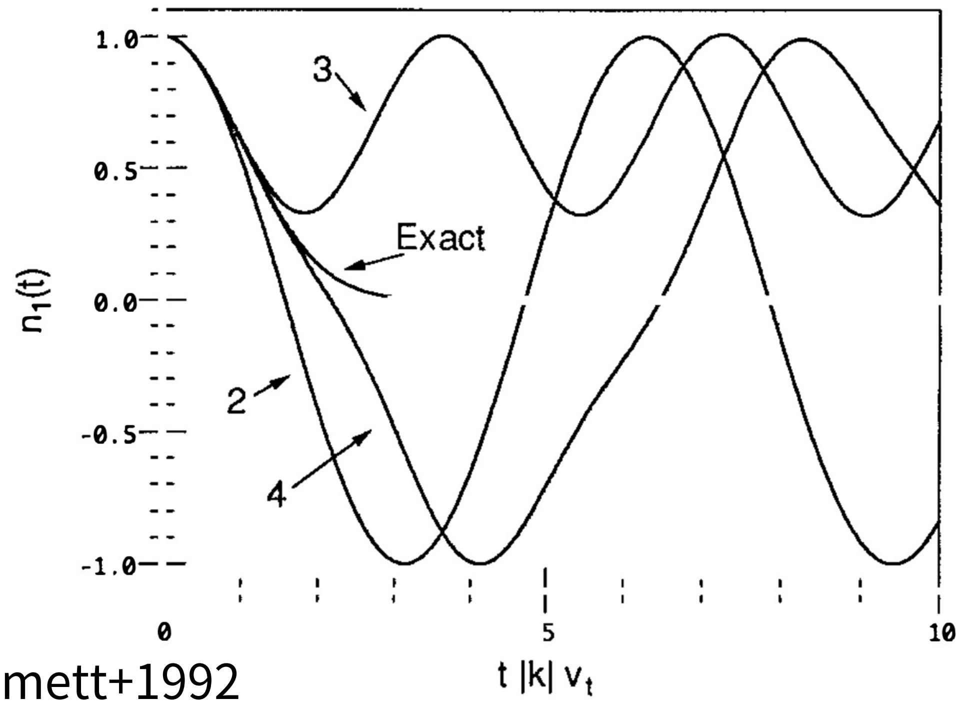
Density perturbation rapidly decays even without any field perturbations:

$$f(x, v, t = 0) \equiv \left(n_0 + n_1 e^{ikx}\right) f_0(x, v)$$

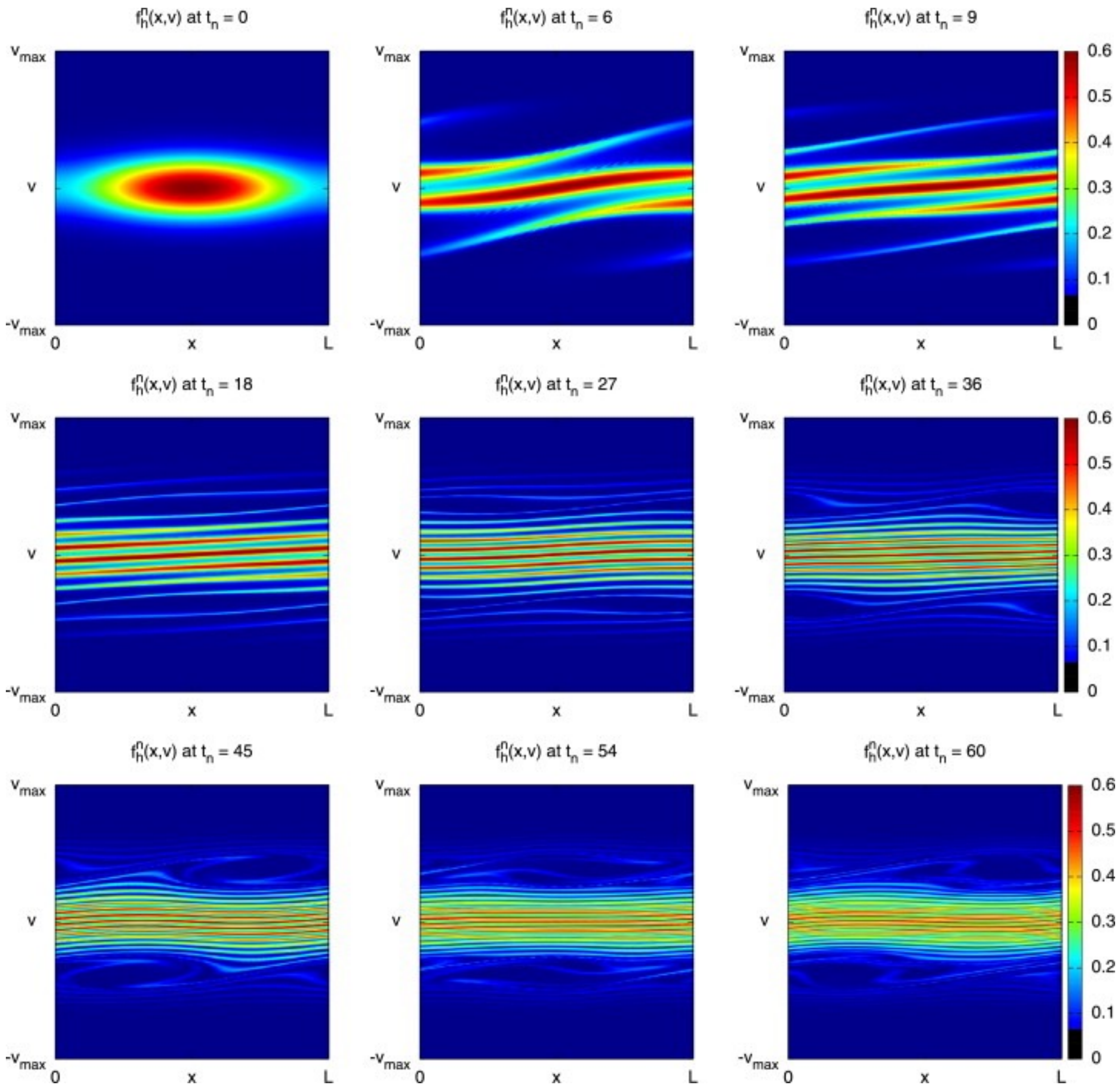
$$\Downarrow$$

$$n_1(t) = n_1(0) e^{-k^2 v_{\rm th}^2 t^2/2}$$

Fluid models never reproduce the correct behavior



Hammett+1992



Pinto+2014

Nonlocal "Landau" Closure

The pressure equation with heat flux:

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(up) + 2p\frac{\partial u}{\partial x} = -\frac{\partial q}{\partial x}$$

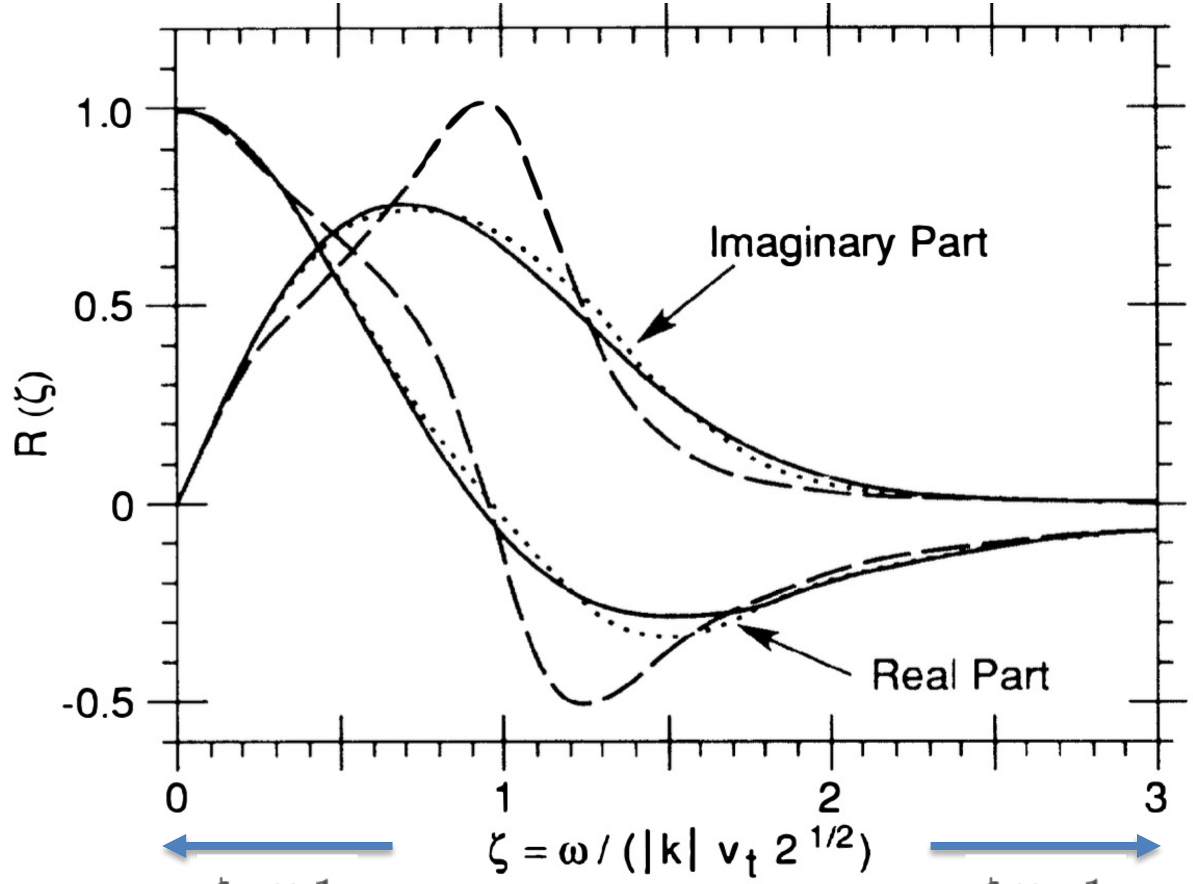
The nonlocal heat flux is defined in k-space:

$$\tilde{q}_k = -n_0 \chi \frac{\sqrt{2}v_{\rm th}}{|k|} ik\tilde{T}_k$$

(which is an integral operator in real space)

The heat flux coefficient is determined by matching the response function to the fully kinetic counterpart in the kinetic limit: $\zeta \ll 1$

$$\frac{\tilde{n}}{n_0} = -\frac{e\tilde{\phi}}{k_B T}R(\zeta) \quad \text{where} \quad \zeta = \frac{\omega}{\sqrt{2}kv_{\rm th}}$$



$\zeta \ll 1$ kinetic

$\zeta \gg 1$ cold

Hammett & Perkins 1990

Collisionless MHD = CGL-MHD + Landau Closure

CGL EoS | Heat flux

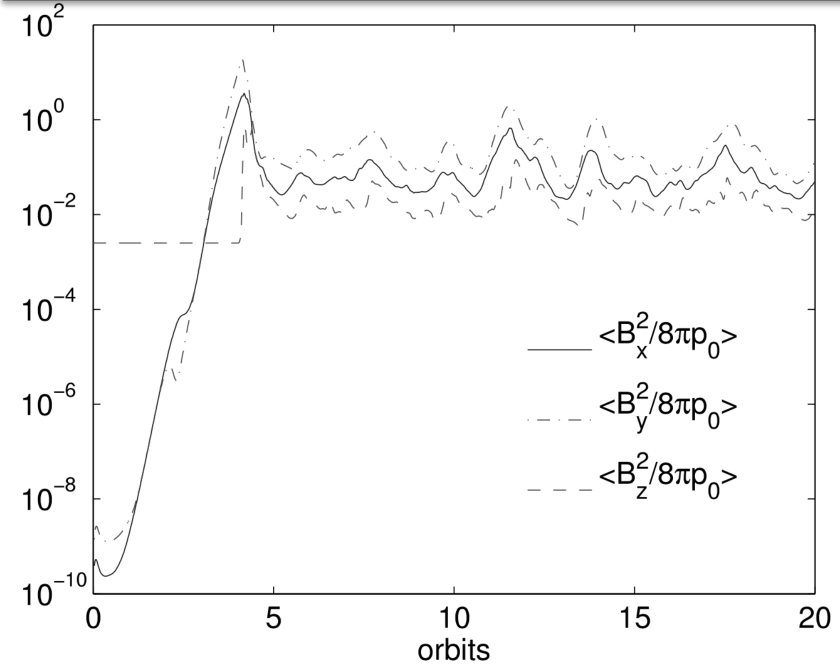
$$\rho B \frac{D}{Dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot\boldsymbol{q}_\perp - q_\perp \nabla\cdot\hat{\boldsymbol{b}},$$

$$\frac{\rho^3}{B^2}\frac{D}{Dt}\left(\frac{p_\parallel B^2}{\rho^3}\right) = -\nabla\cdot\boldsymbol{q}_\parallel + 2q_\perp \nabla\cdot\hat{\boldsymbol{b}},$$

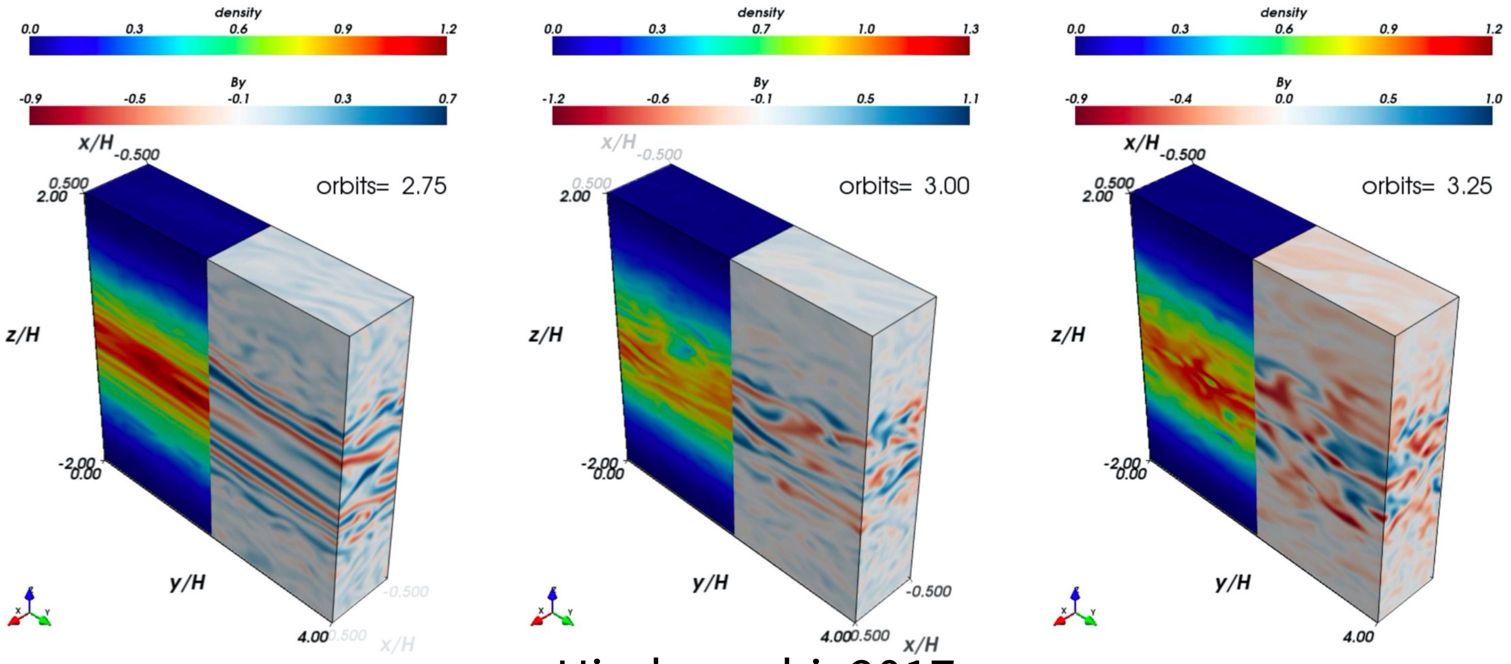
- Reproduces linear mirror and firehose instabilities.
- Not applicable to cyclotron resonance.

[e.g., Snyder+1997, Sharma+2003]

Application to collisionless accretion disks (but with pitch-angle scattering added by hand)



Sharma+2006



Hirabayashi+2017

Physics of Plasmas ARTICLE scitation.org/journal/php

A non-local fluid closure for modeling cyclotron resonance in collisionless magnetized plasmas

Editor's pick

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ABSTRACT

A fluid description for collisionless magnetized plasmas that takes into account the effect of cyclotron resonance has been developed. Following the same approach as the Landau fluid closure, the heat flux components associated with transverse electromagnetic fluctuations are approximated by a linear combination of lower-order moments in wavenumber space. The closure successfully reproduces the linear cyclotron resonance for electromagnetic waves propagating parallel to the ambient magnetic field. In the presence of finite temperature anisotropy, the model gives approximately correct prediction for an instability destabilized via the cyclotron resonance. A nonlinear simulation demonstrates the wave growth consistent with the linear theory followed by the reduction of initial anisotropy, and finally, the saturation of the instability. The isotropization may be understood in terms of quasilinear theory, which is developed within the framework of the fluid model but very similar to its fully kinetic counterpart. The result indicates that both linear and nonlinear collisionless plasma responses are approximately incorporated in the fluid model.

Cyclotron Resonance Closure (CRC)

Linearized equations for transverse perturbation with off-diagonal pressure components:

$$-i\omega\tilde{u}_x + ik\tilde{p}_{xz}/mn_0 - \frac{e}{m}(\tilde{E}_x + \tilde{u}_y B_0) = 0,$$

$$-i\omega\tilde{u}_y + ik\tilde{p}_{yz}/mn_0 - \frac{e}{m}(\tilde{E}_y - \tilde{u}_x B_0) = 0,$$

$$-i\omega\tilde{p}_{xz} + ik(p_0\tilde{u}_x + \tilde{q}_{xzz}) - \frac{eB_0}{m}\tilde{p}_{yz} = 0,$$

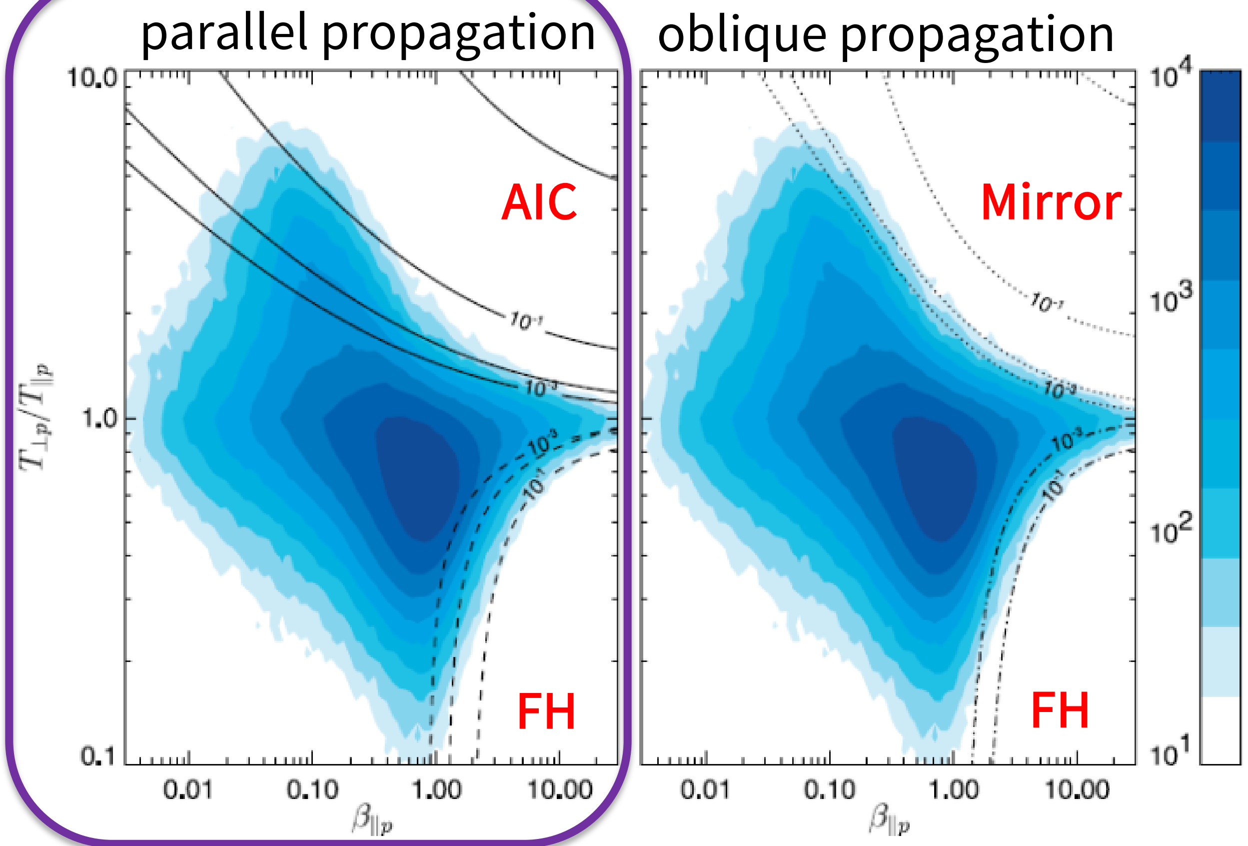
$$-i\omega\tilde{p}_{yz} + ik(p_0\tilde{u}_y + \tilde{q}_{yzz}) + \frac{eB_0}{m}\tilde{p}_{xz} = 0.$$

Heat flux

$$\begin{cases} \tilde{q}_{xzz} = \Pi\tilde{u}_x + \nu\tilde{p}_{xz}, \\ \tilde{q}_{yzz} = \Pi\tilde{u}_y + \nu\tilde{p}_{yz}, \end{cases}$$

linear combination of lower-order moments

Target: consider only k // B



Fast solar wind (Hellinger+2007)

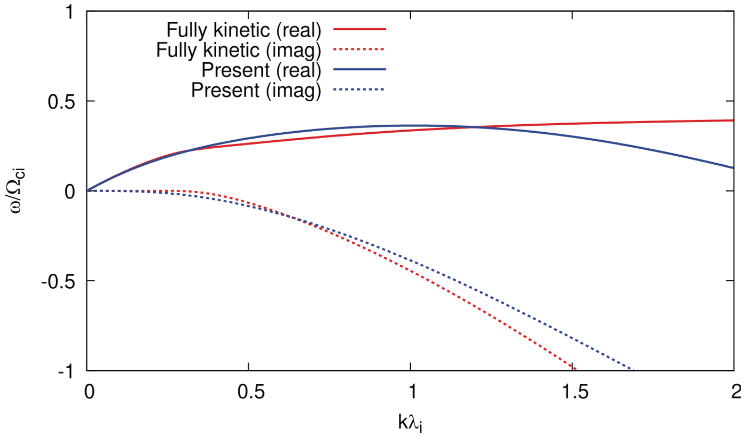
The coefficients in the assumed form of heat flux are free parameters and need to be determined to approximate fully kinetic dispersion relation. [c.f., Hammett & Perkins 1990, 1992]

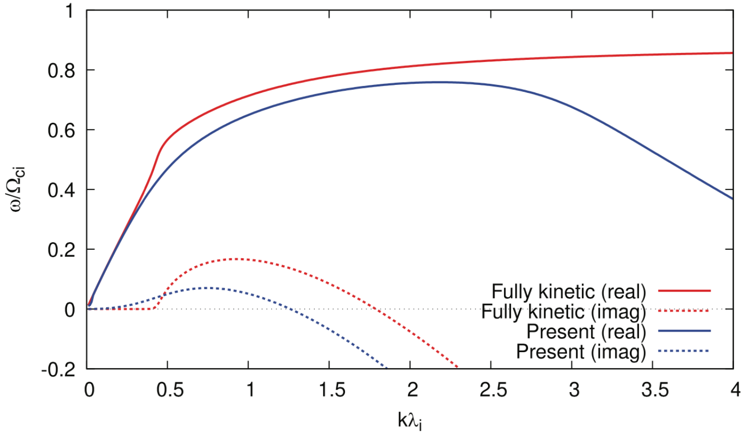
Jikei & Amano 2021

dispersion relation dependent on closure coefficients

$$1 - \frac{c^2k^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega(\omega_s' - \Omega_s')} = 0,$$

$$\begin{cases} \omega' = \omega\left(1 - \dfrac{p_0 + \Pi}{n_0 m}\dfrac{\omega - k\nu}{\omega}\dfrac{k^2}{(\omega - k\nu)^2 - \Omega^2}\right), \\ \Omega' = \Omega\left(1 + \dfrac{p_0 + \Pi}{n_0 m_s}\dfrac{k^2}{(\omega - k\nu)^2 - \Omega^2}\right), \end{cases}$$





Matching condition for the kinetic limit:

$$Z(\zeta) = i\sqrt{\pi} - 2\zeta - \cdots,$$

determines the closure coefficients

$$\begin{cases} \nu = -iv_{\rm th}\dfrac{\sqrt{2\pi}}{\pi - 2}\dfrac{k}{|k|} \\ \Pi = \dfrac{4 - \pi}{\pi - 2}p_0 \end{cases}$$

nonlocality

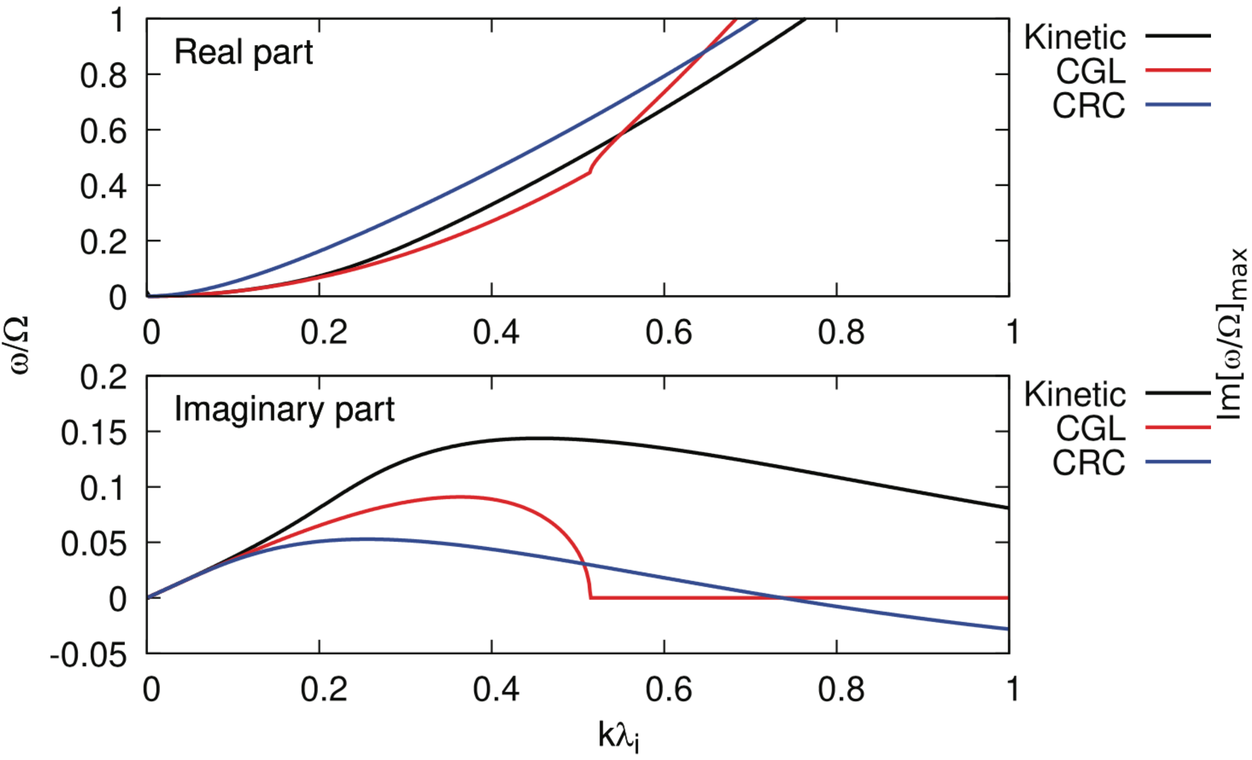
fully kinetic dispersion relation

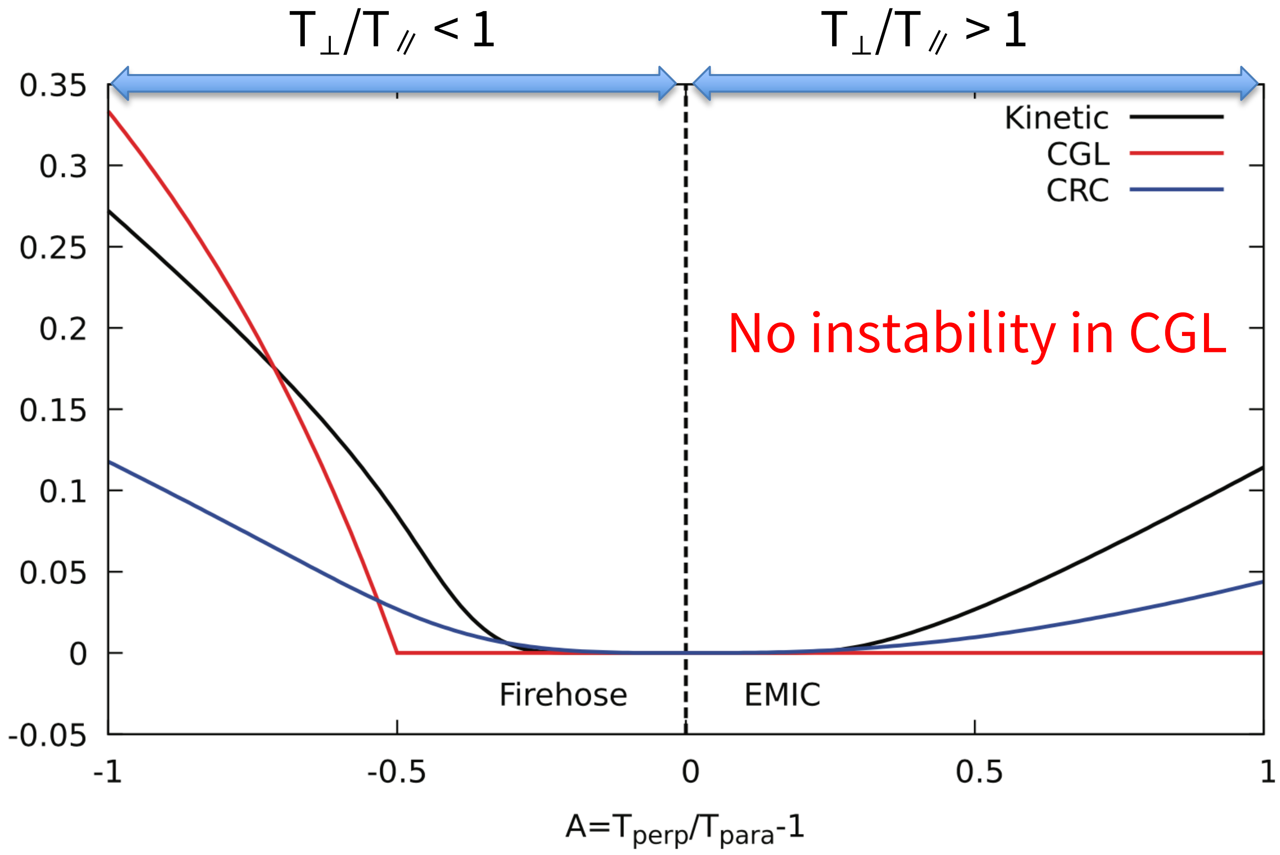
$$1 - \frac{c^2k^2}{\omega^2} + \sum_s \frac{\omega_{ps}^2}{\sqrt{2}\omega|k|v_{{\rm th},s}} Z\left(\frac{\omega - \Omega_s}{\sqrt{2}|k|v_{{\rm th},s}}\right) = 0,$$

Jikei & Amano 2021

[1] FLR1 of Hunana et al. 2019

Comparison with CGL-FLR[1]





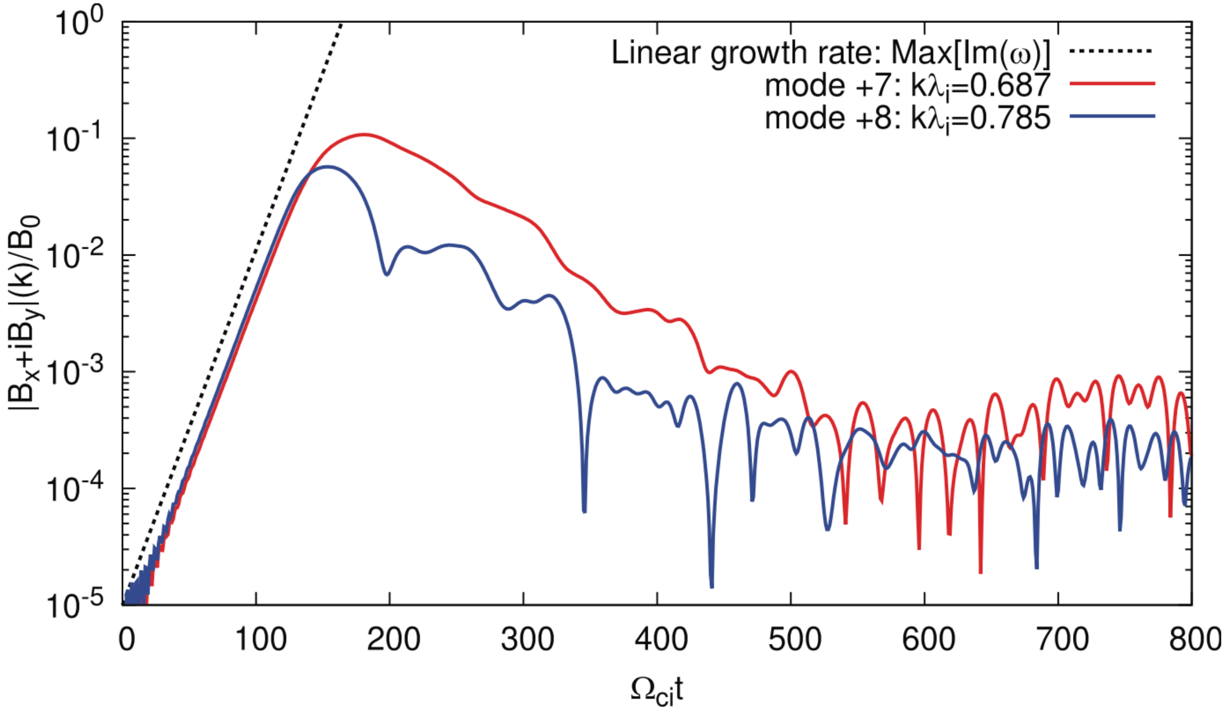
- No instability for A > 0 in CGL even with FLR correction.
- The threshold anisotropy for A < 0 is better described by CRC.
- The growth rate for A < 0 in the strongly driven regime is better described by CGL.

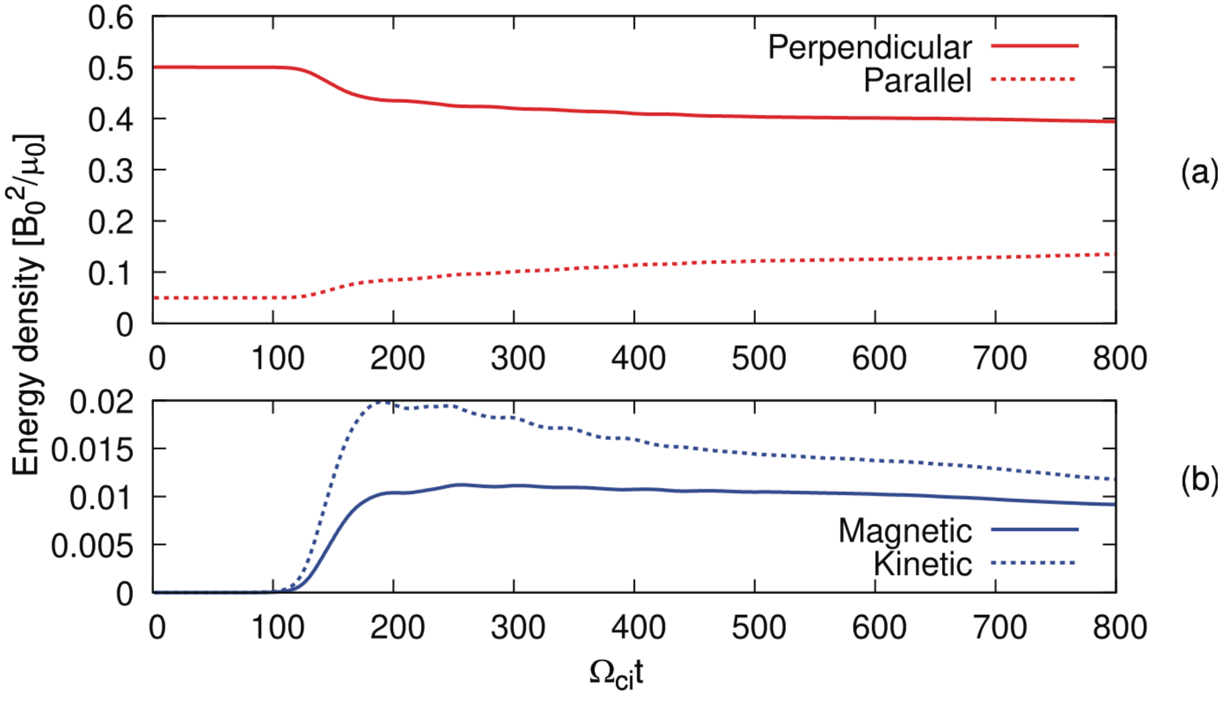
Jikei & Amano 2022

Simulation for Alfven-Ion-Cyclotron Instability

- Including the Hall term in Ohm's law.
- Landau closure for the longitudinal mode.
- Cyclotron resonance closure for the transverse mode.
- Adopt the pseudo spectral method to implement the non-local closure in k-space.

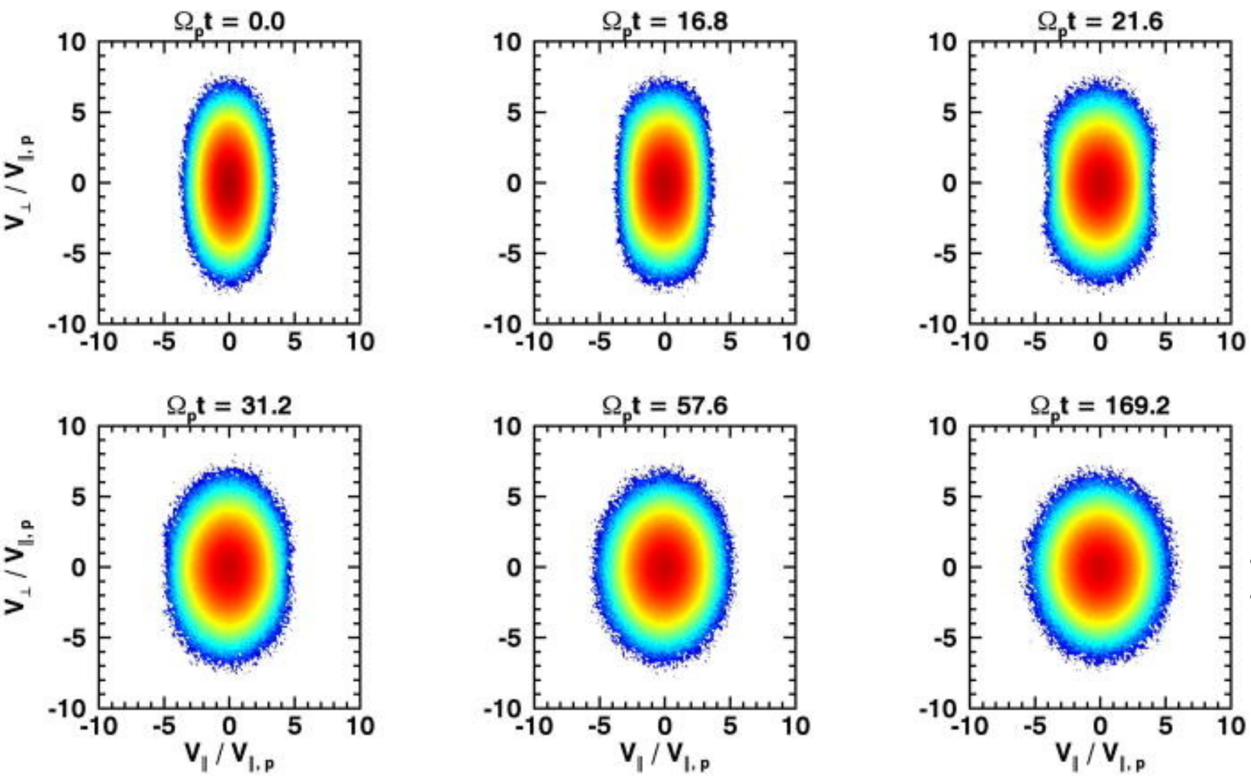
Qualitatively correct nonlinear behaviors with fully kinetic model. Why?



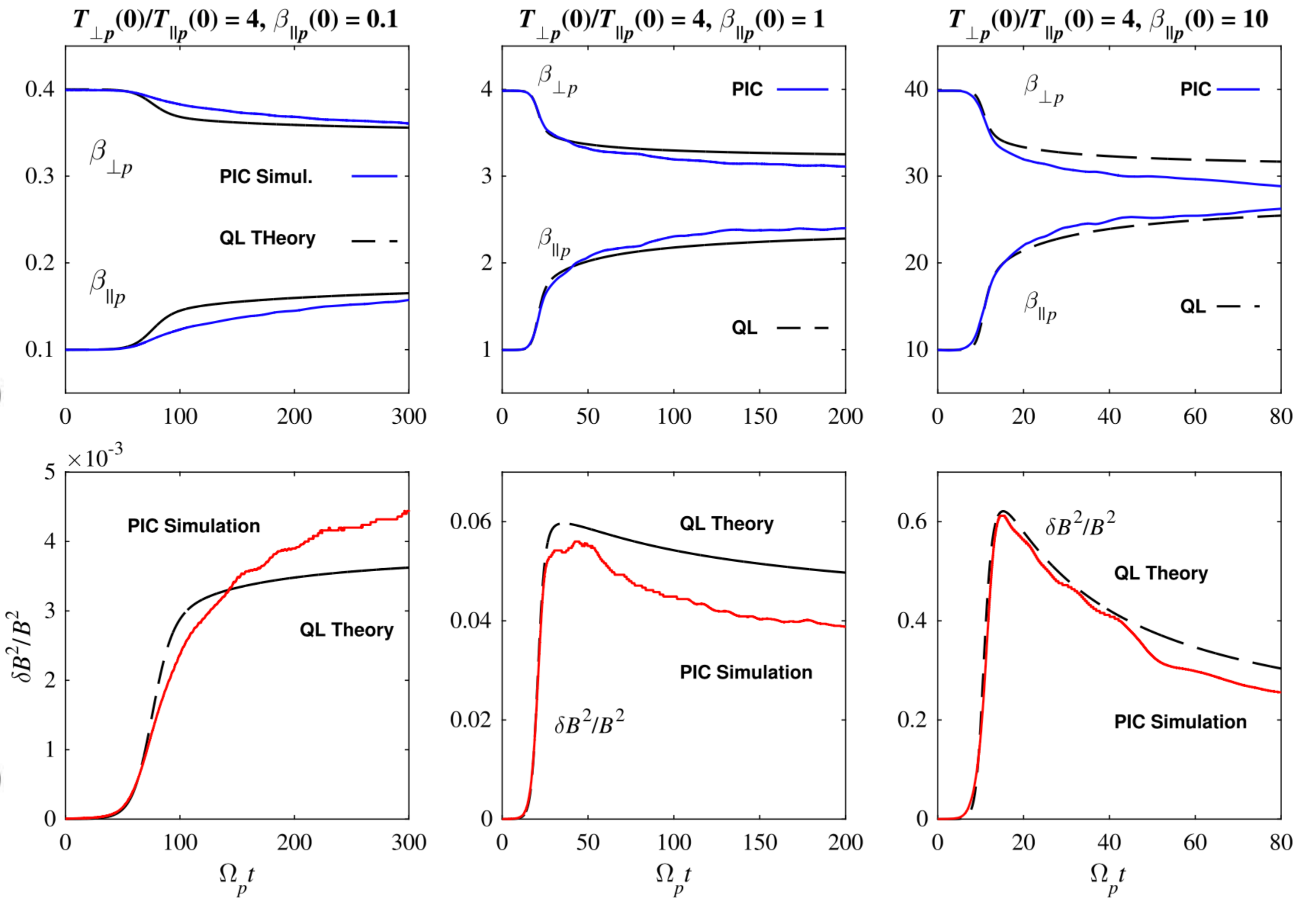


Quasi-linear Isotropization

Seough+2014



Yoon 2017



Quasi-linear theory well (but not always!) approximates the nonlinear evolution of the kinetic anisotropy-driven instability.

Quasi-linear Theory for CRC Model

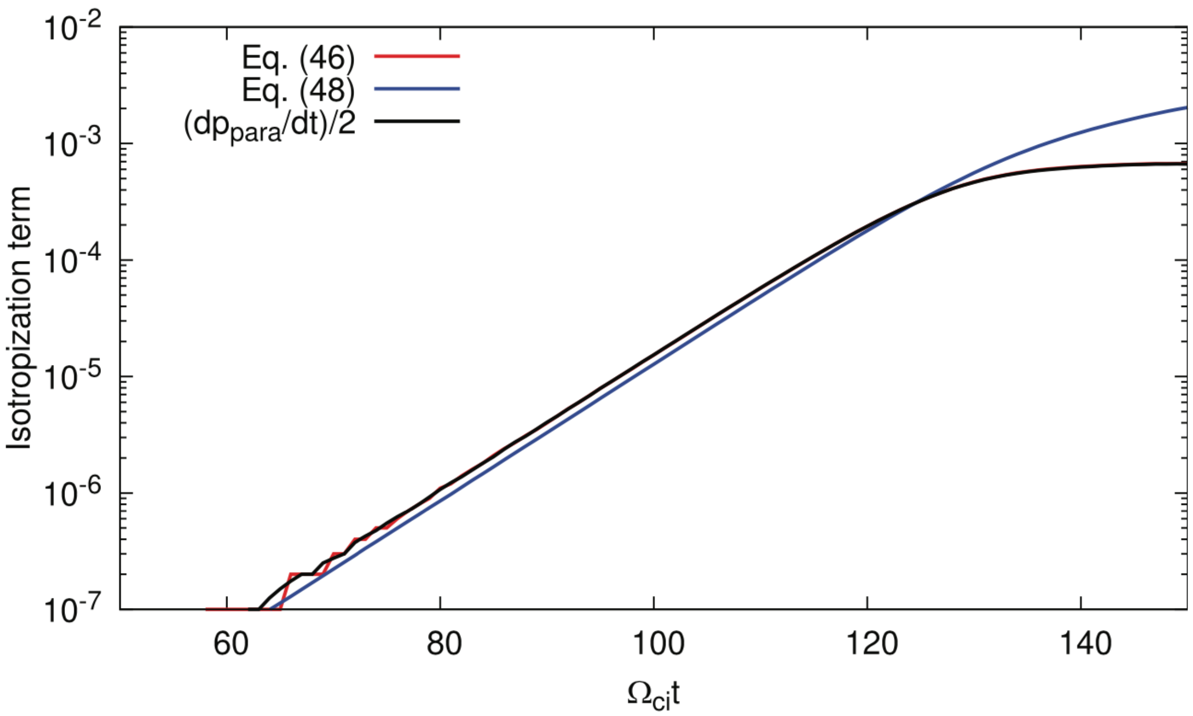
$$\frac{\partial p_\perp}{\partial t} + \frac{\partial}{\partial z}(p_\perp u_z) + p_{xz}\frac{\partial u_x}{\partial z} + p_{yz}\frac{\partial u_y}{\partial z} - \boxed{\frac{e}{m}(B_x p_{yz} - B_y p_{xz})} = 0, \quad (43)$$

$$\frac{\partial p_\parallel}{\partial t} + \frac{\partial}{\partial z}(p_\parallel u_z + q_{zzz}) + 2\left[p_\parallel \frac{\partial u_z}{\partial z} + \boxed{\frac{e}{m}(B_x p_{yz} - B_y p_{xz})}\right] = 0, \quad (44)$$

The off-diagonal pressure components (albeit small in magnitude) play the role for the isotropization.

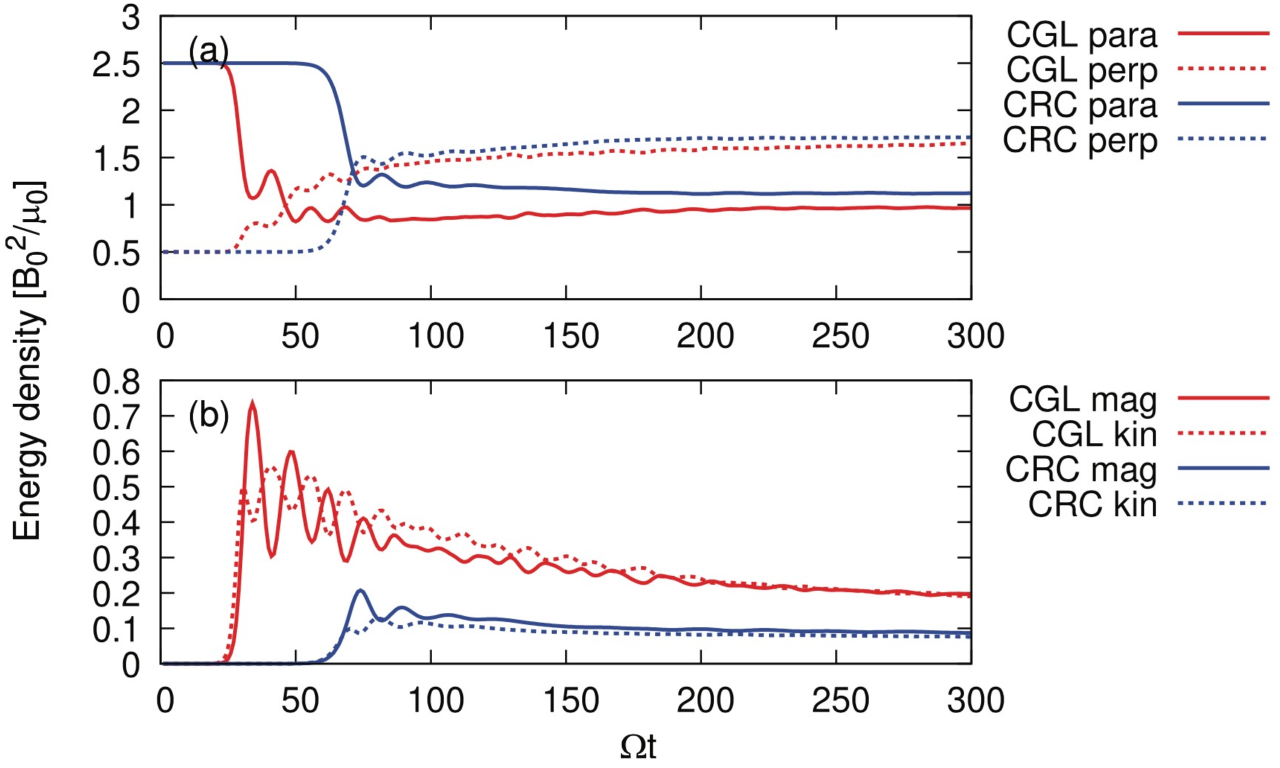
QL evaluation of dominant nonlinear term

$$\frac{e}{m}(B_x p_{yz} - B_y p_{xz})_{k=0} = \left(\frac{e}{m}\right)^2 \int dk' \left|\hat{B}(k')\right|^2 \times \mathrm{Im}\left[\frac{p_{\parallel 0}\left[\omega\left(1+\frac{\Pi}{p_{\parallel 0}}\right) + A(\omega - \Omega)\right]}{(\omega - \Omega - k'\nu)(\omega - \Omega) - k'^2 v_{\mathrm{th}}^2\left(1+\frac{\Pi}{p_{\parallel 0}}\right)}\right].$$

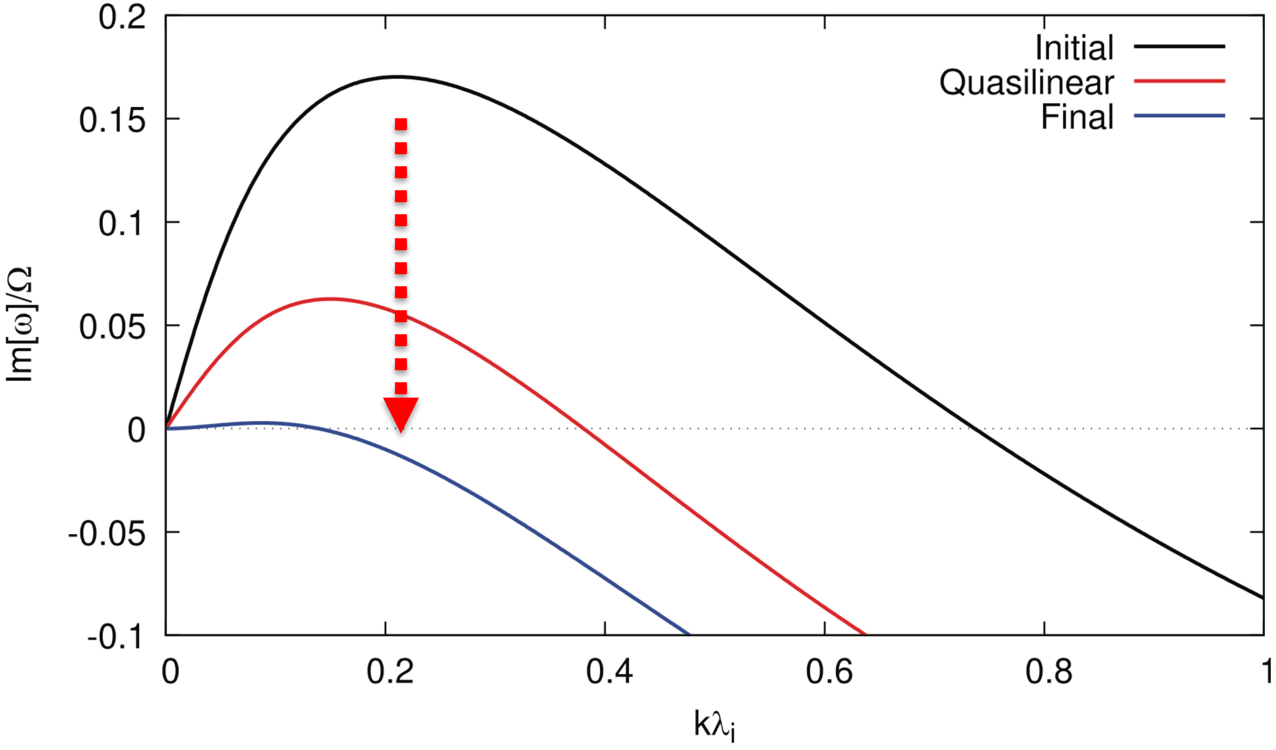


Firehose Instability

$\beta_{/\!/} = 10,\ T_{\perp}/T_{/\!/} = 0.1$

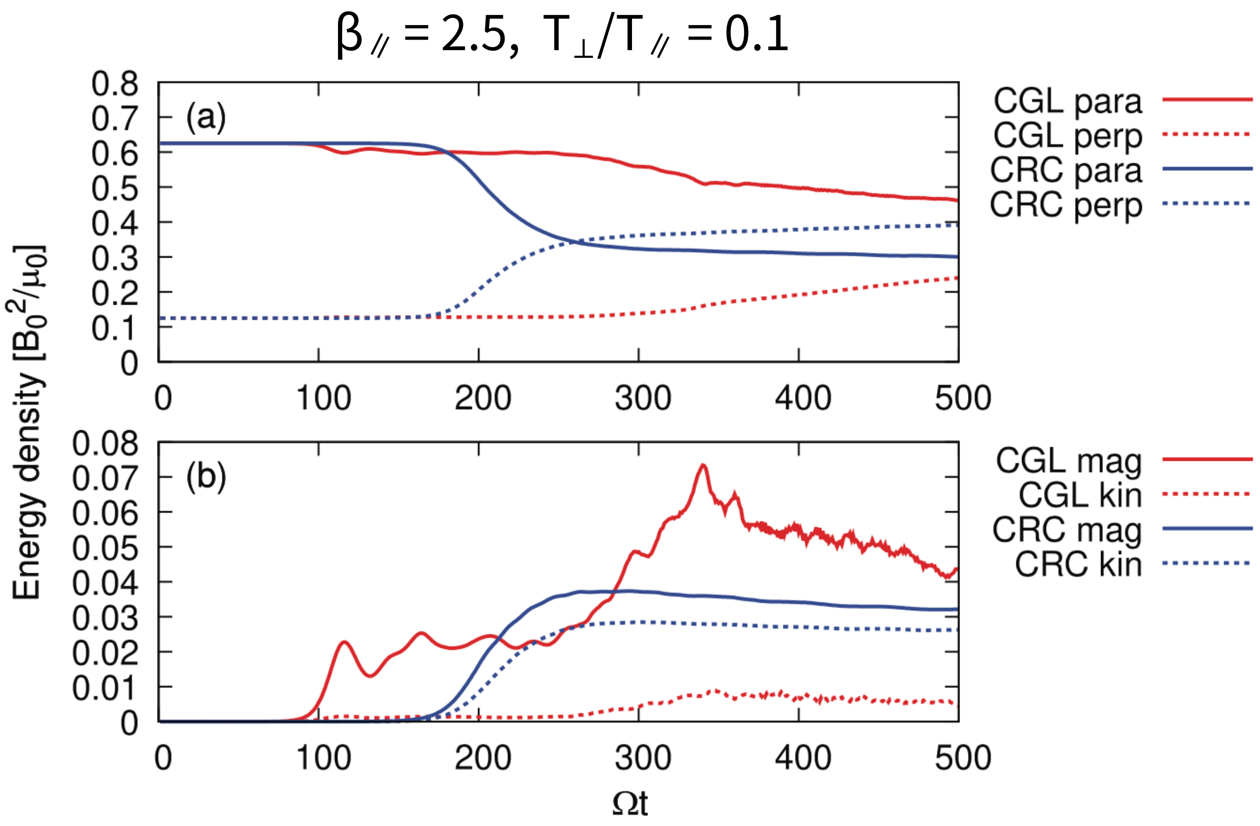


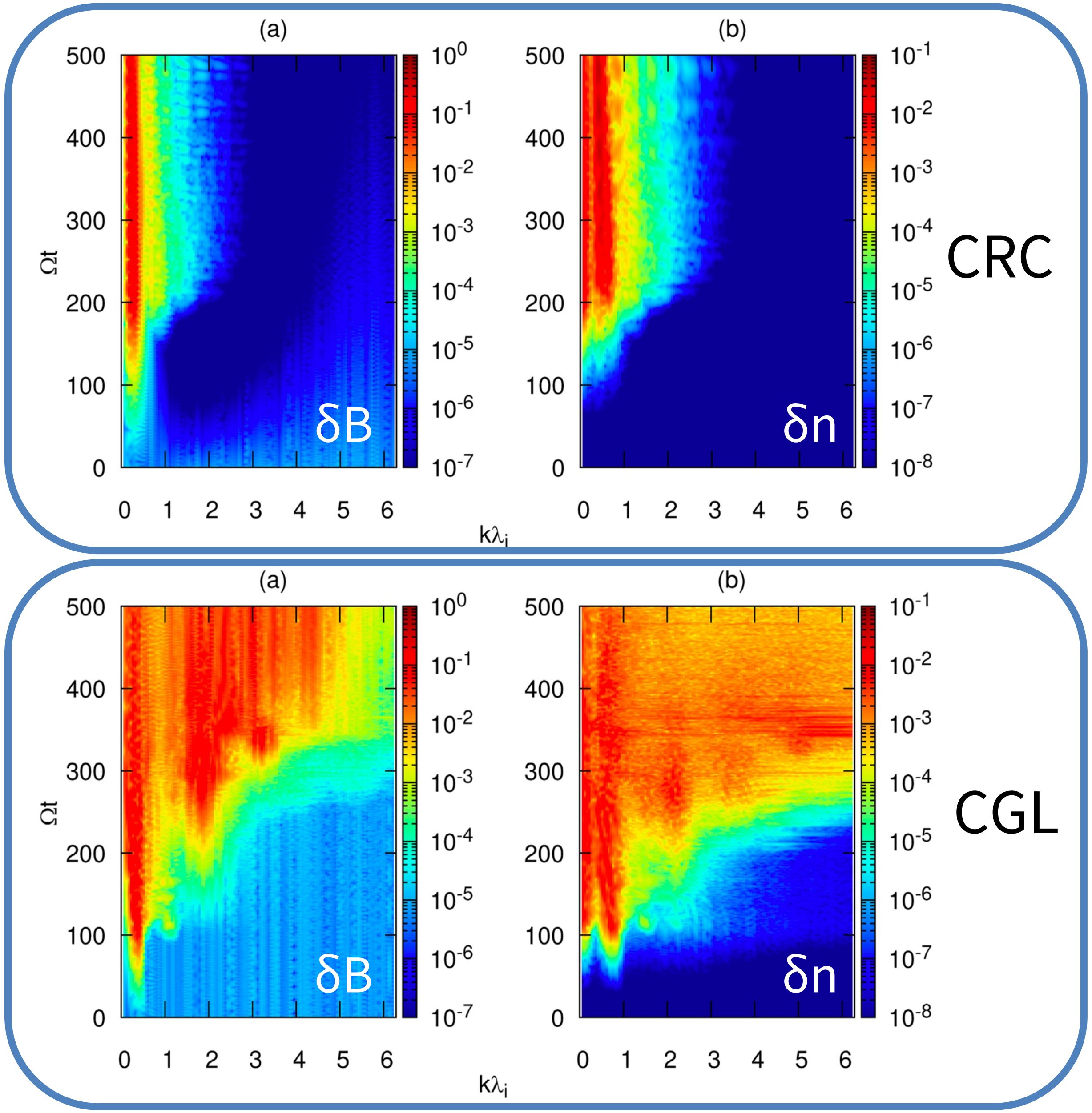
relaxation to marginal stability in CRC



- Initial anisotropy is relaxed quasi-linearly in both CRC and CGL.
- The final quasi-steady state is consistent with the marginal stability in CRC, whereas the CGL shows over-relaxation.

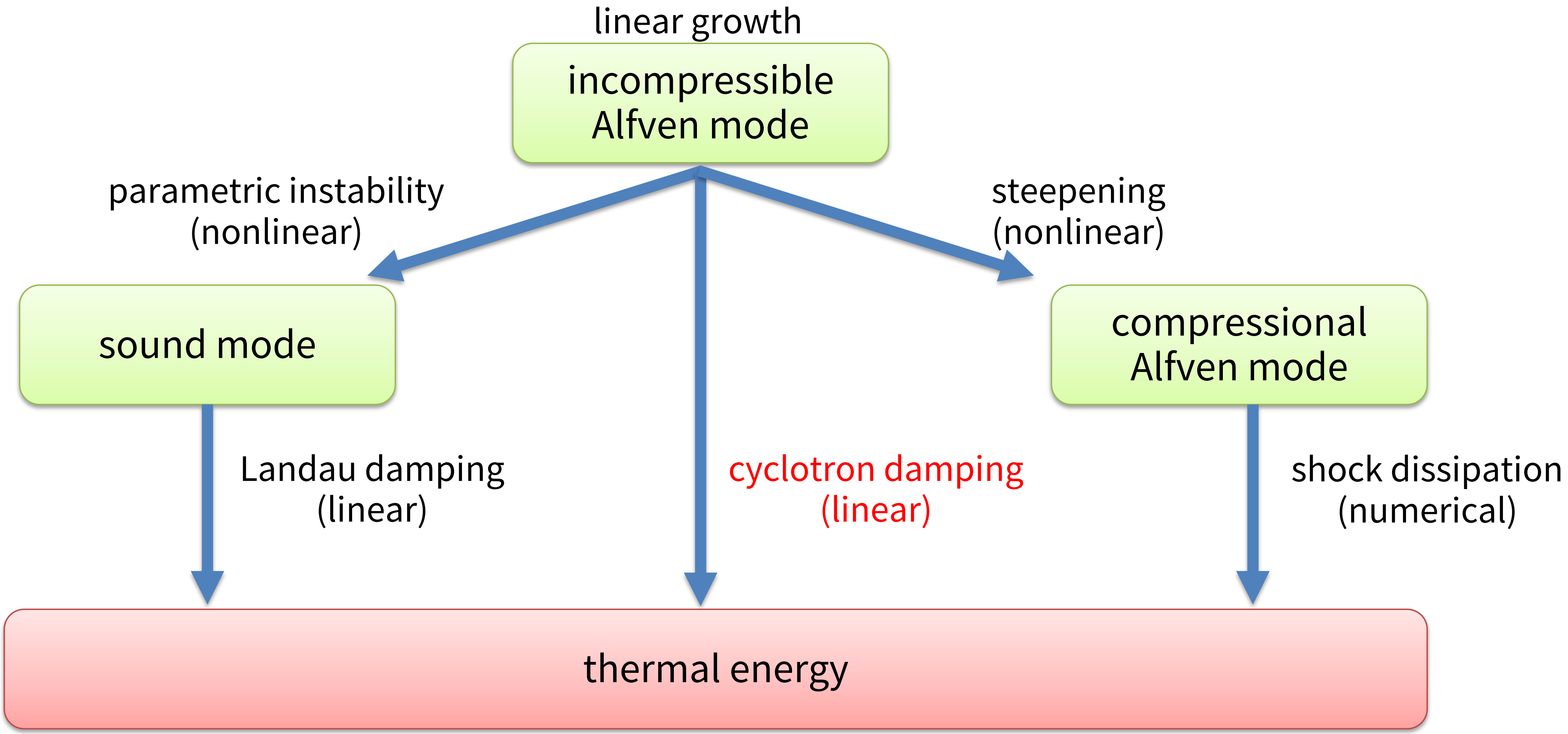
Role of Dissipation





Both magnetic and density wave powers of CGL are substantially higher than CRC at short wavelength, implying an issue of CGL in the nonlinear development.

Energy Dissipation Channel



- The lack of cyclotron damping in CGL induces much stronger fluctuations at short wavelengths.
- High-frequency nature of short wavelength fluctuations violates the assumption of CGL-FLR model.

Conclusions

- The standard MHD has been widely used in many applications. However, the adiabatic equation of state is not appropriate even at macroscopic scales for collisionless and weakly collisional plasma modeling.
- We have introduced for the first time a novel closure in wavenumber space to take into account the linear cyclotron resonance of transverse fluctuations, which may have a wider range of applications than traditional CGL-type models.
- Nonlinear simulation results demonstrate that the cyclotron damping plays an important role in the long-term development of the firehose instability.

References

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