Magnetogenesis via the canonical battery effect

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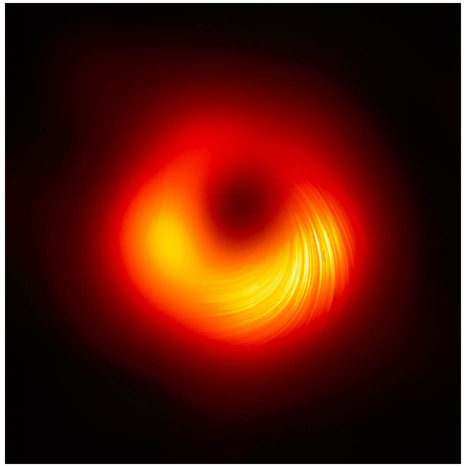
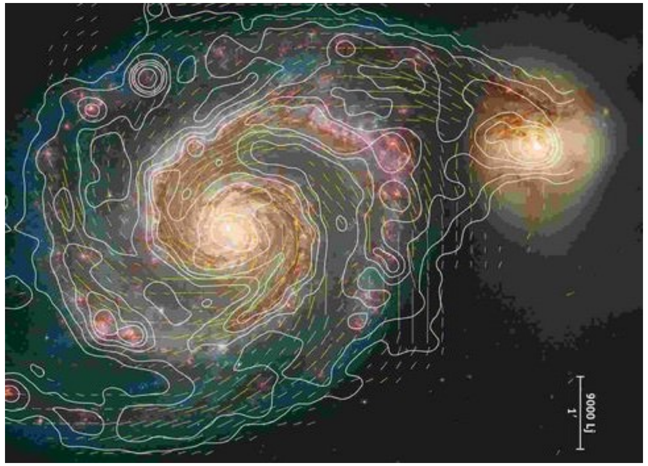
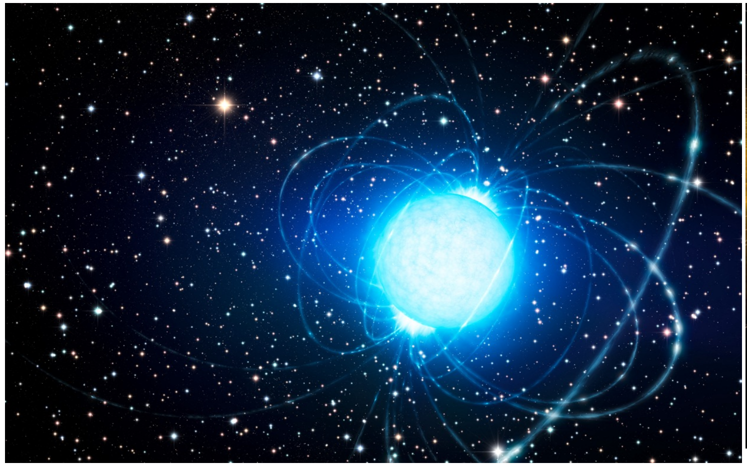
Magnetic fields in the Universe

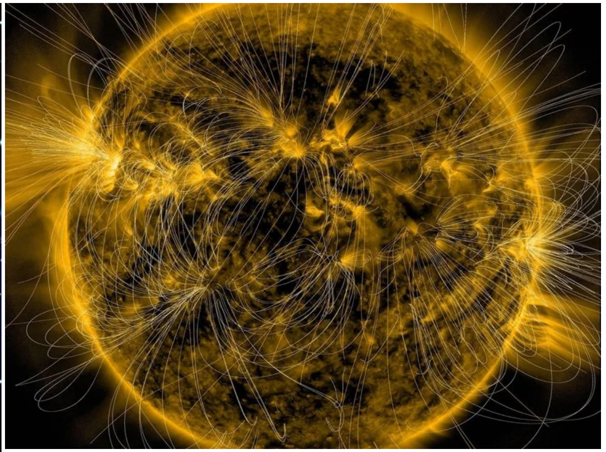
- Magnetic fields are everywhere in the Universe at all scales and strength

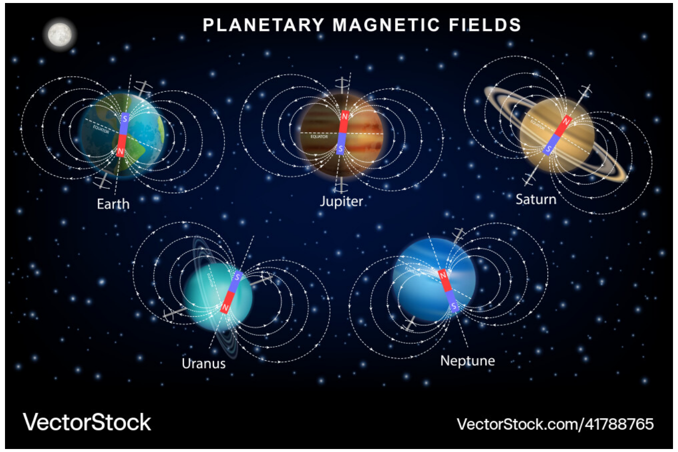

Black Holes



Galaxies


Neutron Stars


Stars



Planets

μG --- Interstellar Medium, Galaxies

G --- Black Holes, Stars, Planets

10^{10}G --- Neutron Star (Magnetars)

Standard cosmology: no magnetic fields were created at the big bang

How were the primordial magnetic fields generated in the first place?

Plasma Dynamo

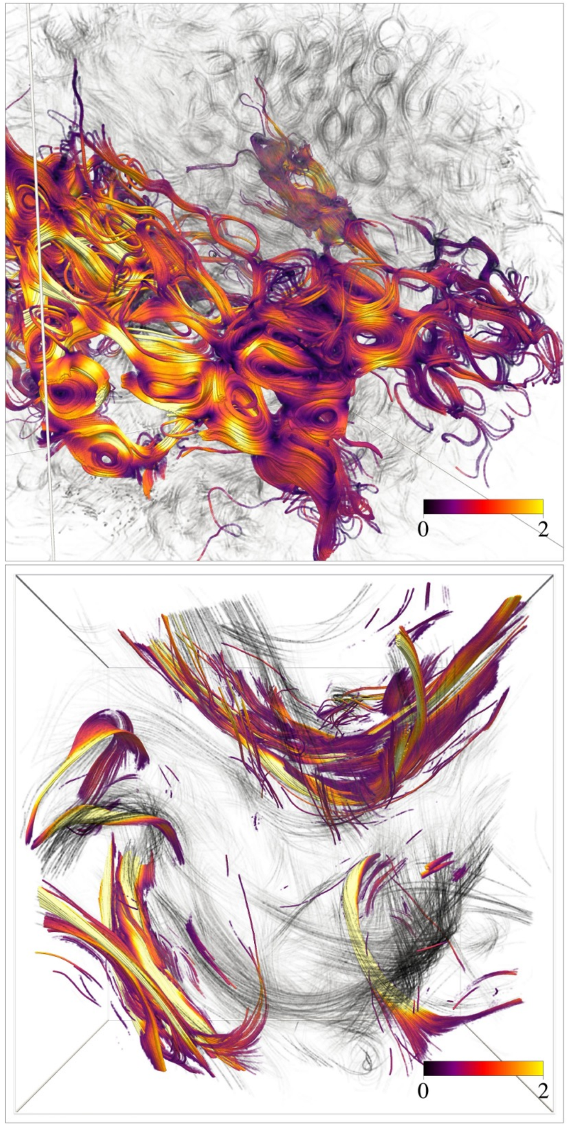
- Plasma induction equation is the underlying equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{U}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B}$$

- If $\boldsymbol{B} = 0$ at $t = 0$,

$$\frac{\partial \boldsymbol{B}}{\partial t} = 0$$

- We need a source for the "seed" magnetic field!



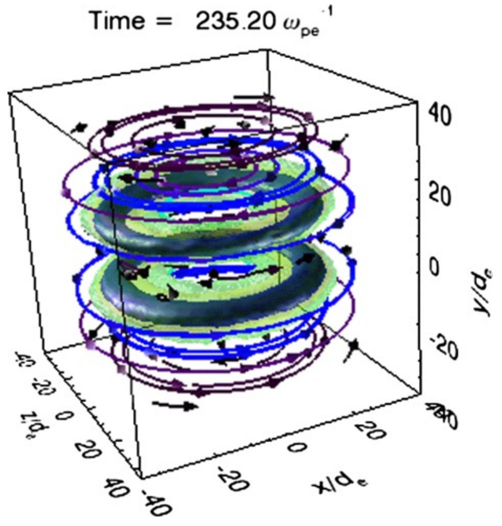
Zhou+, ApJ (2023)

Seed Magnetogenesis

- Two popular mechanisms for seed magnetogenesis
- Biermann battery [Biermann, 1950]
  - Misalignment of density and temperature

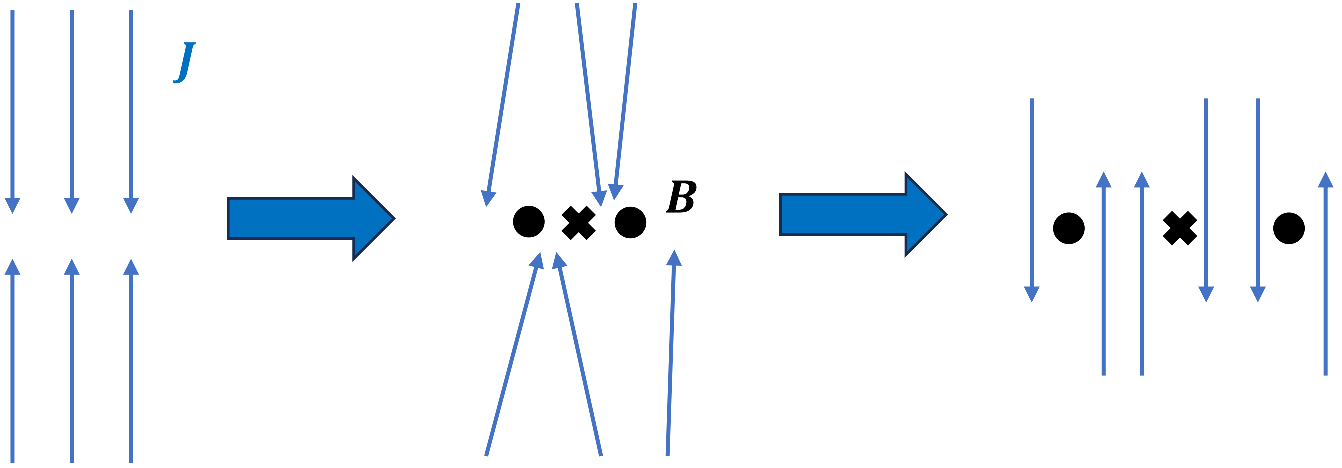
$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{\nabla n \times \nabla p}{e n^2}$$

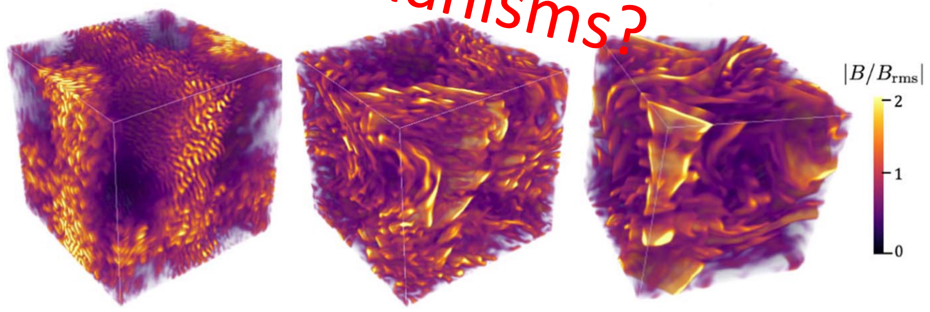
Can we generalize these mechanisms and find new mechanisms?



[Schoeffler+, PRL (2014)]

- Weibel Instability [Weibel, 1959]
  - Anisotropic pressure or, equivalently, counter-propagating beams





[Zhou+, ApJ (2023)]

Canonical Battery Effect

- Take the equation of motion (first moment of Vlasov equation)

$$m_\sigma\left(\frac{\partial \boldsymbol{u}_\sigma}{\partial t}+\boldsymbol{u}_\sigma\cdot\nabla\boldsymbol{u}_\sigma\right) = q_\sigma\boldsymbol{E}+q_\sigma\boldsymbol{u}_\sigma\times\boldsymbol{B}-\frac{\nabla\cdot\boldsymbol{p}_\sigma}{n_\sigma}$$

- Take curl:

$$m_\sigma\left(\frac{\partial\nabla\times\boldsymbol{u}_\sigma}{\partial t}-\nabla\times(\boldsymbol{u}_\sigma\times\nabla\times\boldsymbol{u}_\sigma)\right) = -q_\sigma\frac{\partial\boldsymbol{B}}{\partial t}+q_\sigma\nabla\times(\boldsymbol{u}_\sigma\times\boldsymbol{B})-\nabla\times\left(\frac{\nabla\cdot\boldsymbol{p}_\sigma}{n}\right)$$

- Rearrange:

$$\frac{\partial}{\partial t}(m_\sigma\nabla\times\boldsymbol{u}_\sigma+q_\sigma\boldsymbol{B}) = \nabla\times\left(\boldsymbol{u}_\sigma\times(m_\sigma\nabla\times\boldsymbol{u}_\sigma+q_\sigma\boldsymbol{B})\right)-\nabla\times\left(\frac{\nabla\cdot\boldsymbol{p}_\sigma}{n_\sigma}\right)$$

Define canonical vorticity $\boldsymbol{Q}_\sigma = m_\sigma\nabla\times\boldsymbol{u}_\sigma+q_\sigma\boldsymbol{B}$

- Canonical induction equation

$$\frac{\partial\boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{U}\times\boldsymbol{B}) \quad\Longleftrightarrow\quad \frac{\partial\boldsymbol{Q}_\sigma}{\partial t}=\underbrace{\nabla\times(\boldsymbol{u}_\sigma\times\boldsymbol{Q}_\sigma)}_{\text{Convection}}-\underbrace{\nabla\times\left(\frac{\nabla\cdot\boldsymbol{p}_\sigma}{n_\sigma}\right)}_{\text{“Canonical Battery”}}$$

Canonical Battery Effect

- Let us restrict ourselves to electrons for now

$$\frac{\partial \boldsymbol{Q_e}}{\partial t} = \nabla\times(\boldsymbol{u_e}\times\boldsymbol{Q_e}) - \nabla\times\left(\frac{\nabla\cdot\boldsymbol{p_e}}{n_e}\right) = \boldsymbol{\mathcal{C}} + \boldsymbol{\mathcal{B}}$$

- What is $\boldsymbol{Q_e}$?

Canonical Vorticity $\boldsymbol{Q_e} = m_e\nabla\times\boldsymbol{u_e} + q_e\boldsymbol{B} \simeq q_e(d_e^2\nabla^2\boldsymbol{B} + \boldsymbol{B})$

- $\boldsymbol{Q_e}$ is a proxy for $\boldsymbol{B}$
  - $\boldsymbol{Q_e} \simeq q_e d_e^2\nabla^2\boldsymbol{B}$ for $L < d_e$
  - $\boldsymbol{Q_e} \simeq q_e\boldsymbol{B}$ for $L > d_e$

- When $\boldsymbol{Q_e} = 0$,

$$\frac{\partial \boldsymbol{Q_e}}{\partial t} = -\nabla\times\left(\frac{\nabla\cdot\boldsymbol{p_e}}{n_e}\right) = \boldsymbol{\mathcal{B}}$$

- Canonical battery is the only term that can spontaneously generate $\boldsymbol{Q_e}$ and thus $\boldsymbol{B}$!

Canonical Battery Effect

- So what is canonical battery?
- Assume isotropic pressure, i.e., $\boldsymbol{p_e} = p_e \boldsymbol{I}$

Biermann Battery!

$$\boldsymbol{\mathcal{B}} = -\nabla\times\frac{\nabla\cdot\boldsymbol{p_e}}{n_e} = -\nabla\times\frac{\nabla p_e}{n_e} = \boxed{\frac{\nabla n_e\times\nabla p_e}{n_e^2}}$$

- So canonical battery is a generalization of Biermann battery
- Can canonical battery generalize Weibel instability as well?
- Assume 1D, i.e., $\nabla = \hat{x}\partial/\partial x$

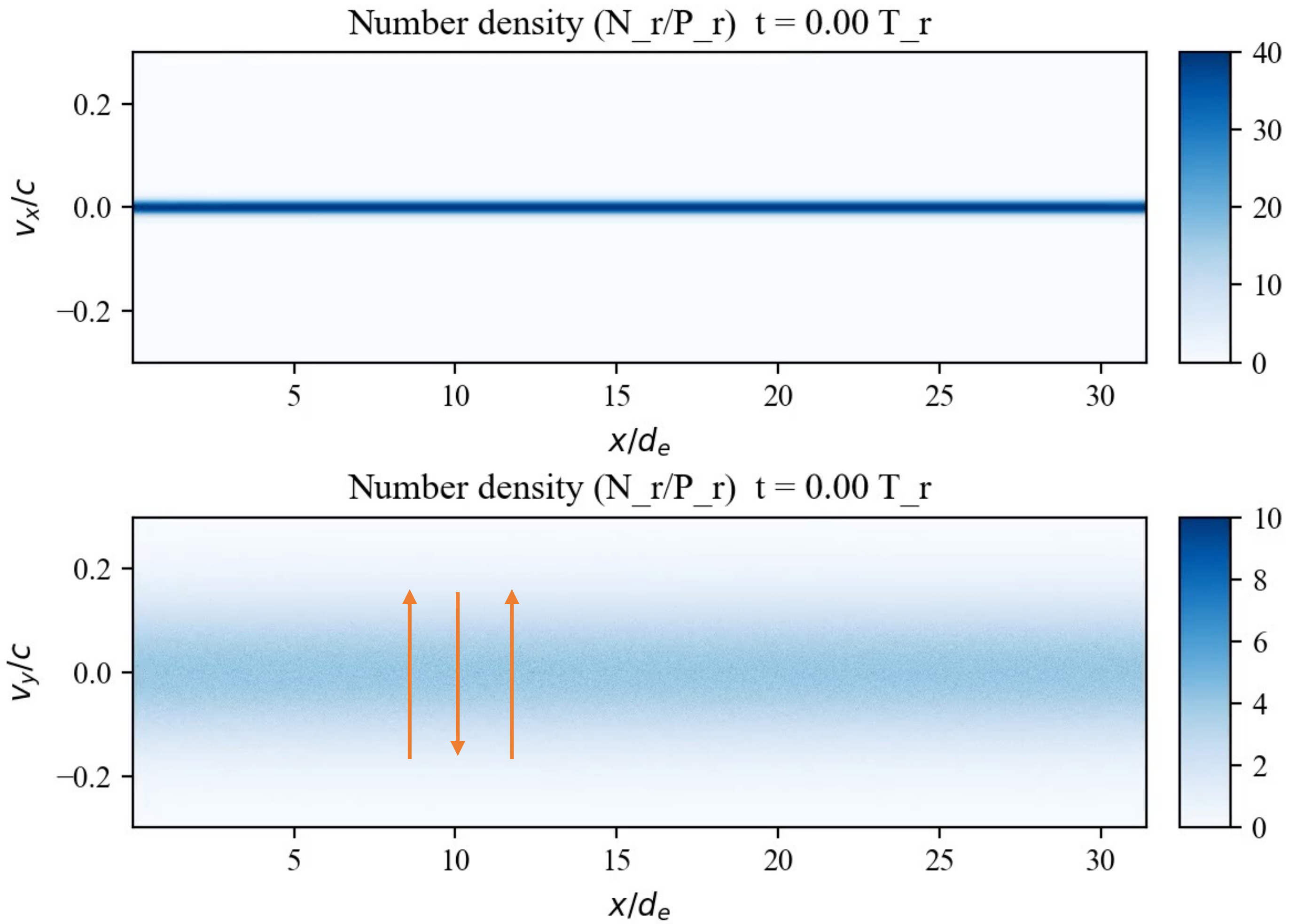
$$\mathcal{B}_z = -\left(\nabla\times\frac{\nabla\cdot\boldsymbol{p_e}}{n_e}\right)\cdot\hat{z} = -\frac{1}{n_e}\frac{\partial^2 p_{exy}}{\partial x^2}$$

- Mixing between p_{exx} and p_{eyy}

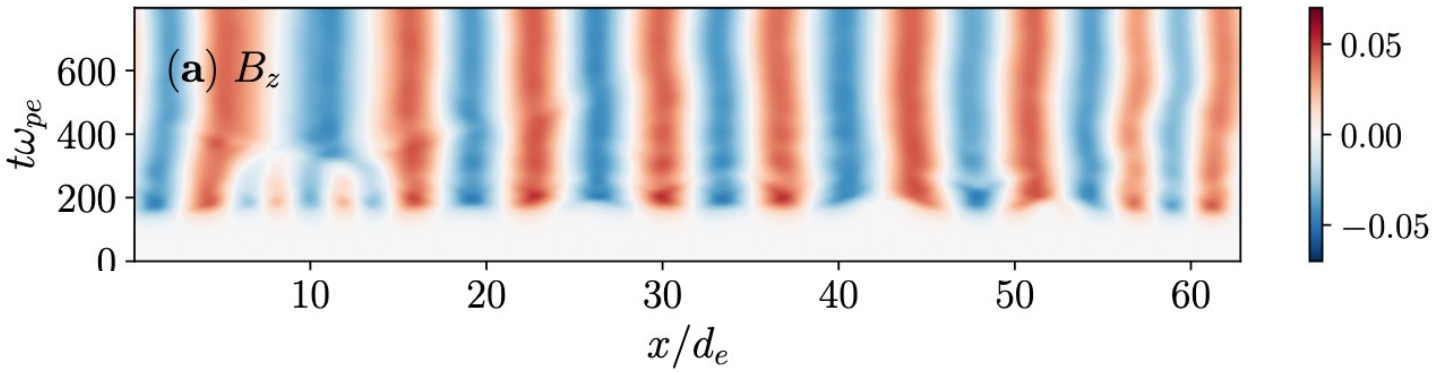
Weibel Instability

- Particle-in-cell simulations of the Weibel instability

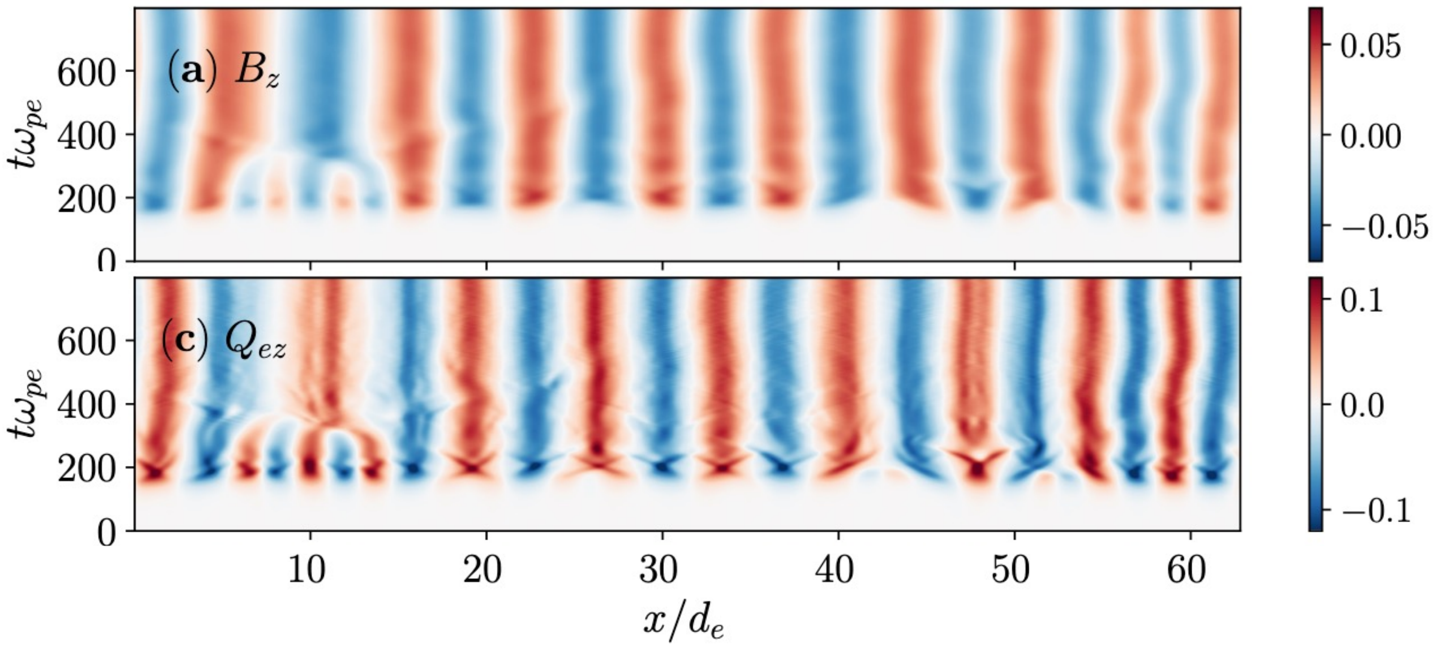
Initially, $T_{eyy} \gg T_{exx}$



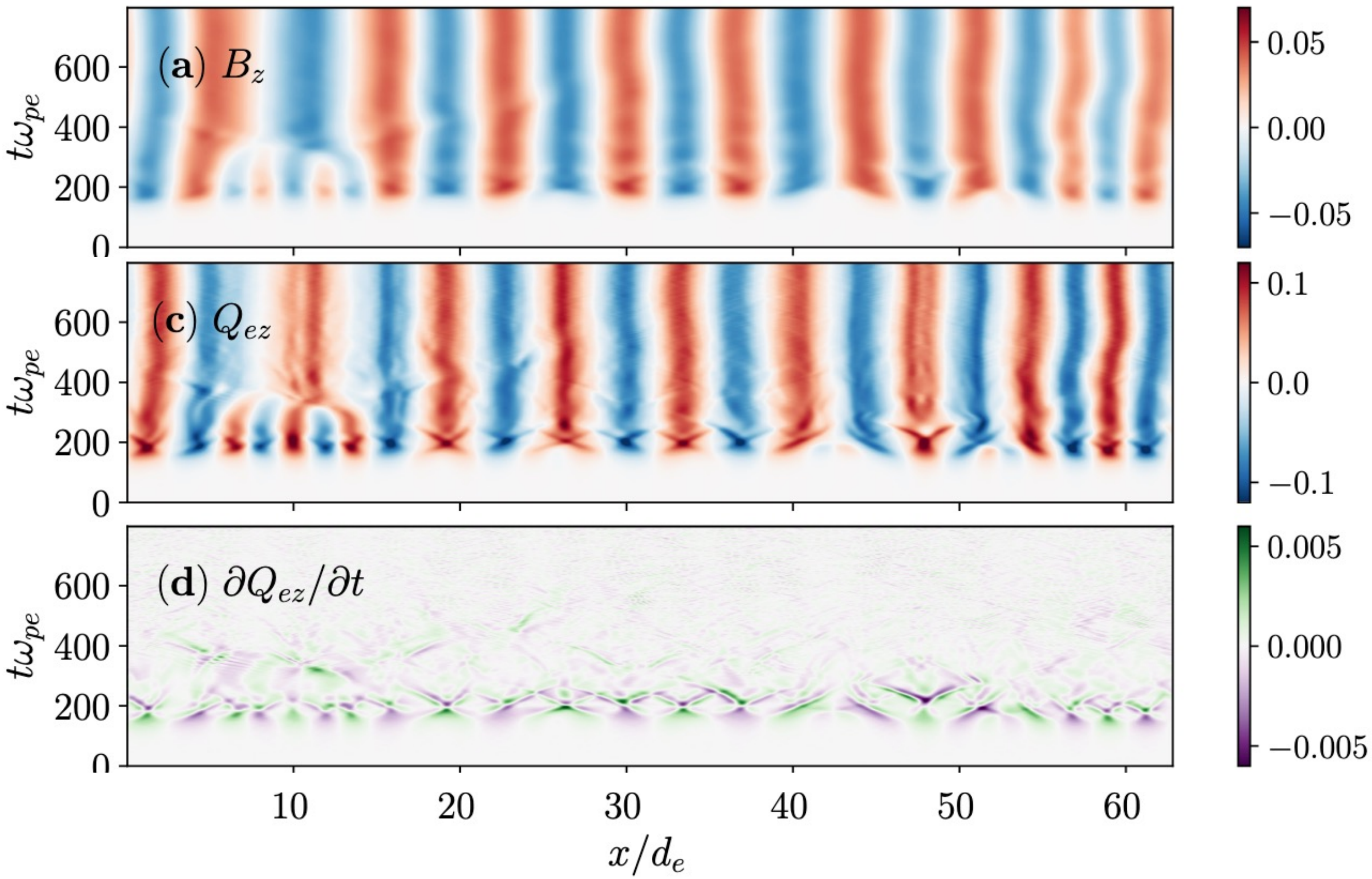
Weibel Instability



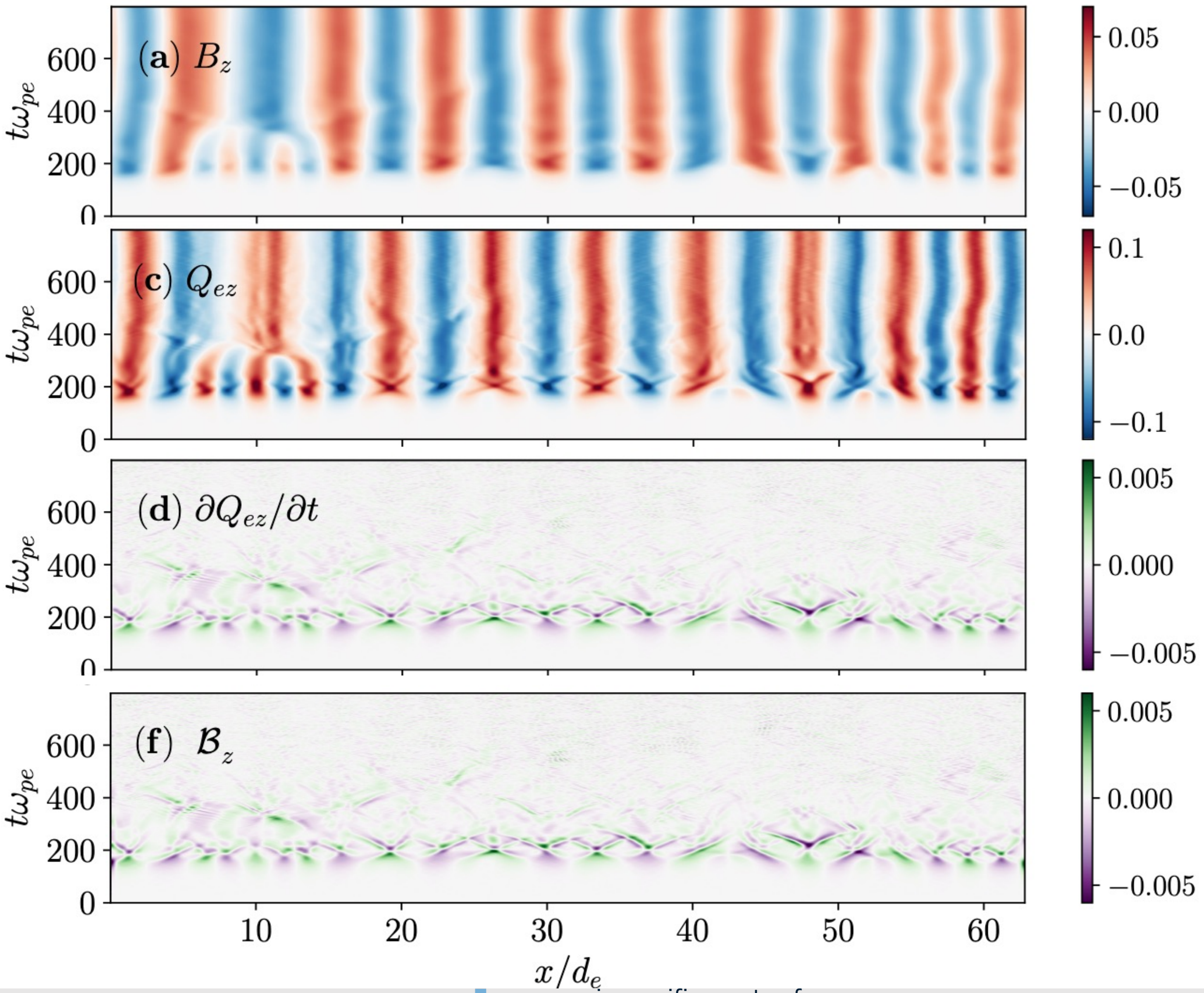
Weibel Instability



Weibel Instability



Weibel Instability



$$-\frac{1}{n_e}\frac{\partial^2 p_{exy}}{\partial x^2}$$

Canonical battery is entirely responsible for magnetogenesis

New Mechanisms?

- Canonical battery generalizes Biermann battery ✅
- Canonical battery generalizes Weibel instability ✅
- Can there be other mechanisms?
- Assume 2D ($\frac{\partial}{\partial z} = 0$) and examine the z-component

$$\mathcal{B}_z = -\left(\nabla\times\frac{\nabla\cdot\boldsymbol{p}_e}{n_e}\right)\cdot\hat{z}$$

$$= \hat{z}\cdot\left[-\nabla\left(\frac{1}{n_e}\right)\times\nabla\cdot\boldsymbol{p}_e\right]$$

Biermann-like term

New Mechanisms?

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$$\mathcal{B}_z = -\left(\nabla\times\frac{\nabla\cdot\boldsymbol{p}_e}{n_e}\right)\cdot\hat{z}$$

$$= \hat{z}\cdot\left[-\nabla\left(\frac{1}{n_e}\right)\times\nabla\cdot\boldsymbol{p}_e\right] + \frac{1}{n_e}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)p_{exy}$$

Biermann-like term　　　Weibel term

New Mechanisms?

- Canonical battery generalizes Biermann battery ✅
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- Can there be other mechanisms?
- Assume 2D ($\frac{\partial}{\partial z} = 0$) and examine the z-component

$$\mathcal{B}_z = -\left(\nabla\times\frac{\nabla\cdot\boldsymbol{p}_e}{n_e}\right)\cdot\hat{z}$$

$$= \hat{z}\cdot\left[-\nabla\left(\frac{1}{n_e}\right)\times\nabla\cdot\boldsymbol{p}_e\right] + \frac{1}{n_e}\left(\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial x^2}\right)p_{exy} + \frac{1}{n_e}\frac{\partial^2}{\partial x\partial y}\left(p_{exx}-p_{eyy}\right)$$

Biermann-like term — Weibel term — New term (2D-localized pressure anisotropy)

2D-localized Pressure Anisotropy

- Last term

$$\mathcal{B}_z = \frac{1}{n_e} \frac{\partial^2}{\partial x \partial y} \left(p_{exx} - p_{eyy} \right)$$

- Assume

$$p_{exx} - p_{eyy} \sim \exp\left(-\frac{x^2 + y^2}{\sigma^2} \right)$$

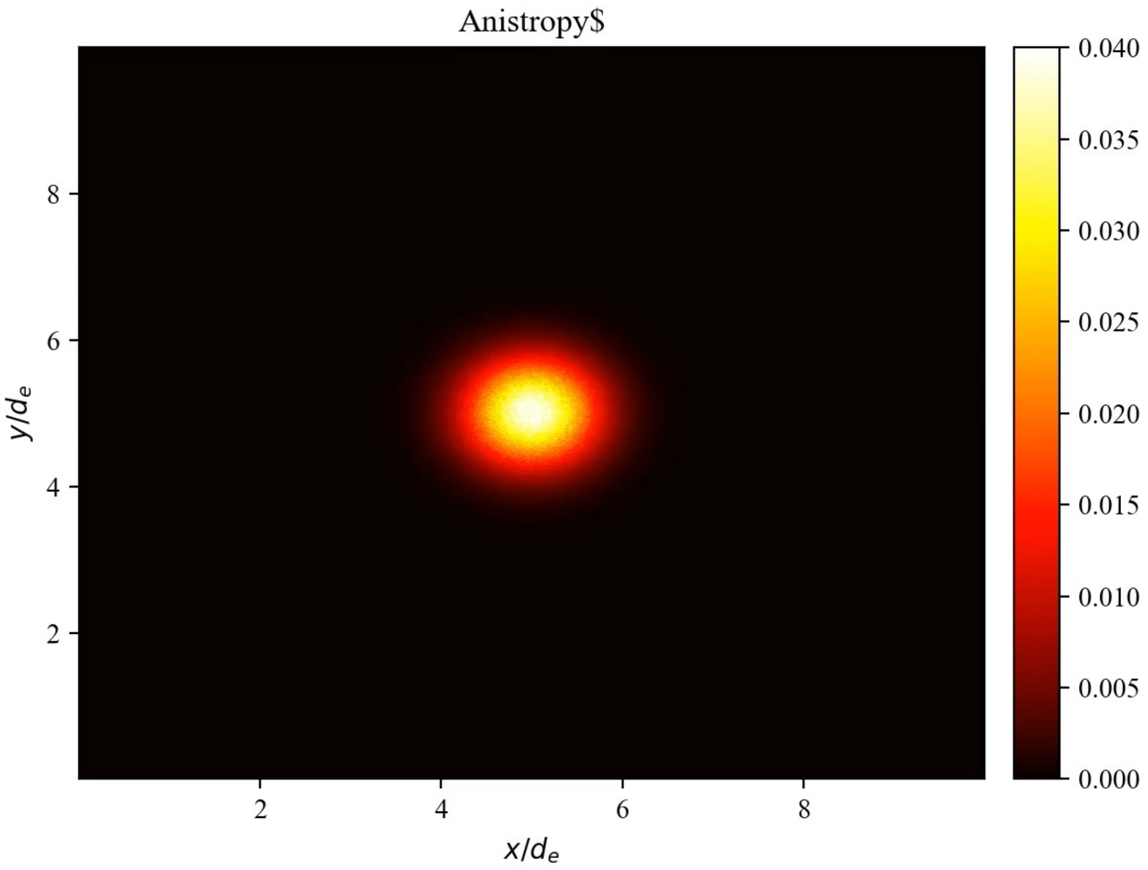
- Then

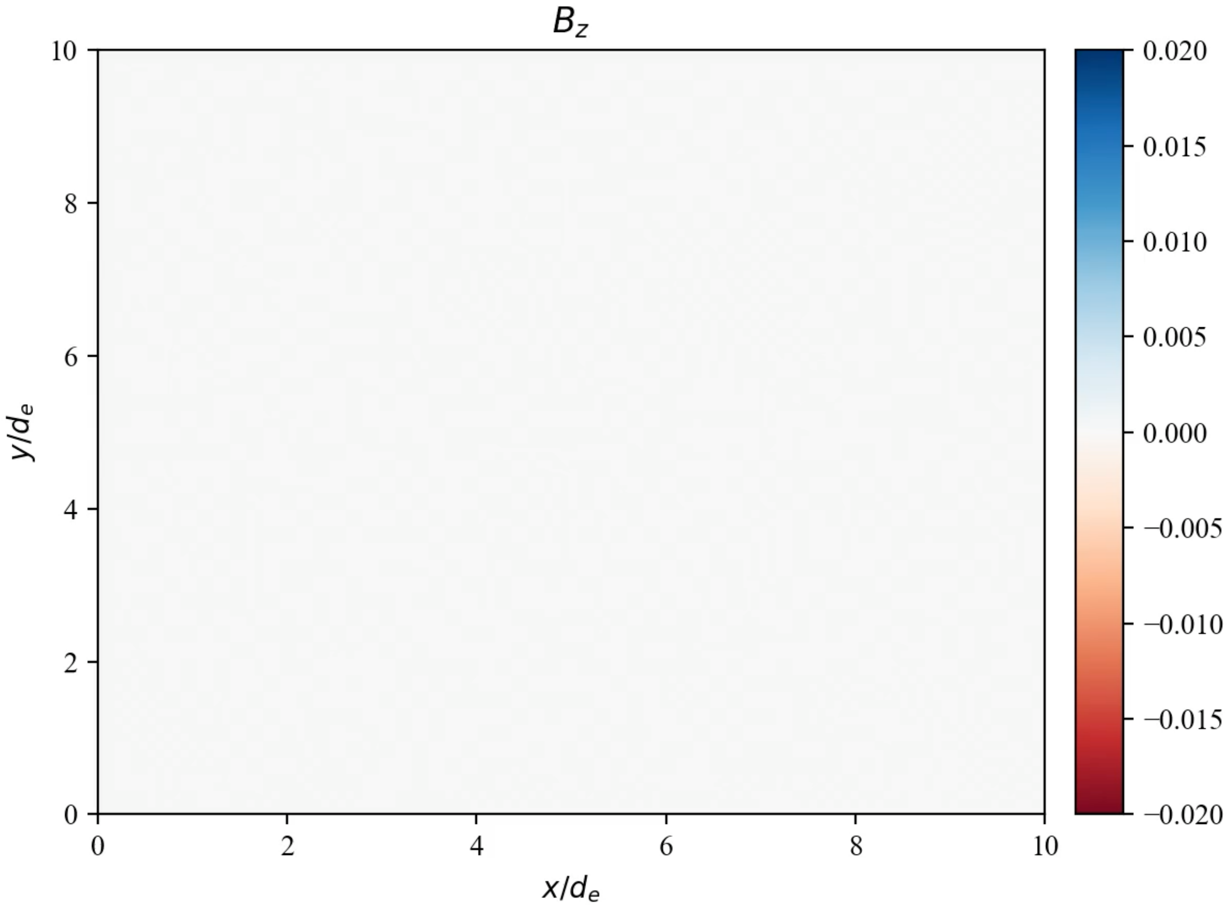
$$\mathcal{B}_z \sim xy$$

- Quadrupole magnetic fields expected

2D-localized Pressure Anisotropy

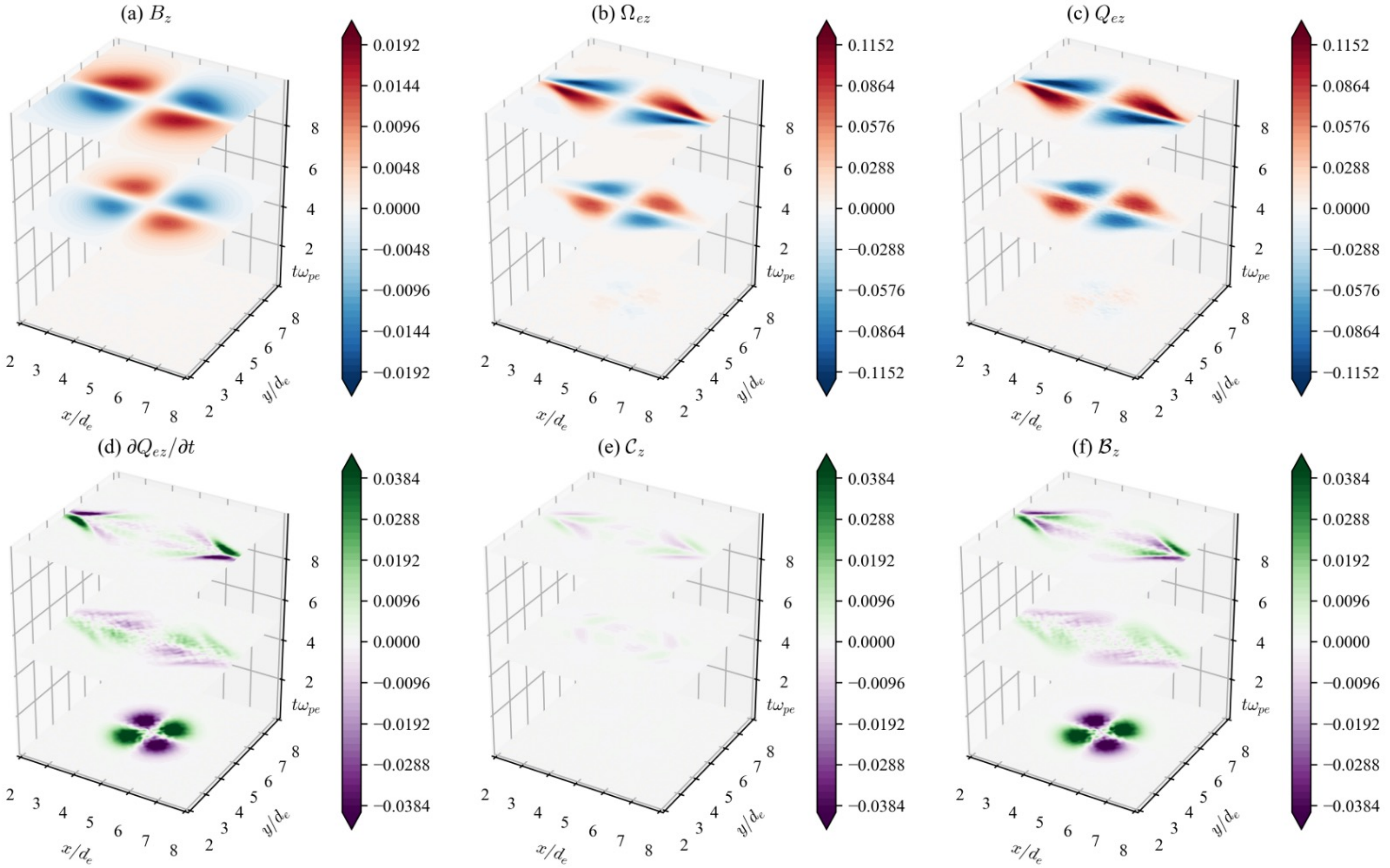
- Again, verify with particle-in-cell simulations





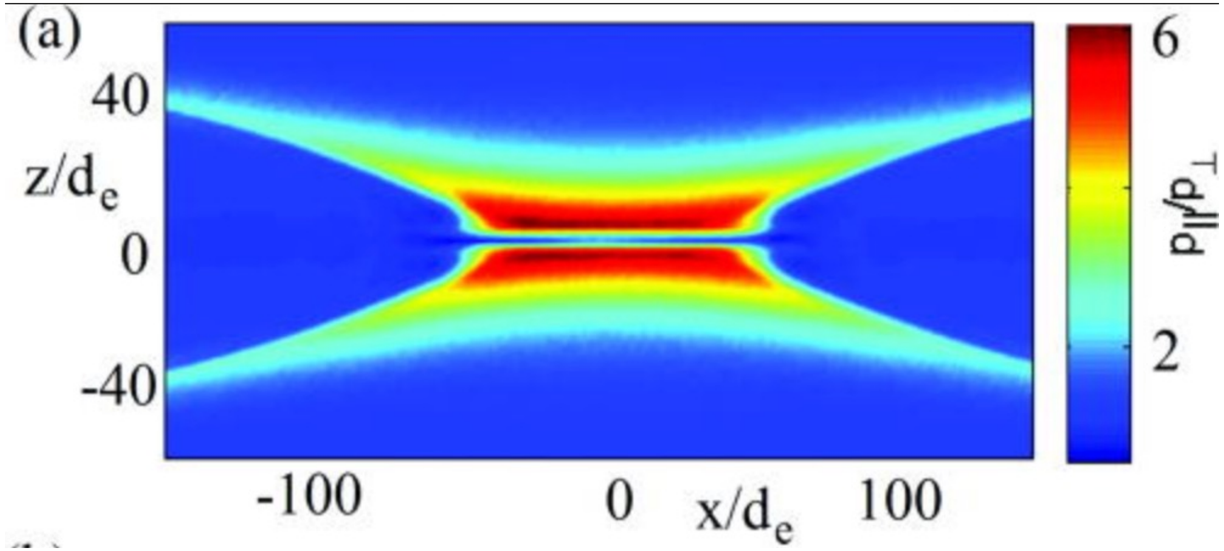
2D-localized Pressure Anisotropy

- Again, canonical battery term is confirmed to be the source!

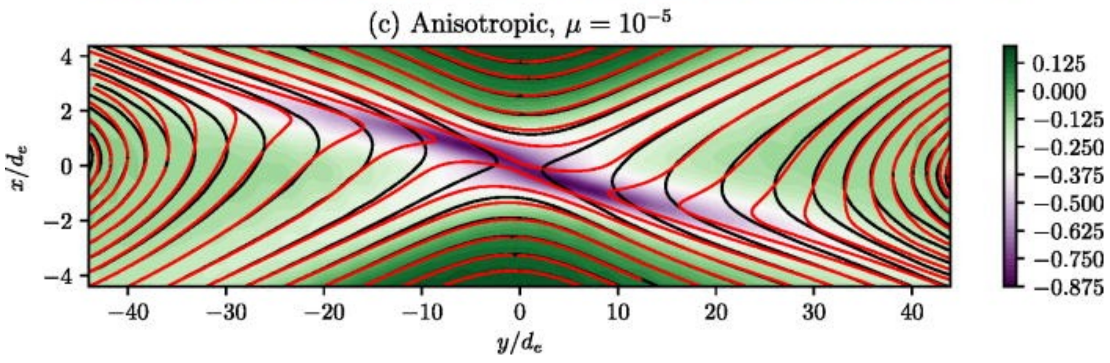


2D-localized Pressure Anisotropy

- Important in magnetic reconnection as well



Egedal+, PoP (2013)



Yoon and Bellan, PoP (2019)

What next?

- Relativistic regime: one more term solely due to relativity

$$\frac{\partial Q_{ez}}{\partial t} = \hat{z} \cdot \nabla \times (\boldsymbol{u_e} \times \boldsymbol{Q_e}) - \hat{z} \cdot \nabla \times \frac{\nabla \cdot \boldsymbol{p_e}}{n_e} + \sum_{i=x,y} [u_i, p_i]$$

- 3D terms when $\frac{\partial}{\partial z} \neq 0$?

- Competition between convective and battery terms: magnetogenesis to dynamo

$$\frac{\partial \boldsymbol{Q_e}}{\partial t} = \nabla \times (\boldsymbol{u_e} \times \boldsymbol{Q_e}) - \nabla \times \left(\frac{\nabla \cdot \boldsymbol{p_e}}{n_e} \right)$$

[Zhou+, ApJ (2023); Sironi+, PRL (2023)]

- Ion canonical battery?

Summary

- Dynamo requires a seed magnetic field
- Canonical battery generalizes known mechanisms for seed magnetogenesis
- Canonical battery also predicts new mechanisms: 2D-localized pressure anisotropy in 2D
- Lots of interesting things to do within the framework