# “The Hough Transform Search for Continuous Gravitational Waves”:  20 Years Later

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(on behalf of A.Sintes, M.A.Papa,
B.F.Schutz, S.Frasca, C.Palomba)

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# Hough transform search for continuous gravitational waves

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This paper describes an incoherent method to search for continuous gravitational waves based on the *Hough transform*, a well-known technique used for detecting patterns in digital images. We apply the Hough transform to detect patterns in the time-frequency plane of the data produced by an earth-based gravitational wave detector. Two different flavors of searches will be considered, depending on the type of input to the Hough transform: either Fourier transforms of the detector data or the output of a coherent matched-filtering type search. We present the technical details for implementing the Hough transform algorithm for both kinds of searches, their statistical properties, and their sensitivities.

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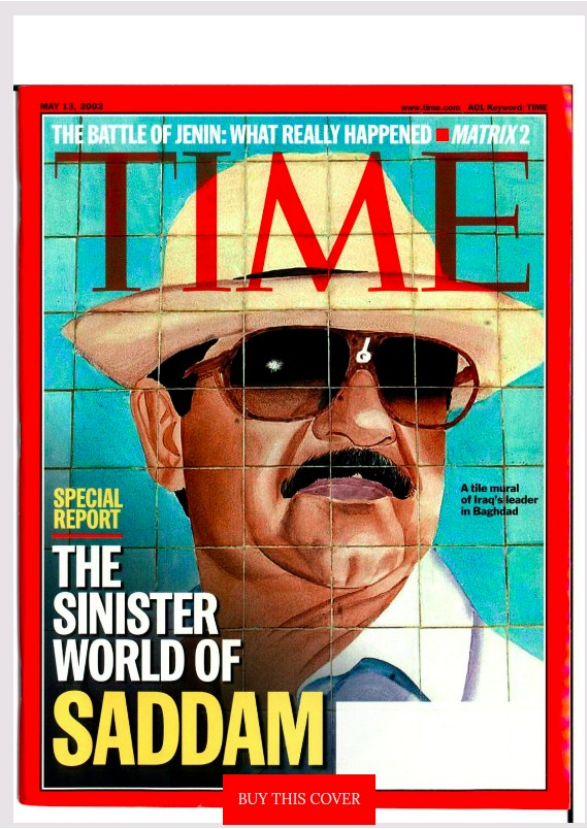
# The Context in 2002

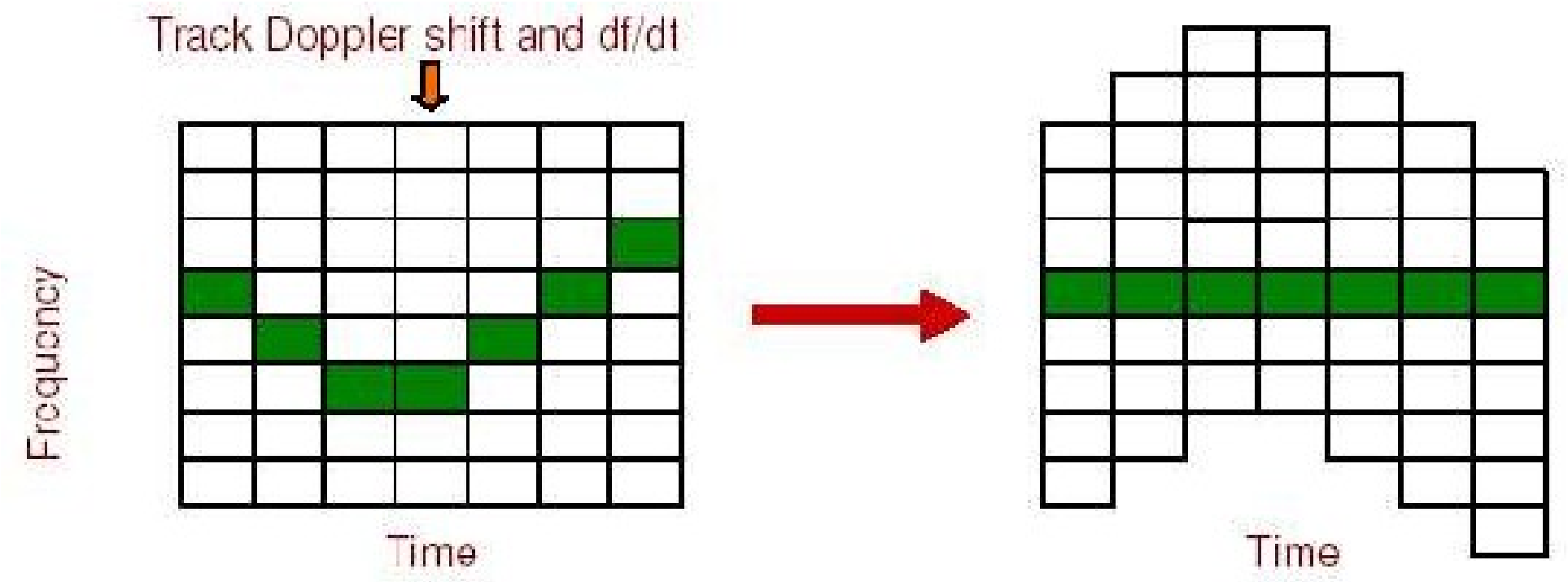

# The Context in 2002

- August 2002: I started my postdoc in MAP's group at AEI Potsdam
- Stimulating atmosphere at AEI Potsdam with diverse research groups – Ed Seidel was the other group leader in Bernard's department (Ehlers and Nicolai were the other directors)
- Struggles with "LALDemod" during the winter → this later became ComputeFStatistic (Reinhard joined MAP's group in 2003)
- MAP decides to send me off to Palma in early 2003 to learn about the Hough transform from Alicia

# The Context in 2002

- The LIGO Detectors had started collecting data
- S1: 17 Days starting from August 23– September 09, 2002
- The CW group was led by MAP and Mike Landry
- The first published observational result was a targeted search for PSR J1939+2134 (using both LIGO and GEO data) – arXiv submission on August 14, 2003 (the first BNS result came a week later)
- This used both the F-statistic and a time domain Bayesian method
- It was clear that we need other methods for large parameter space searches

# The Context in 2002



# The Context in 2002

- The data analysis methods, software and also the collaboration were being formed
- Recall: even SFTs were not accepted as a standard data format at that time, and LDAS was supposed to be the platform for searches
- Our CW software was primarily hosted on a CVS at AEI Potsdam, LAL was being developed, and Jolien was the librarian for the pulgroup repository
- The Hough Transform was one of the semi-coherent CW methods under development (along with PowerFlux and StackSlide)

# Development of the Hough analysis

- The Hough Transform was one of these methods -- led by MAP and Alicia
- Builds on previous work in Rome (see talk by Sergio Frasca)
- The basic routines/functions for the Hough transform were available (along with Alicia's very extensive notes and diagrams!)
- My task: Put together these functions into a search pipeline, understand the statistics and thresholds, and apply it to S2 data (and write a methods paper)
- First trip to Palma in 2003 was very important for this project

# The First LIGO Results using Hough – S2 and S4

PHYSICAL REVIEW D **72,** 102004 (2005)

**First all-sky upper limits from LIGO on the strength of periodic gravitational waves using the Hough transform**

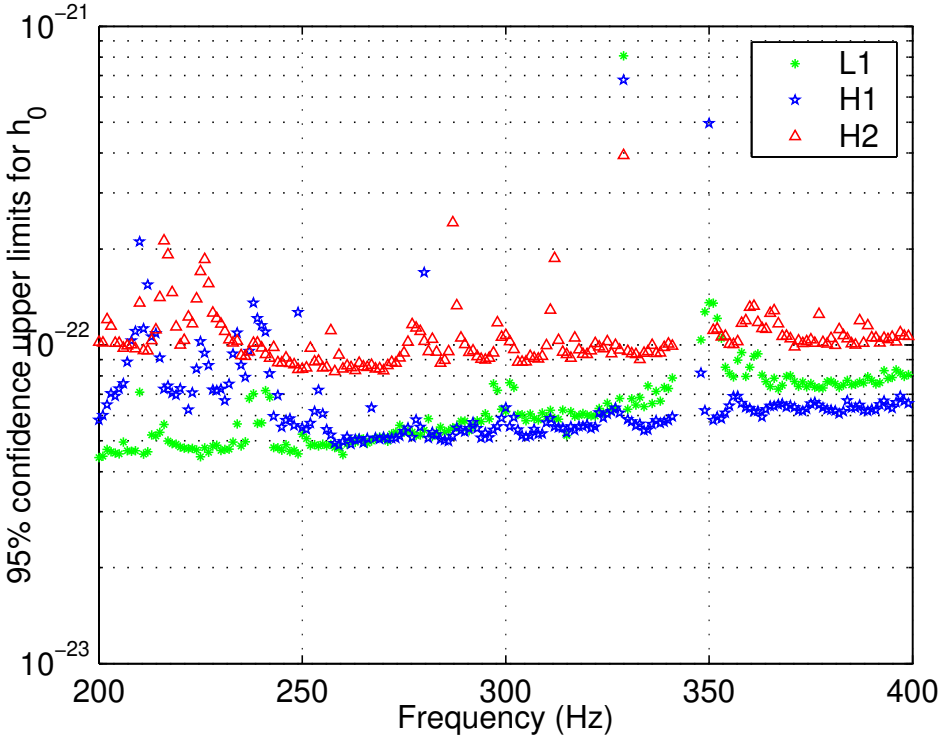
B. Abbott,[12] R. Abbott,[15] R. Adhikari,[12] A. Ageev,[20,27] J. Agresti,[12] B. Allen,[39] J. Allen,[13] R. Amin,[16] S. B. Anderson,[12] W. G. Anderson,[29] M. Araya,[12] H. Armandula,[12] M. Ashley,[28] F. Asiri,[12,a] P. Aufmuth,[31] C. Aulbert,[1] S. Babak,[7] R. Balasubramanian,[7] S. Ballmer,[13] B. C. Barish,[12] C. Barker,[14] D. Barker,[14] M. Barnes,[12,b] B. Barr,[35] M. A. Barton,[12] K. Bayer,[13] R. Beausoleil,[26,c] K. Belczynski,[23] R. Bennett,[35,d] S. J. Berukoff,[1,e] J. Betzwieser,[13] B. Bhawal,[12] I. A. Bilenko,[20] G. Billingsley,[12] E. Black,[12] K. Blackburn,[12] L. Blackburn,[13] B. Bland,[14] B. Bochner,[13,f] L. Bogue,[15] R. Bork,[12] S. Bose,[41] P. R. Brady,[39] V. B. Braginsky,[20] J. E. Brau,[37] D. A. Brown,[12] A. Bullington,[26] A. Bunkowski,[2,31] A. Buonanno,[6,g] R. Burgess,[13] D. Busby,[12] W. E. Butler,[38] R. L. Byer,[26] L. Cadonati,[13] G. Cagnoli,[35] J. B. Camp,[21] J. Cannizzo,[21] K. Cannon,[39] C. A. Cantley,[35] L. Cardenas,[12] K. Carter,[15] M. M. Casey,[35] J. Castiglione,[34] A. Chandler,[12] J. Chapsky,[12,b] P. Charlton,[12,h] S. Chatterji,[12] S. Chelkowski,[2,31] Y. Chen,[1] V. Chickarmane,[16,i] D. Chin,[36] N. Christensen,[8] D. Churches,[7] T. Cokelaer,[7] C. Colacino,[33] R. Coldwell,[34] M. Coles,[15,j] D. Cook,[14] T. Corbitt,[13] D. Coyne,[12] J. D. E. Creighton,[39] T. D. Creighton,[12] D. R. M. Crooks,[35] P. Csatorday,[13] B. J. Cusack,[3] C. Cutler,[1] J. Dalrymple,[27] E. D'Ambrosio,[12] K. Danzmann,[31,2] G. Davies,[7] E. Daw,[16,k] D. DeBra,[26] T. Delker,[34,l] V. Dergachev,[36] S. Desai,[28] R. DeSalvo,[12] S. Dhurandhar,[11] A. Di Credico,[27] M. Díaz,[29] H. Ding,[12] R. W. P. Drever,[4] R. J. Dupuis,[12] J. A. Edlund,[12,b] P. Ehrens,[12] E. J. Elliffe,[35] T. Etzel,[12] M. Evans,[12] T. Evans,[15] S. Fairhurst,[39] C. Fallnich,[31] D. Farnham,[12] M. M. Fejer,[26] T. Findley,[25] M. Fine,[12] L. S. Finn,[28] K. Y. Franzen,[34] A. Freise,[2,m] R. Frey,[37] P. Fritschel,[13] V. V. Frolov,[15] M. Fyffe,[15]

PHYSICAL REVIEW D **77,** 022001 (2008)

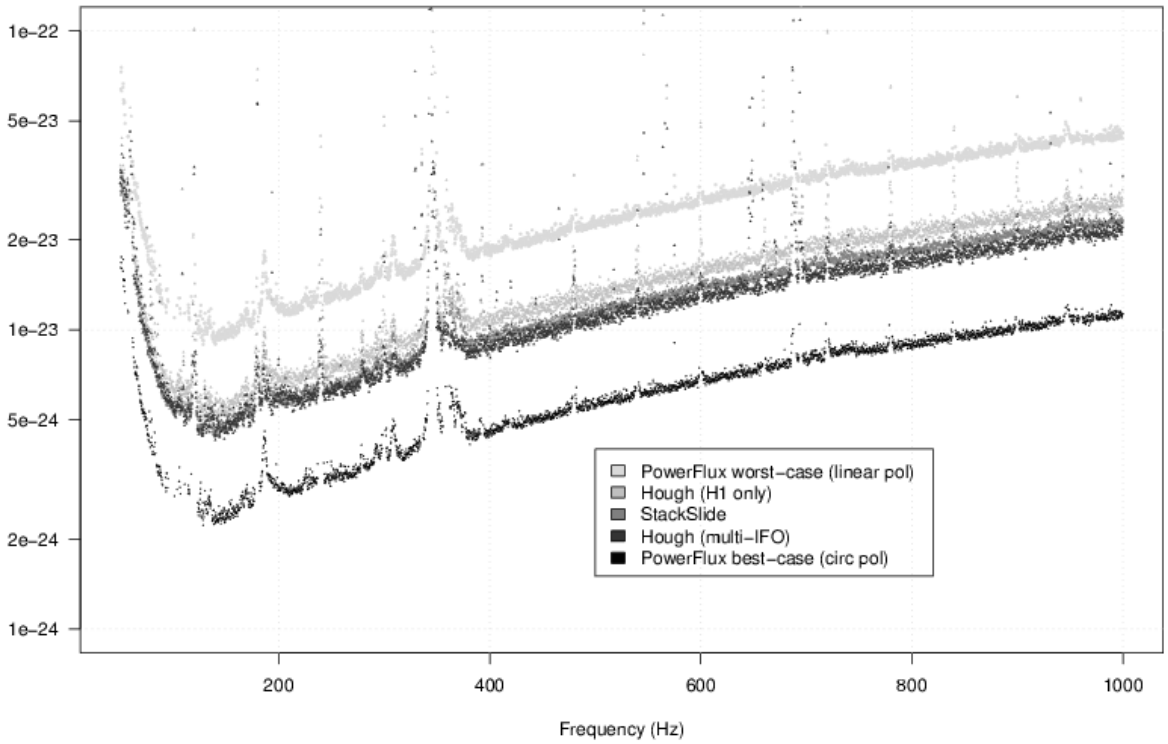
**All-sky search for periodic gravitational waves in LIGO S4 data**

B. Abbott,[15] R. Abbott,[15] R. Adhikari,[15] J. Agresti,[15] P. Ajith,[2] B. Allen,[2,50] R. Amin,[19] S. B. Anderson,[15] W. G. Anderson,[50] M. Arain,[39] M. Araya,[15] H. Armandula,[15] M. Ashley,[4] S. Aston,[38] P. Aufmuth,[14] C. Aulbert,[1] S. Babak,[1] S. Ballmer,[15] H. Bantilan,[8] B. C. Barish,[15] C. Barker,[16] D. Barker,[16] B. Barr,[40] P. Barriga,[49] M. A. Barton,[40] K. Bayer,[18] K. Belczynski,[25] J. Betzwieser,[18] P. T. Beyersdorf,[28] B. Bhawal,[15] I. A. Bilenko,[22] G. Billingsley,[15] R. Biswas,[50] E. Black,[15] K. Blackburn,[15] L. Blackburn,[18] D. Blair,[49] B. Bland,[16] J. Bogenstahl,[40] L. Bogue,[17] R. Bork,[15] V. Boschi,[15] S. Bose,[51] P. R. Brady,[50] V. B. Braginsky,[22] J. E. Brau,[43] M. Brinkmann,[2] A. Brooks,[37] D. A. Brown,[15,6] A. Bullington,[31] A. Bunkowski,[2] A. Buonanno,[41] O. Burmeister,[2] D. Busby,[15] R. L. Byer,[31] L. Cadonati,[18] G. Cagnoli,[40] J. B. Camp,[23] J. Cannizzo,[23] K. Cannon,[50] C. A. Cantley,[40] J. Cao,[18] L. Cardenas,[15] M. M. Casey,[40] G. Castaldi,[46] C. Cepeda,[15] E. Chalkley,[40] P. Charlton,[9] S. Chatterji,[15] S. Chelkowski,[2] Y. Chen,[1] F. Chiadini,[45] D. Chin,[42] E. Chin,[49] J. Chow,[4] N. Christensen,[8] J. Clark,[40] P. Cochrane,[2] T. Cokelaer,[7] C. N. Colacino,[38] R. Coldwell,[39] R. Conte,[45] D. Cook,[16] T. Corbitt,[18] D. Coward,[49] D. Coyne,[15] J. D. E. Creighton,[50] T. D. Creighton,[15] R. P. Croce,[46] D. R. M. Crooks,[40]

# The S2 Upper Limits

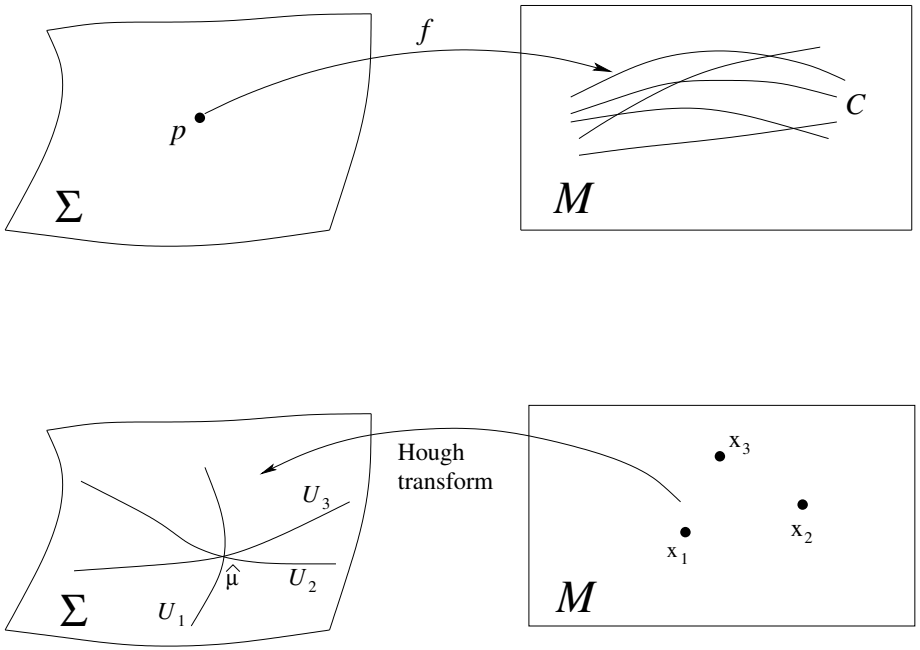


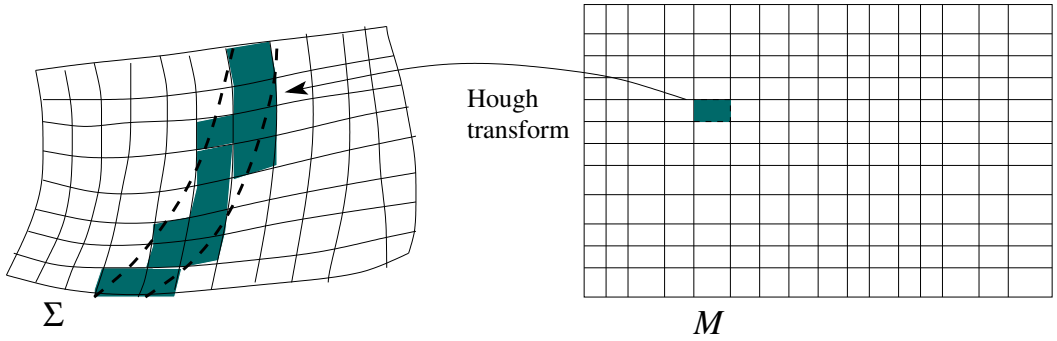
# The S4 Upper Limits



# The Hough Methods Paper

- Start with a geometrical formulation of the method



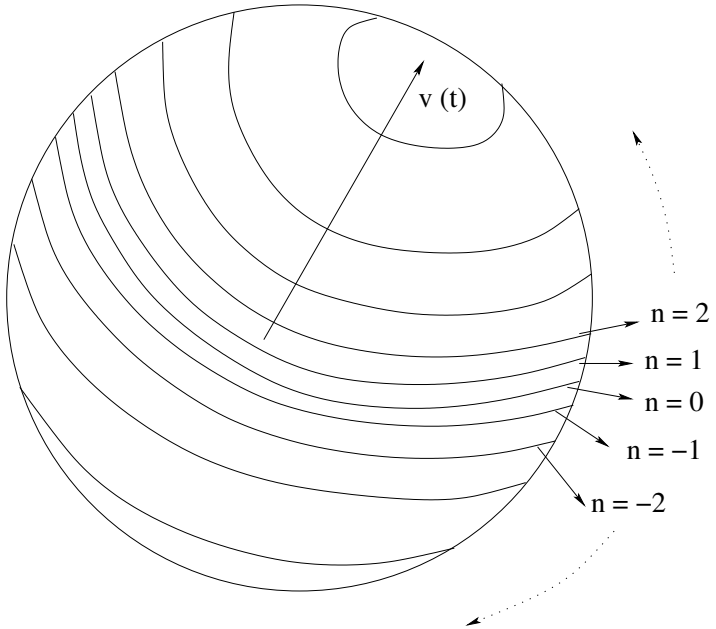



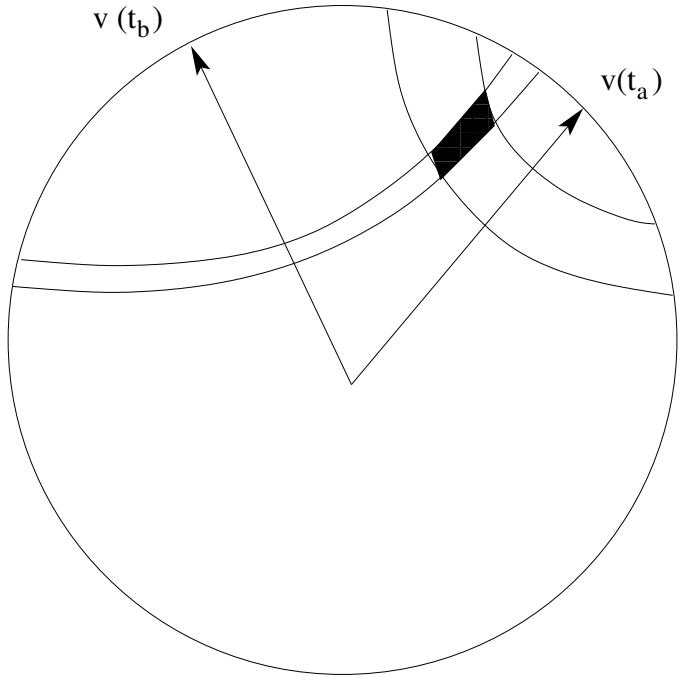
# The Hough Methods Paper

- Two flavors of the Hough transform – Non-Demodulated and demodulated
- Non-demodulated: get peaks from SFT power
- Demodulated: get peaks from F-statistic

# Circles in the sky

- The demodulated case – we get “circles in the sky”



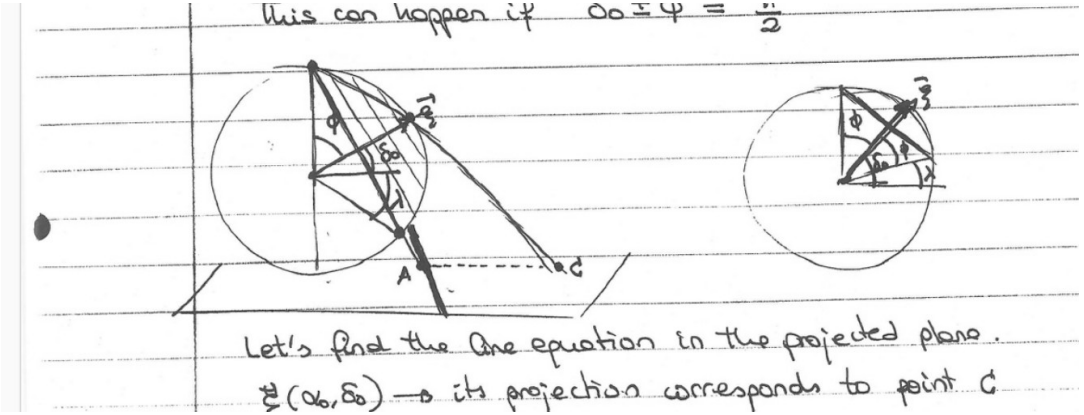



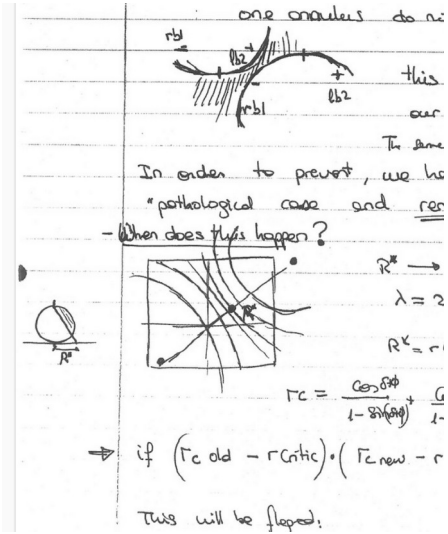
# The Hough Methods Paper

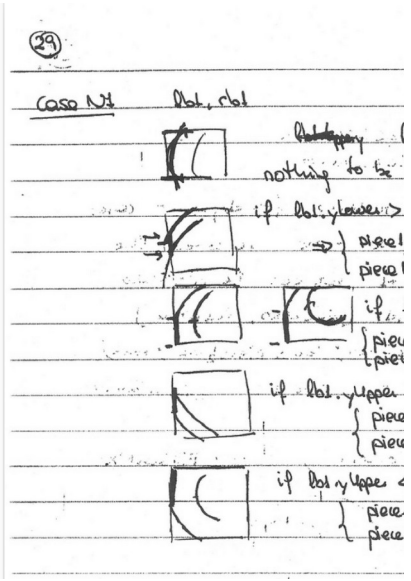
- Heuristic estimates of some important quantities:
- SFT duration
- Width of annuli in sky
- Sky resolution (intersection of annuli)
- Spindown resolution

# The (in)famous look-up-tables

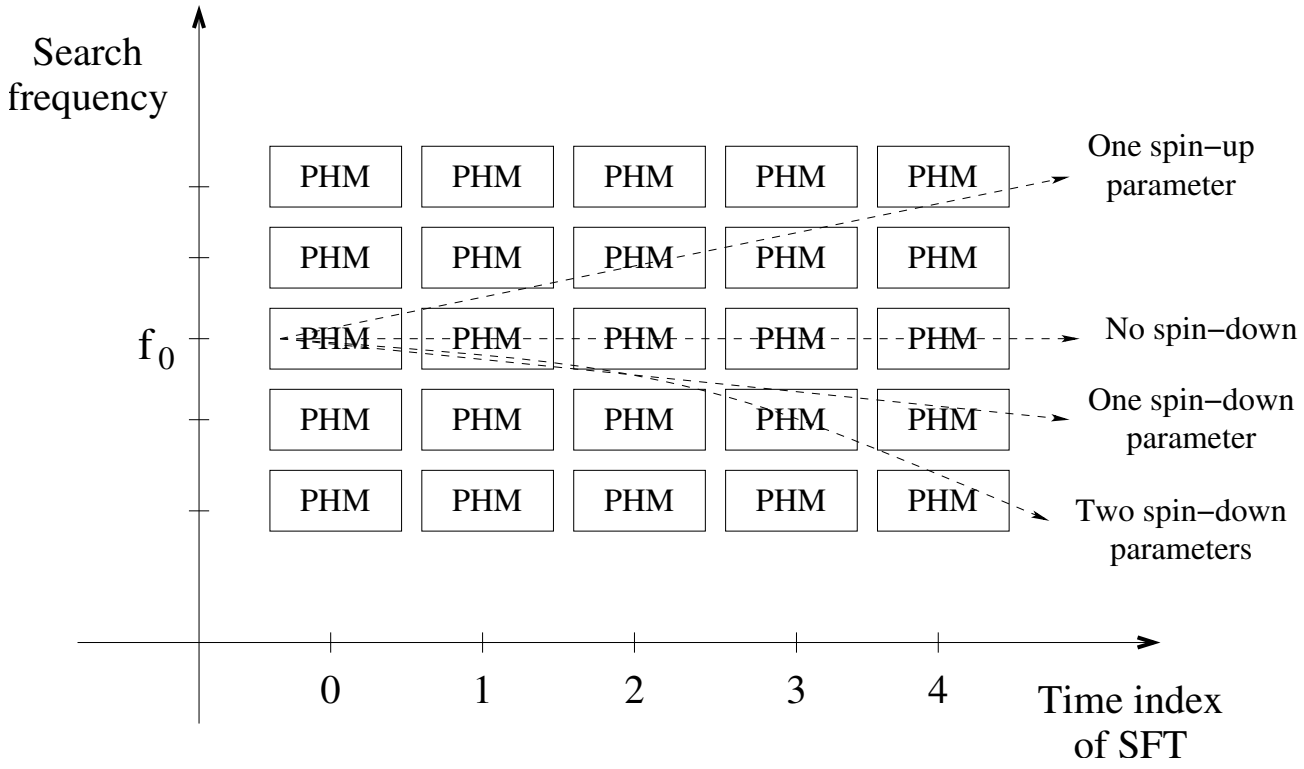
- The annuli can be re-used for several frequencies
- Needs a careful construction of the annuli (in the stereographic plane) – Alicia's notes!



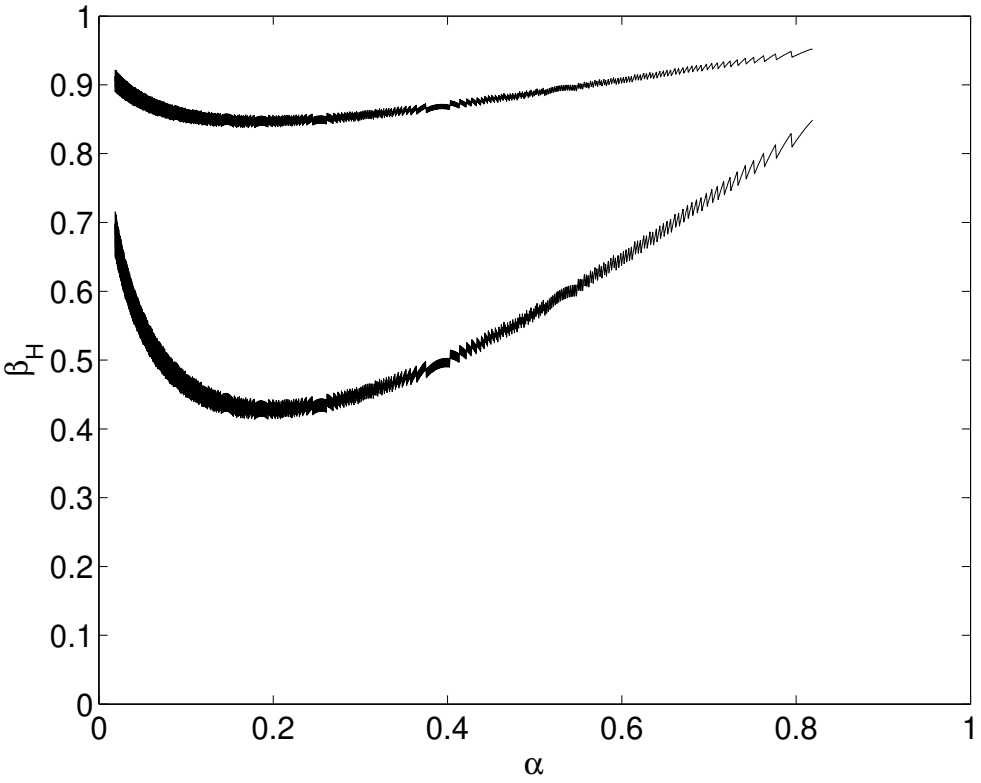
# The (in)famous look-up-tables

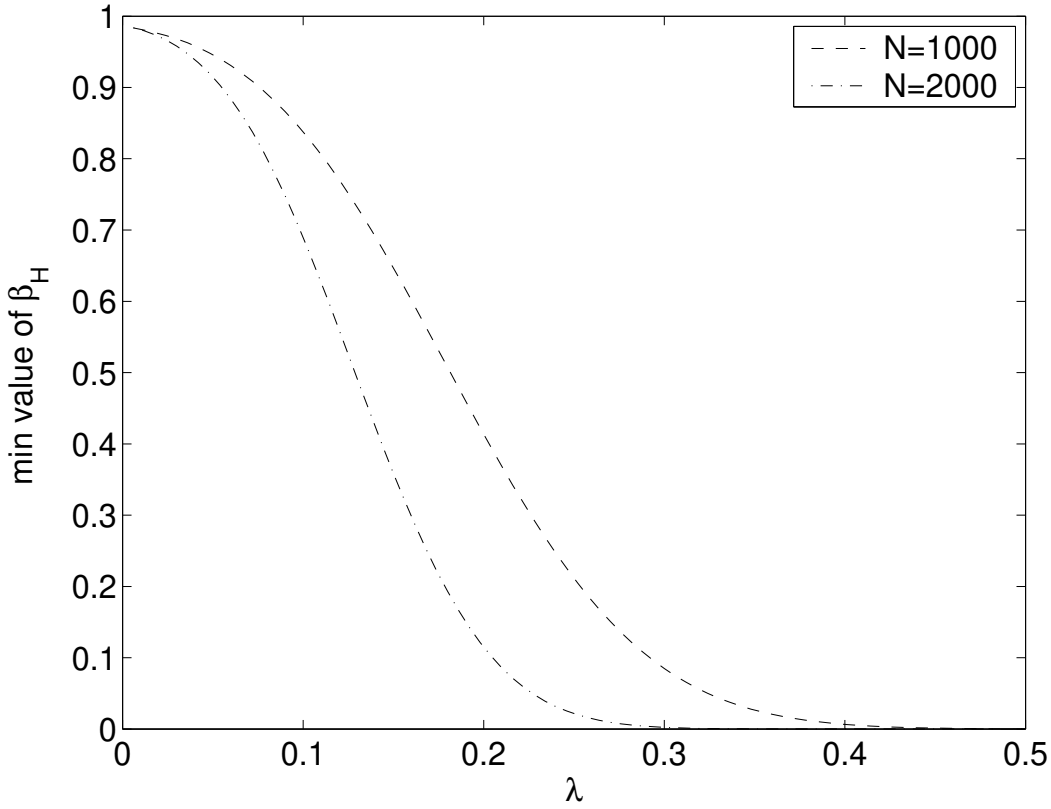


# Thresholds and sensitivity estimates

- There are two thresholds in the problem: for selecting peaks and for final number count
- Two methods: using either the Neyman-Pearson criterion, or miminizing the “critical ratio” (i.e. expected mean/standard deviation)
- Sensitivity estimates were the most detailed calculations in the paper – used and compared several approximations
- The final sensitivity estimate when averaging over sources was in fact biased (fixed by Dreissigacker, Prix & Wette, 2018)

# Thresholds and sensitivity estimates



# Thresholds and sensitivity estimates

- Leads to estimate of smallest signal that can be detected
- For FA 1% and FD 10%

$$\lambda \approx \frac{S}{\rho_{\mathrm{th}}^{\star}} \sqrt{\frac{8(1-\alpha^{\star})}{N\alpha^{\star}}} \approx \frac{9.02}{\sqrt{N}}$$

where

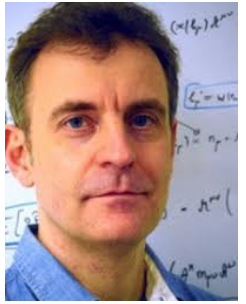
$$S := \mathrm{erfc}^{-1}(2\alpha_H^{\star}) + \mathrm{erfc}^{-1}(2\beta_H^{\star})$$

- Leads to final sensitivity estimate after averaging over sources

$$h_0 = \frac{8.54}{N^{1/4}} \sqrt{\frac{S_n}{T_{\mathrm{coh}}}} = 8.54 N^{1/4} \sqrt{\frac{S_n}{T_{\mathrm{obs}}}}.$$

# Further development: using weights

- The Hough method is inherently less sensitive than PowerFlux because of thresholding on SFT power
- It should be more robust, but we do need to improve sensitivity for quiet frequencies
- Weighing scheme motivated by Power Flux and Vladimir Dergachev (very important to compete with smart people!) -- used in S4 analysis
- Code written for dealing with multiple detectors with important input from Reinhard (very important to work with smart people!)



# Hough with “demodulated” data

- The notion of “demodulation” in the paper likely predates the F-statistic
- It provides heuristic estimates and the statistics of the hough maps for this case – several new features arise but we again have annuli in the sky (see also Prix & Itoh, 2005)
- Applied to S5 data using Einstein@Home -- was in fact the first implementation of a semi-coherent search using the F-statistic
- Segments can now be arbitrarily long – needs a further optimization for a multi-stage hierarchical search (Cutler, Gholami & Krishnan, 2005) (Prix & Shaltev 2012)

# Hough with “demodulated” data

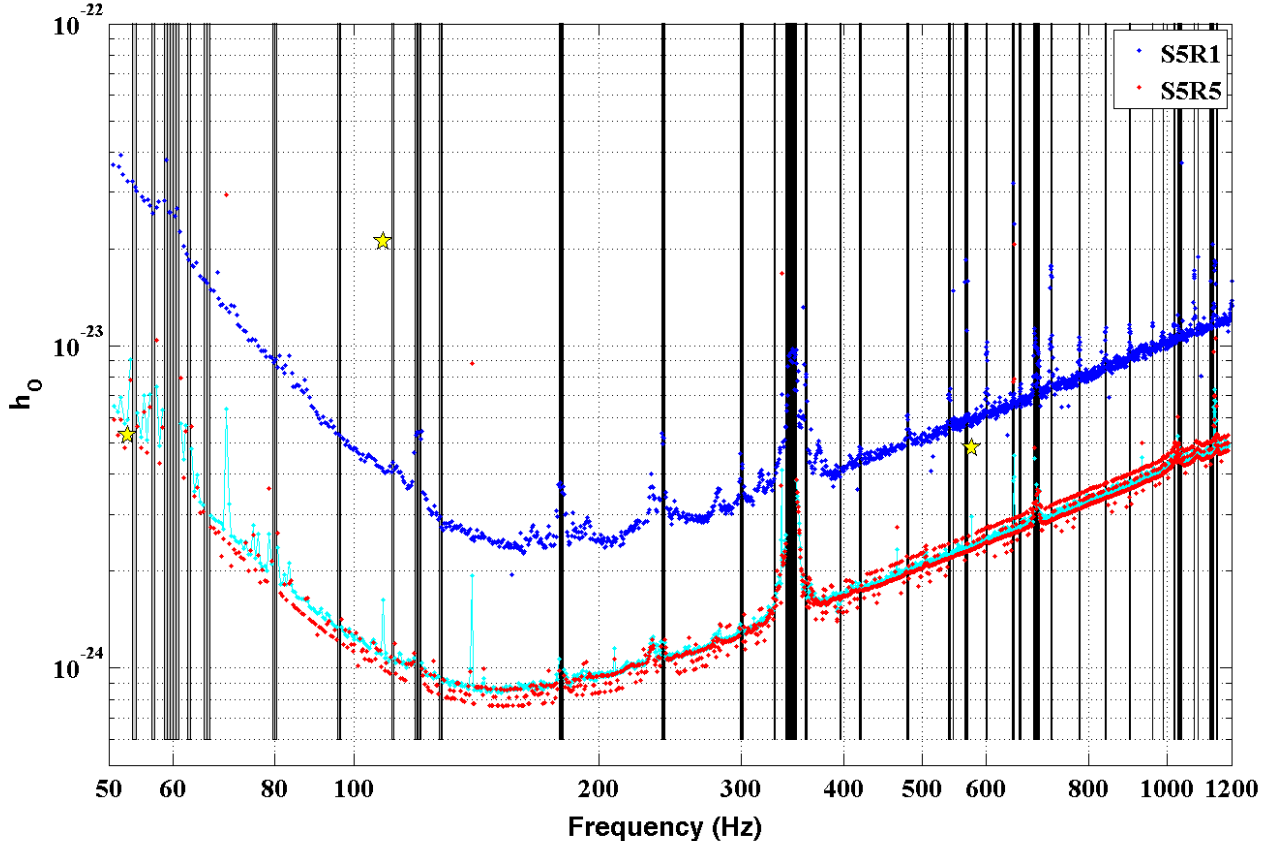
PHYSICAL REVIEW D **87,** 042001 (2013)

**Einstein@Home all-sky search for periodic gravitational waves in LIGO S5 data**

J. Aasi,[1] J. Abadie,[1] B. P. Abbott,[1] R. Abbott,[1] T. D. Abbott,[2] M. Abernathy,[3] T. Accadia,[4] F. Acernese,[5a,5c] C. Adams,[6] T. Adams,[7] P. Addesso,[8] R. Adhikari,[1] C. Affeldt,[9,10] M. Agathos,[11a] K. Agatsuma,[12] P. Ajith,[1] B. Allen,[9,10,13] A. Allocca,[14a,14c] E. Amador Ceron,[13] D. Amariutei,[15] S. B. Anderson,[1] W. G. Anderson,[13] K. Arai,[1] M. C. Araya,[1] S. Ast,[9,10] S. M. Aston,[6] P. Astone,[16a] D. Atkinson,[17] P. Aufmuth,[9,10] C. Aulbert,[9,10] B. E. Aylott,[18] S. Babak,[19] P. Baker,[20]

- First time coincidences between coherent segments was done on the host machines of Einstein@Home
- Precursor to GCT (Pletsch & Allen, 2009) and Weave (Wette, Walsh, Prix & Papa, 2018)
- Data from Jan – Oct 2009, 121 coherent segments of 25 hours

# The S5 “Hierarchical Search” Upper Limits



# Conclusions

- The development of the Hough transform method is part of the general problem of wide parameter space CW searches
- The Hough methods paper aimed to provide a general description of the method, optimal thresholds, and sensitivity estimates
- Used in observational results with S2, S4 and S5 Data
- These include the first semi-coherent search using the F-statistic
- Recent developments include the frequency-Hough version (see Sergio’s talk)