Modelling continuous gravitational waves from neutron stars

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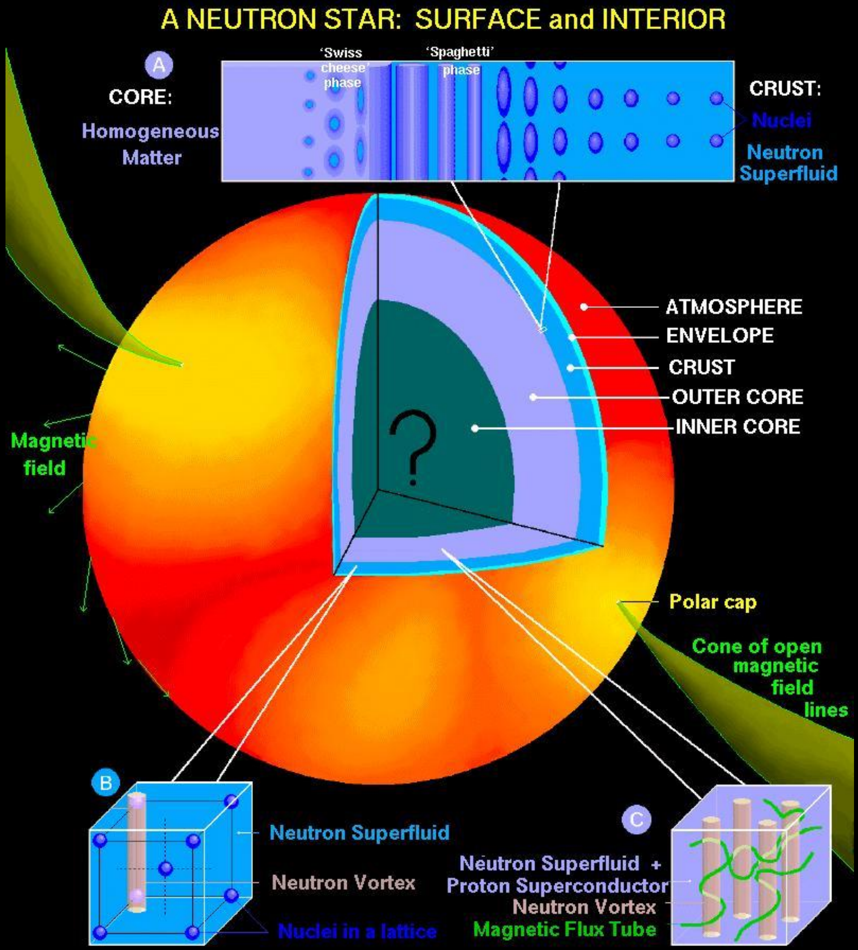
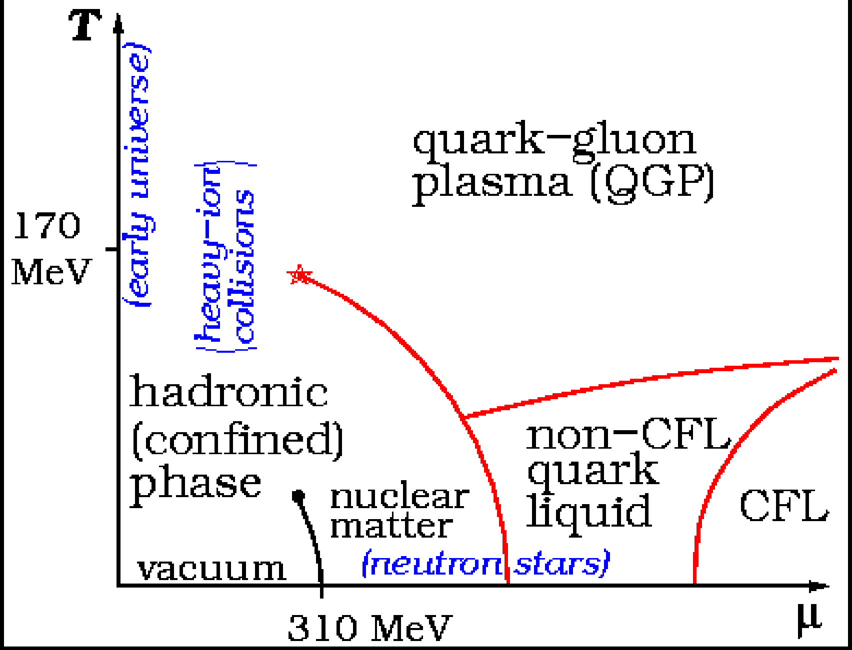
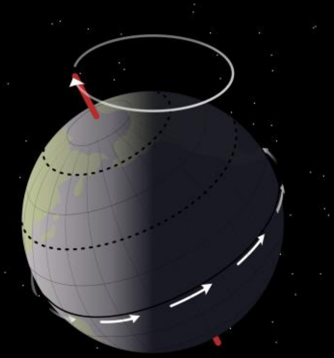
Neutron stars probe QCD



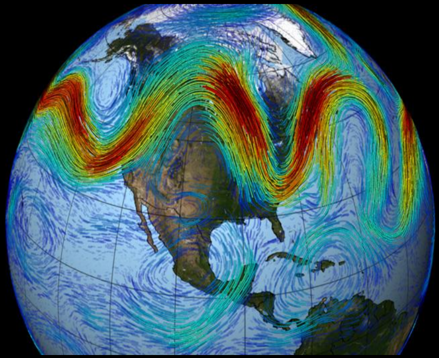


GW emission mechanisms for neutron stars

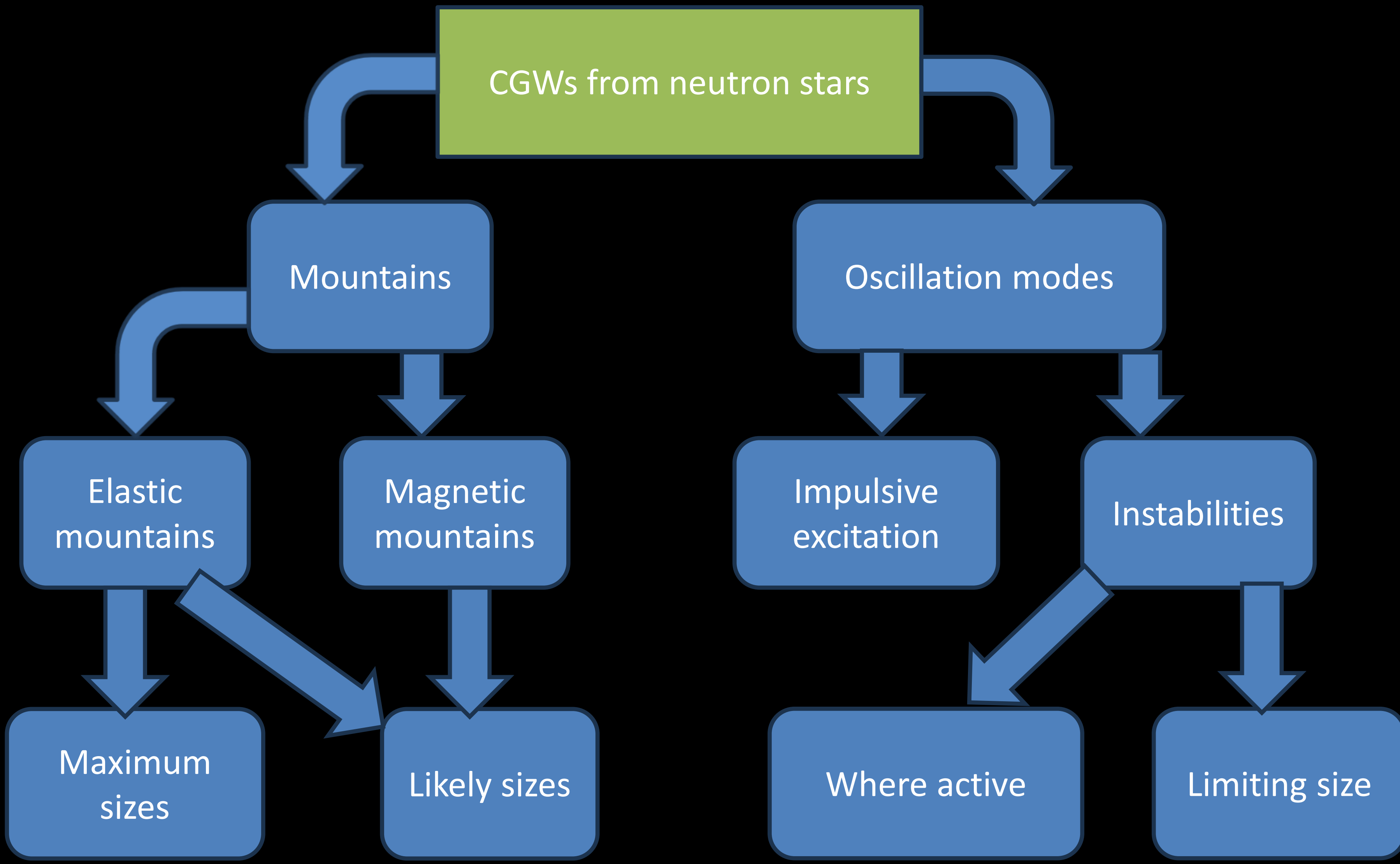
"Mountains" – non-axisymmetric deformation



"Wobble" - free precession
Most general motion of rigid body



Fluid oscillations – many possible sorts.



The Inverse Problem: What Do We Learn?

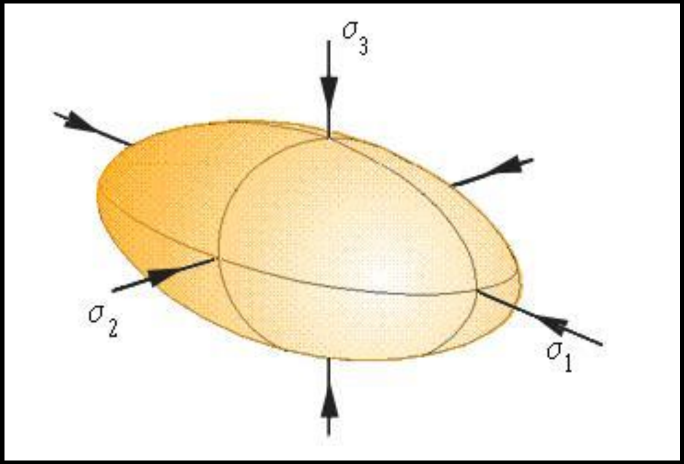
Geophysics is a useful guide

Phenomenon	Earth	Neutron Star
Deformed shape	✓	Not yet observed
Quakes	✓ Earthquakes	✓(but more complicated)
Oscillations	✓ Mainly elasto-gravitational	✓ (can be elasto-magneto-gravitational)
Rossby waves	✓	Not yet observed
Free precession	✓ ("Chandler wobble")	May have been observed

Elastic Mountains

Gravitational waves from mountains

A *triaxial* neutron star, rotating steadily, emits gravitational waves:



$$h = 3 \times 10^{-28} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}}\right)^2 \left(\frac{1\,{\rm kpc}}{r}\right)$$

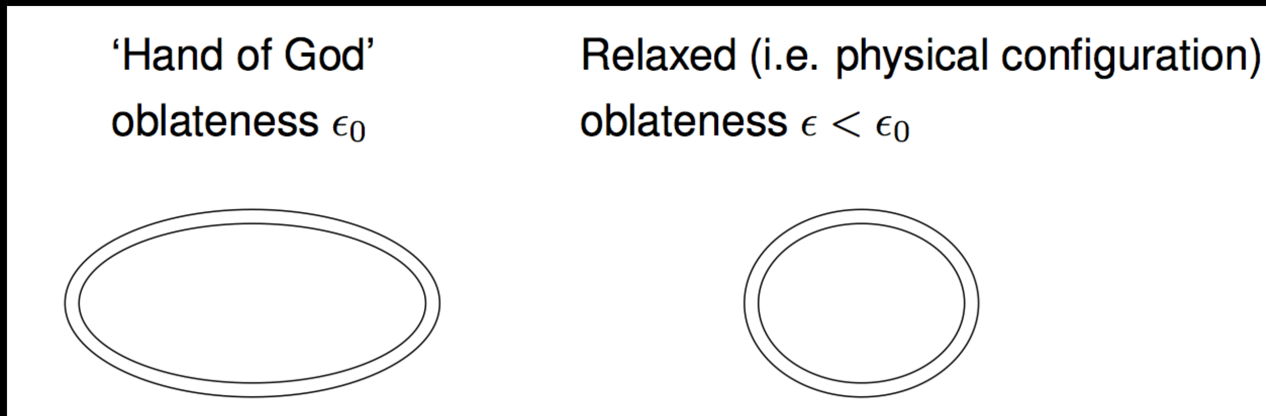
$$\epsilon = \frac{I_{yy} - I_{xx}}{I_{zz}}$$

Dimensionless asymmetry in moment of inertia tensor

Spin frequency

Distance to source

Elastic mountains: maximum

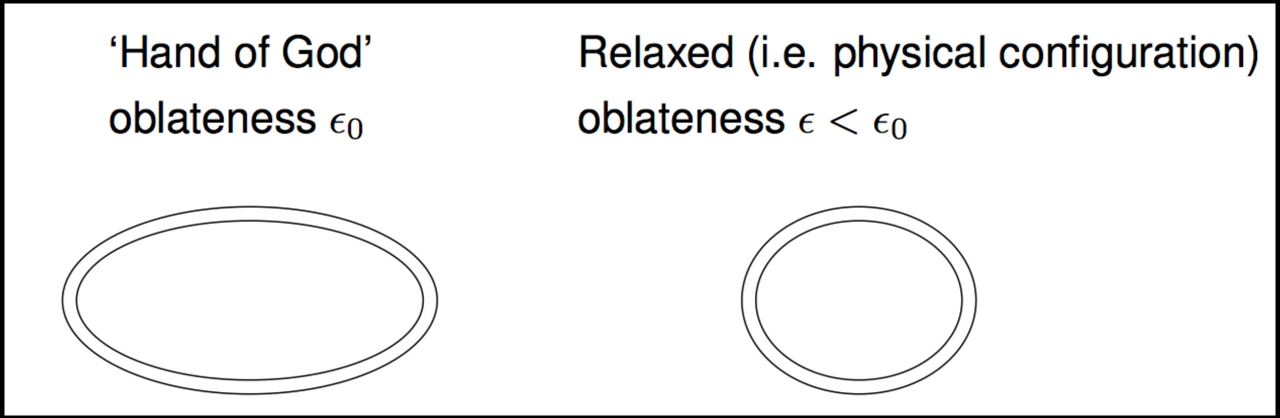


Energy of equilibrium star: $E = A\,\epsilon^2 + B\,(\epsilon_0 - \epsilon)^2$

$A \sim$ grav. binding energy $\sim \frac{G\,M^2}{R}$ $\qquad$ $B \sim$ elastic binding energy $\sim \int_V \mu\, dV$

Minimise E w.r.t. ϵ at fixed ϵ_0: $\left.\frac{\partial E}{\partial \epsilon}\right|_{\epsilon_0} = 0 \quad \Rightarrow \quad \epsilon = \frac{B}{A+B}\,\epsilon_0$

Elastic mountains: maximum cont…



For realistic NS, $B \ll A$, $\epsilon \sim 10^{-5}\, \epsilon_0$

Strain in crust $\sim |\epsilon_0 - \epsilon| \approx \epsilon_0$

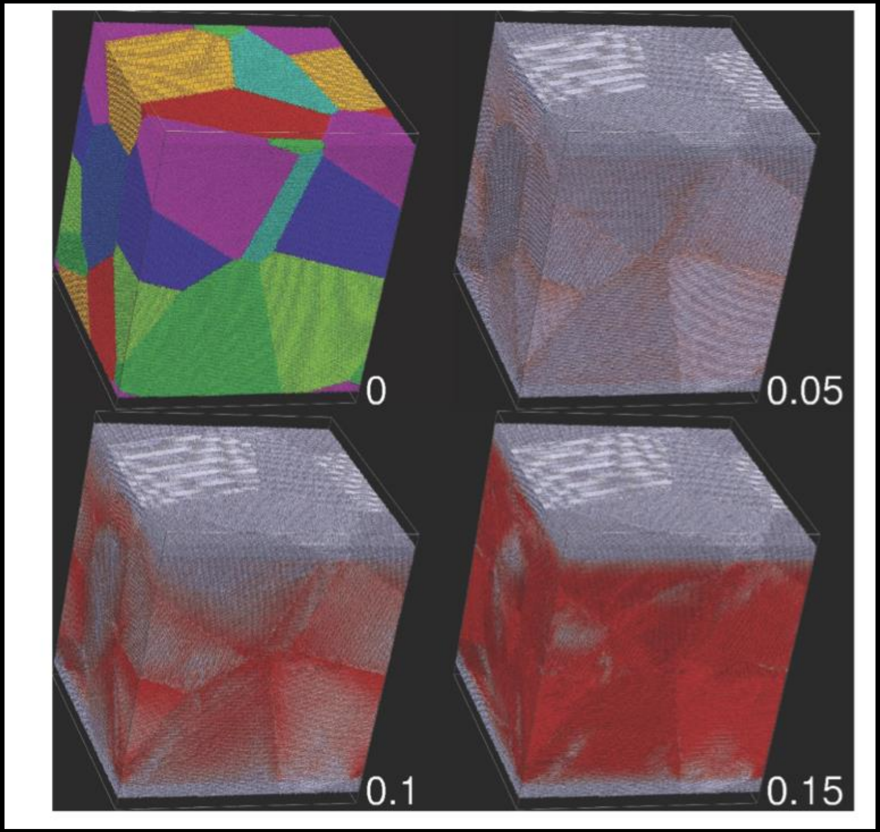
For a maximum breaking strain u_{break}:

$$\epsilon_{max} = \frac{B}{A+B} u_{break} = 10^{-6} \left(\frac{B/A}{10^{-5}}\right) \left(\frac{u_{max}}{0.1}\right)$$

ϵ_{max} could be $\sim 10^{-4}$ for exotic QCD phases; see e.g. Owen (2005)

Elastic mountains: maximum cont …

Molecular dynamics of Horowitz & Kadau (2009) indicate high breaking strain.



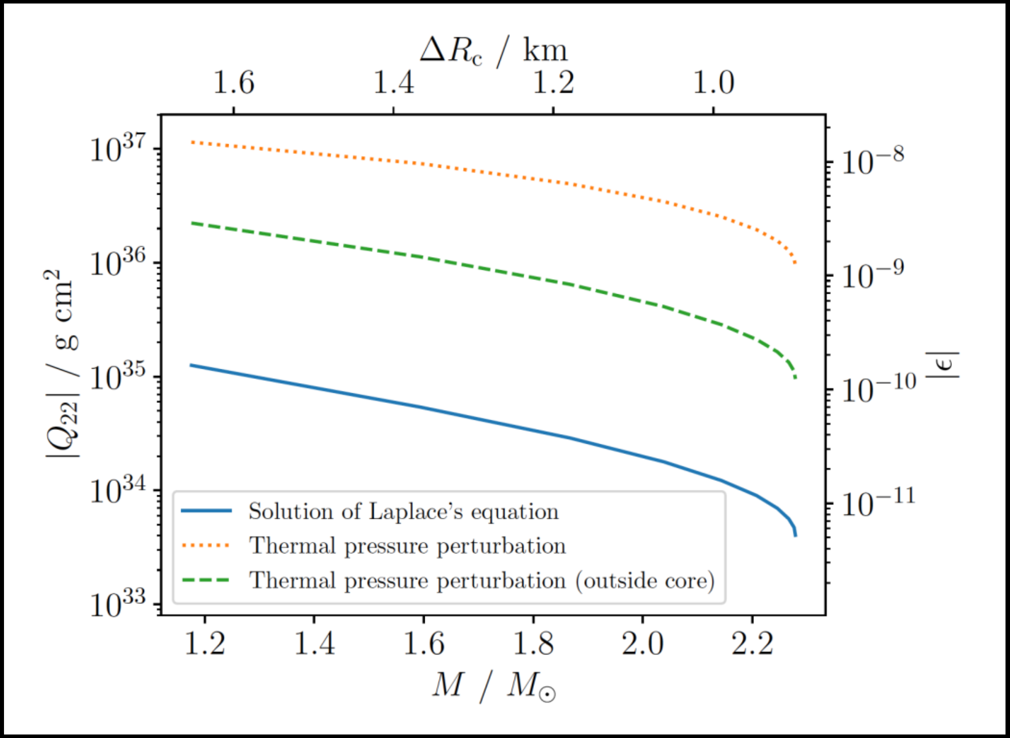
Plastic flow may relax crust on longer timescales (Chugunov & Horowitz 2010)

Elastic mountains: numerical calculations

Accuracy of rough estimate verified by numerical calculations of Ushomirsky, Cutler & Bildsten (2000).

Several follow up calculations since, using different set-ups and making different assumptions.

E.g. Gittins, Andersson & DIJ (2021), Gittins & Andersson (2021):



Realistic mountain sizes…

…or, how to you build a mountain on a neutron star?

"Thermoelastic" mountains

Bildsten (1998) proposed building mountain via temperature/composition asymmetry in accreting stars.

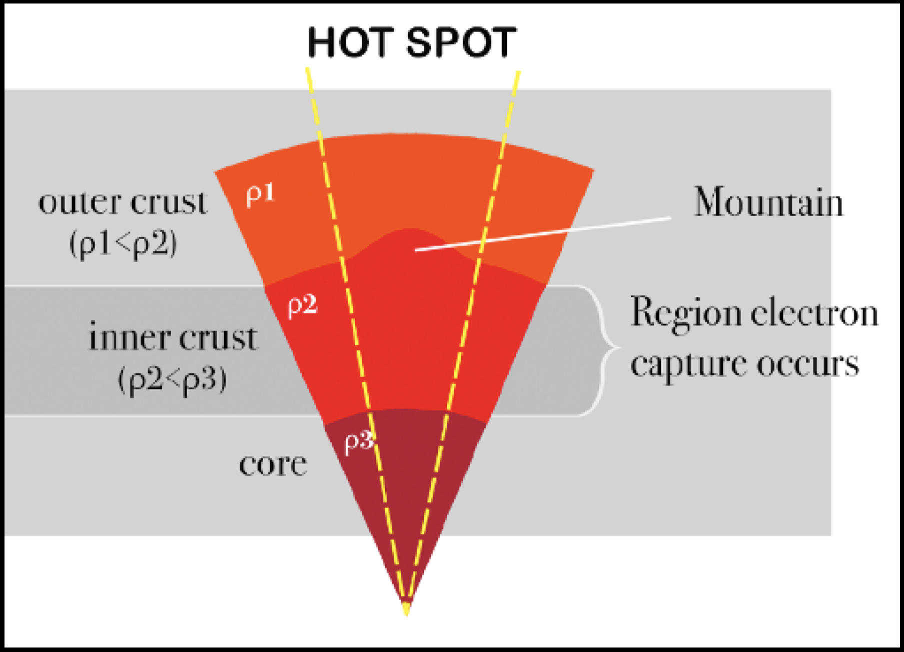


Figure credit: Emma Osborne (2020)

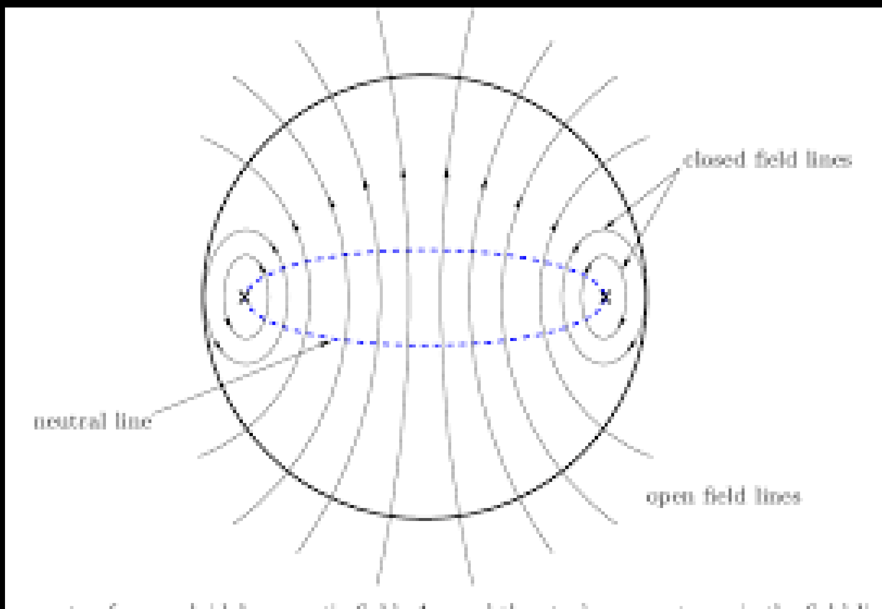
Viability of mechanism confirmed by Ushomirsky, Cutler & Bildsten (2000).

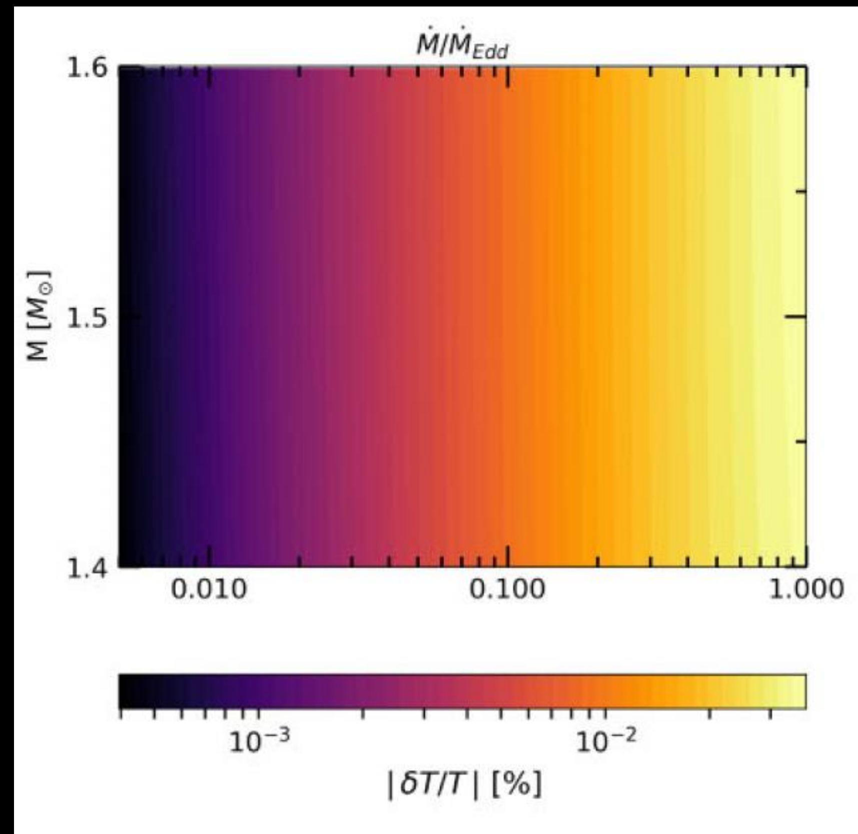
Possible evidence for mechanism at work proposed (Haskell & Patruno 2017).

But key unknown is likely level of temperature asymmetry. Need ~ 1% variation…

“Magneto- thermoelastic” mountains

See Osborne & DIJ (2020) and Hutchins & DIJ (2023) for concrete proposal: anisotropic heat conduction in magnetic field.



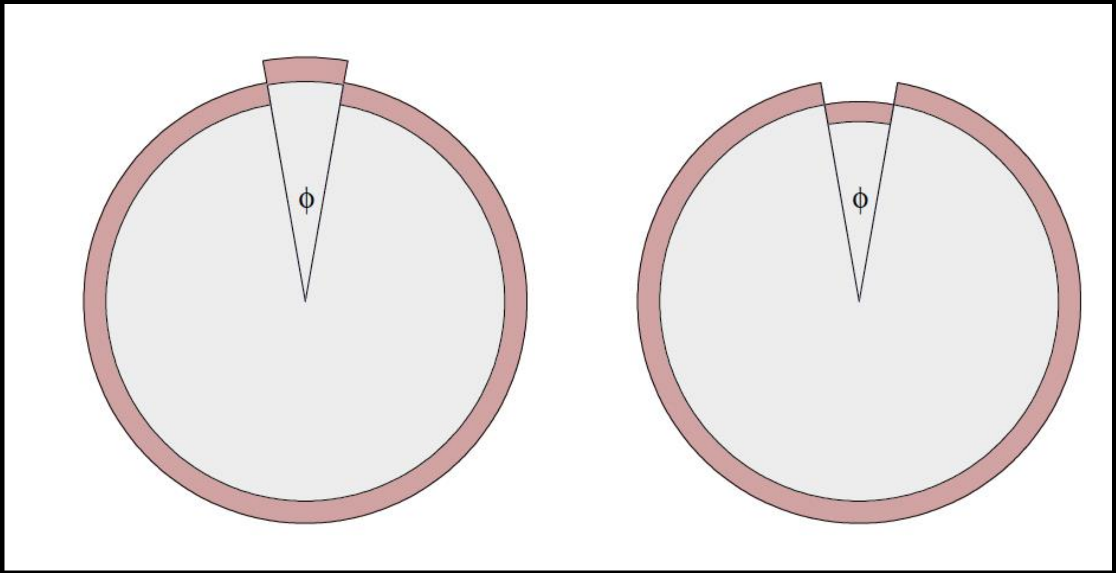


Hutchins & DIJ (2023); $B_{tor} = 10^8$ G

Would need very strong internal toroidal fields for this mechanism to work.

Mountains from crustquakes

Fattoyev & Horowitz (2018) suggested mountains may form in *crustquakes*



Particular example of this examined in Giliberti & Cambiotti (2022)

Mountains from crustquakes cont …

Gangwar & DIJ (2024) used energy model to model mountain formation in crustquake in spinning up star:

$$E = \underbrace{\frac{J^2}{2\, I_s\,(1 + \epsilon_{20})}}_{\text{Kinetic energy}} + \underbrace{A_{20}\epsilon_{20}{}^2 + B_{20}\,(\epsilon_{20,0}}_{\text{Axisymmetric piece}} - \epsilon_{20}\,)^2 + A_{22}\,\epsilon^2 + \underbrace{B_{22}\,(\epsilon_{22,0}}_{\text{"Mountain" piece}} - \epsilon_{22}\,)^2$$

Can allow zero strain shape to change while conserving energy and angular momentum.

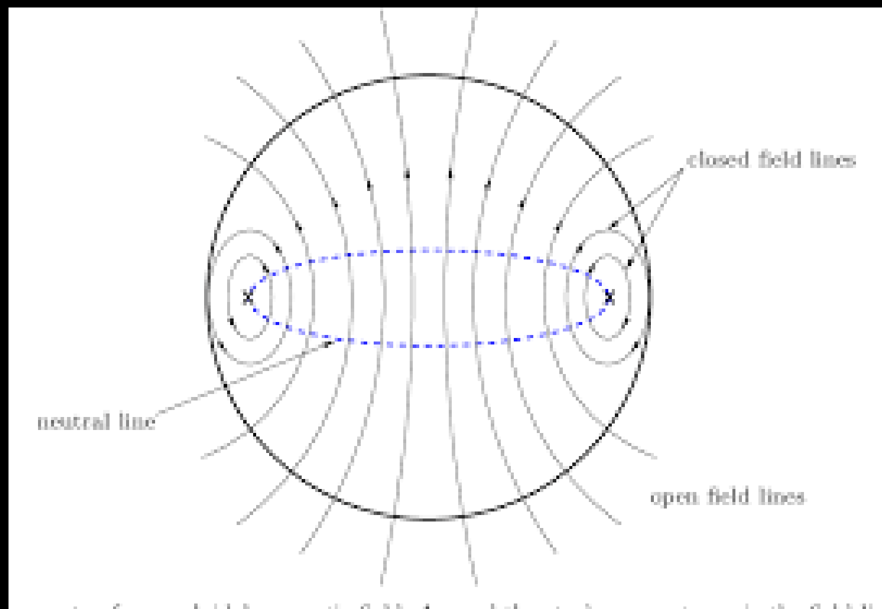
Found largest mountain you can build is

$$\epsilon_{22}^{max} = \frac{\sqrt{B_{20}B_{22}}}{A_{22}}\, u_{break}$$

Magnetic Mountains

Magnetic mountains: back-of-the-envelope

Magnetic field lines have an effective tension, and deform star.



$$\epsilon \sim \frac{\int B^2 \, dV}{GM^2/R} \sim 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2$$

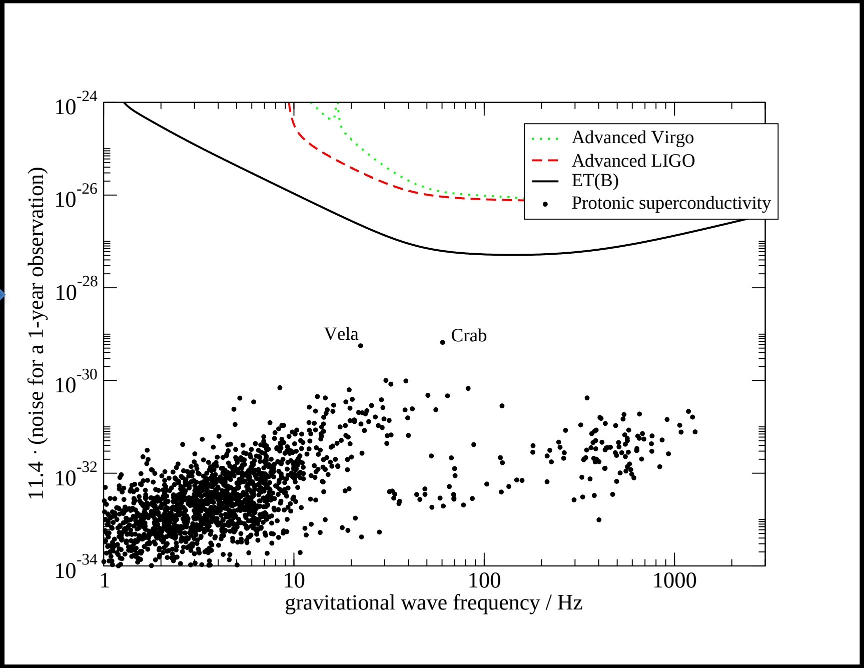
Not very big for typical pulsar magnetic field strengths

Magnetic mountains: back-of-the-envelope

If protons form type II superconductor, magnetic field confined to *fluxtubes*. Effect of this is to increase tension by a factor of H_c/B, where $H_c \sim 10^{15}$ G, increasing ellipticity:

$$\epsilon \sim 10^{-9} \frac{B}{10^{12}\,\mathrm{G}}$$

But even then, the GW emission from known pulsars is not detectable.

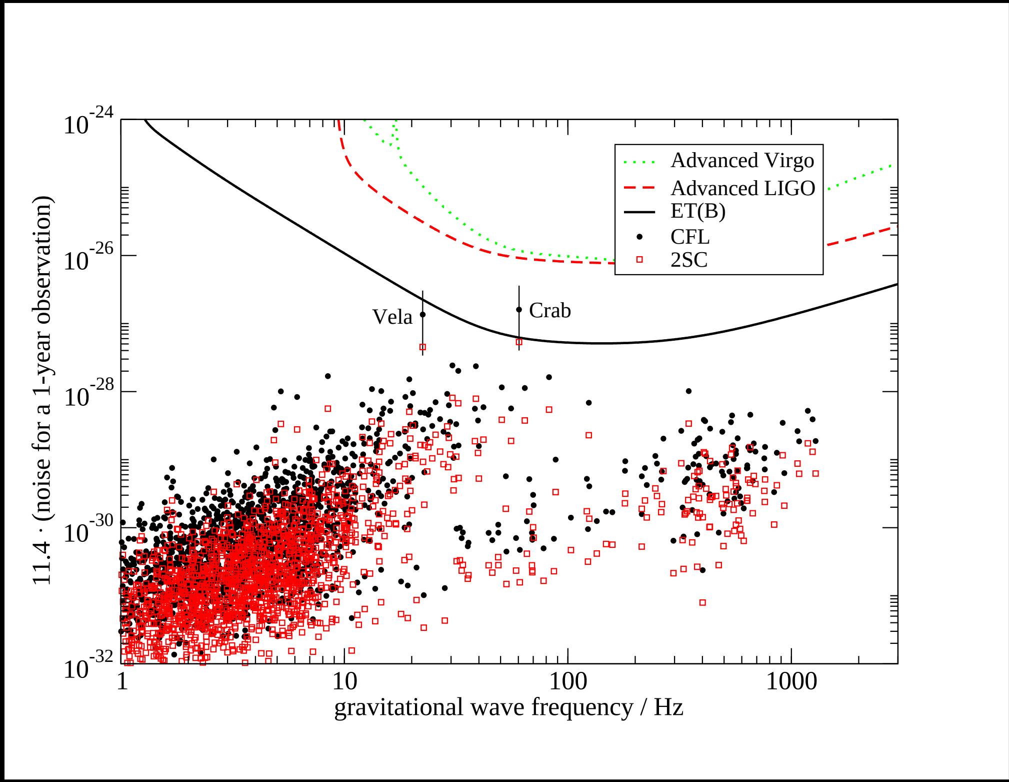


Can get stronger emission from local field burial in accreting systems (Haskell et al 2015).

"Exotic" magnetic mountains: detectability

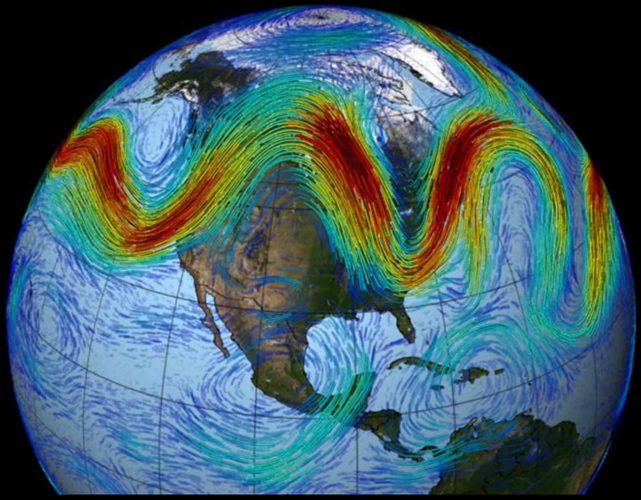
If CFL or 2SC phases occur in neutron star cores, can get *colour-magnetic flux tubes* (Iida & Baym 2002, Iida 2005, Alford & Sedrakian 2010).

This leads to flux tube tension ~ 10^3 larger than in protonic superconductivity case. Glampedakis, DIJ & Samuelsson (2012) modelled this:



Fluid oscillations

Rossby waves



Class of oscillations connected with rotation of a fluid ball.

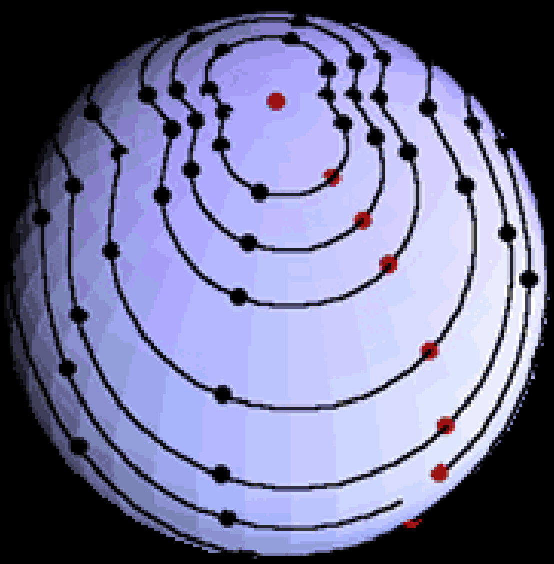
Named after Carl-Gustaf Rossby, who identified them in atmosphere in 1939.

Also found in the Earth’s oceans.

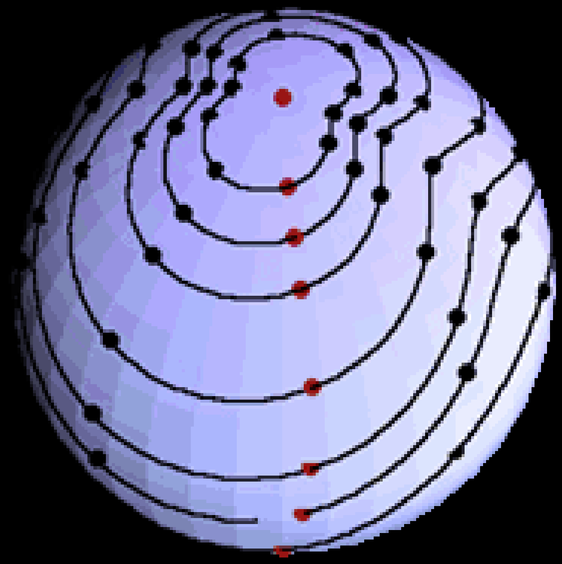
Restoring force is the Coriolis force.

Instability: the CFS mechanism

GW emission can drive the r-mode unstable, *a two-stream instability*.



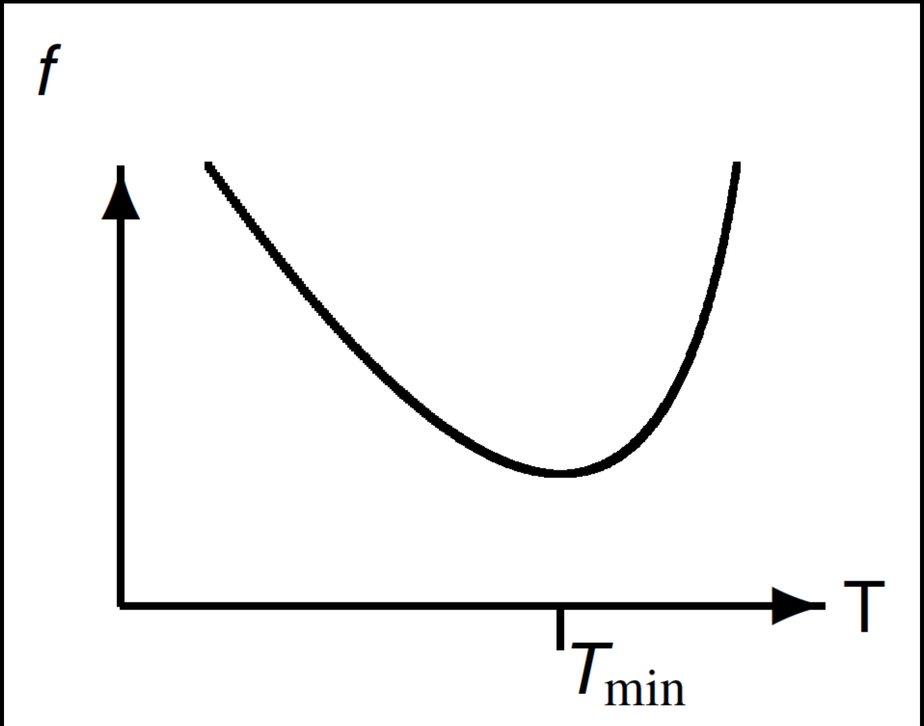
View from the inertial frame.



View from the corotating frame

Movie credit: Hanna & Owen

R-mode instability curve



Not all rotating fluid bodies are unstable…

… gravitational wave instability must compete with stabalizing effects of *dissipation*.

Dissipation rate temperature-dependent, giving rise to an *instability window*.

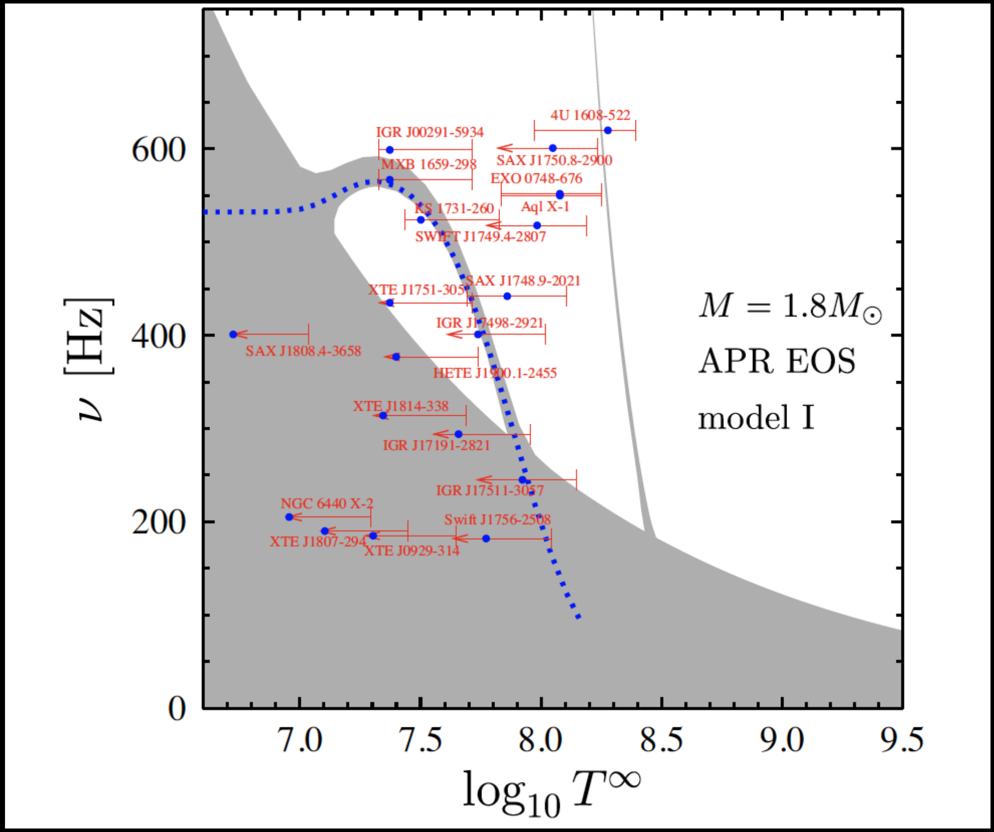
Active area of research: relating instability curve to stellar model.

Realistic instability curves

Need to take many pieces of physics into account, including elastic crust, magnetic field, superfluidity, …

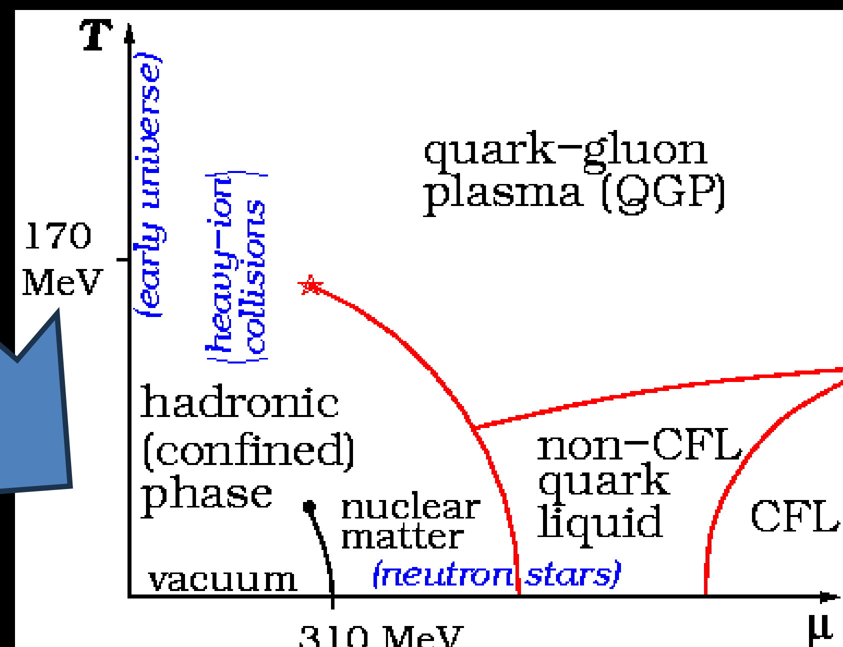
Large literature; see Glampedakis & Gualtieri (2018) review.

Example: Kantor+ (2020) consider "superfluid" modes, finding "stability spikes":



The inverse problem

Or, in the event of a detection, what do we actually learn?



Inverse problem

Ability to solve inverse problem depends in large part of what extra (i.e. EM) Information is available.

If you only have GW signal, then harmonic content and frequency evolution can be useful, e.g. braking index

$$n \equiv \frac{f\,\ddot{f}}{\dot{f}^2}$$

n=5 for mountains, n=7 for r-modes, but only if:

1. There are no non-CGW energy losses
2. The ellipticity/mode amplitude are constant

These are both strong assumptions!

Inverse problem

If $f_{GW}\ and\ f_{spin}$ measured separately, can use ratio:

$$\frac{f_{GW}}{f_{spin}} \approx \begin{cases} 2 & \text{for mountains} \\ \frac{4}{3} & \text{for r-modes} \\ 1 & \text{For small-angle free precession} \end{cases}$$

Inverse problem: mountains

If only GW amplitude h_0 measured, get value of $\epsilon\, I/d$.

If CGW signal is pure $n = 5$ mountain, observed frequency evolution only *partially* breaks the degeneracy; can then measure the quantities

$$\frac{\sqrt{I}}{d} \quad \text{or} \quad \epsilon^2\, I \quad \text{or} \quad \epsilon\, d$$

Need a measurement of d or use of theoretical estimate of I to completely break the degeneracy. CGWs are *"not quite standard sirens" (Sienikawska & DIJ, 2022)*

BUT, there is no obvious way to distinguish between elastic and magnetic mountains

This is a BIG problem

Inverse problem: mountains

To distinguish between elastic and magnetic mountains, could maybe rely on "circumstantial evidence", e.g.

- reject magnetic case if inferred ratio of internal/external magnetic field strength too far from unity?

- Look at relative phasing of CGW and EM (e.g. radio pulsar) emission?

See Lu+ (2023) and Hua+ (2024) for more on inverse problem.

Inverse problem: r-modes

If f_{GW} and f_{spin} both measured, departure from 4/3 gives information on mass and radius, which directly connects with the high density equation of state.

Caride+ (2019) give fitting formula (see also Ghost+ (2023)):

$$\frac{f_{GW}}{f_{spin}} \approx \underbrace{A}_{} - \underbrace{B\left(\frac{f_{spin}}{f_{spin,K}}\right)^2}_{}$$

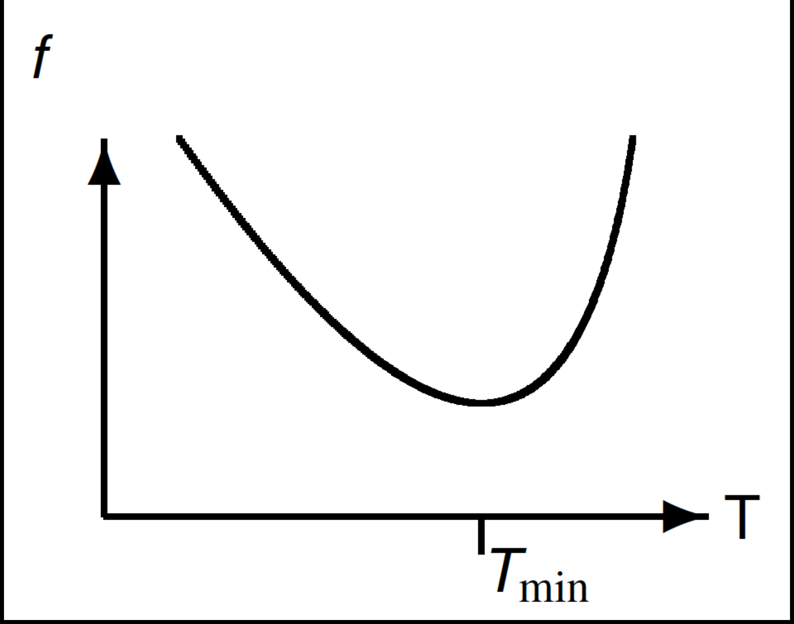
A= 4/3 in Newtonian limit.
Depends mainly upon
compactness M/R

Rotational correction.
Depends mainly upon
Average density M/R^3

Adding in other bits of physics (e.g. resonance with crustal oscillations) can spoilt this.

Inverse problem: r-modes cont …

If temperature can be measured and/or estimated, can also map out the instability curve.



This places an upper limit on the sum of *all* dissipative process that damp the r-mode.

Summary

- LIGO-Virgo observations beginning to probe regimes of astrophysical interest, but detection likely requires:
  - Elastic mountains close to maximally strained, or…
  - ... elastic mountains from exotic phases, or …
  - … exotic magnetic field configuration, or …
  - … excitation of oscillation mode, probably via instability.
- Not clear when first detections will be made.

Inverse problem severely hampered by DEGENERACIES.