Detection possibility of continuous gravitational waves from isolated rotating magnetized neutron stars

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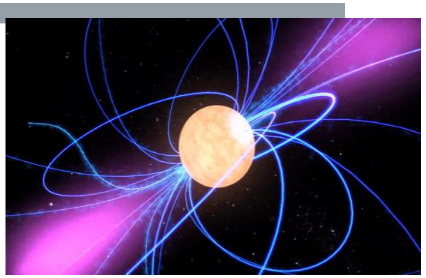
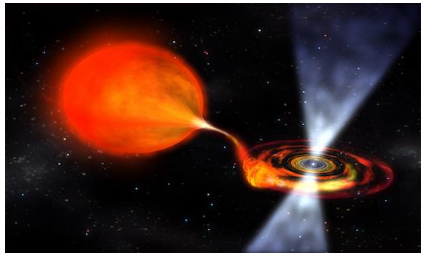
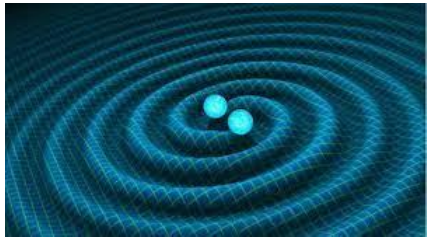
Detection of Neutron stars (NSs):



- **Electromagnetic radiation**: **Pulsar** (magnetized rotating NSs) emits beams of electromagnetic radiation out of its magnetic poles, which does not coincide with the rotational axis, so the emission beams are detected as pulse (Lighthouse effect).



- **Accreting NSs**: locate in binary systems and manifest themselves as X-ray sources.



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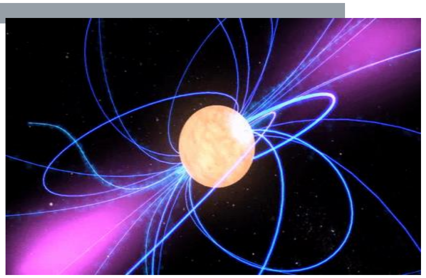

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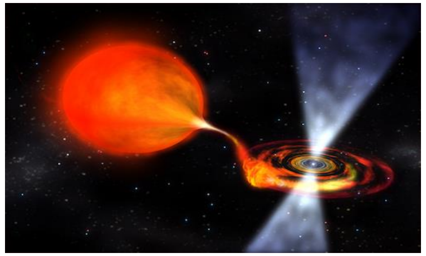

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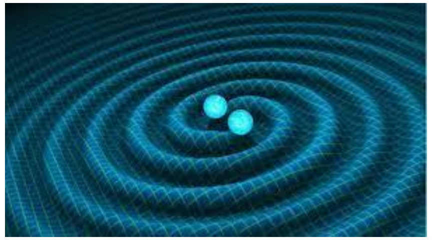

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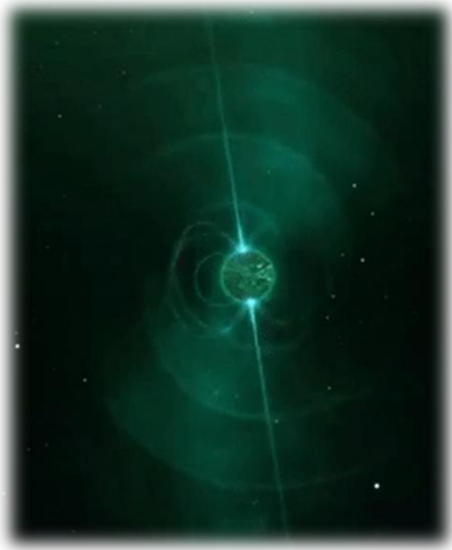
How to Detect 'Isolated' or 'Invisible' NSs?

Isolated NSs (not in binary/ negligible dipole radiation) ➡ tiny size & thermal energy ➡ lowly luminous (or pulse not directing towards earth) thus Invisible

difficult to detect in electromagnetic (EM) surveys, such as SDSS, Kepler, Gaia

But isolated NSs and WDs can produce **continuous GW!**

emitted continuously, as long as star is magnetized and spinning (like a singer holding a single note for a long time).



Direct detection of invisible stars! ➡ idea about mass, magnetic field, spin and equation of state.

Introduction

- NSs are generally born with mass $1.4 - 1.6 M_{\odot}$. GW from merger GW190814 ($M = 2.50 - 2.67 M_{\odot}$;Abbott et al. 2020) suggests the possible existence of NSs with $M > 2M_{\odot}$, similar to pulsar timing study of PSR J0740 + 6620 (Cromartie et al. 2020).
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- **Origin of strong magnetic field:** Collapse of highly magnetised star → Flux freezing →Dynamo (α-Ω)→ amplify up to ~10^{17-18}G for NS at **core: Surface** field could be 10^{15-16}G (Thompson & Duncan 1993)
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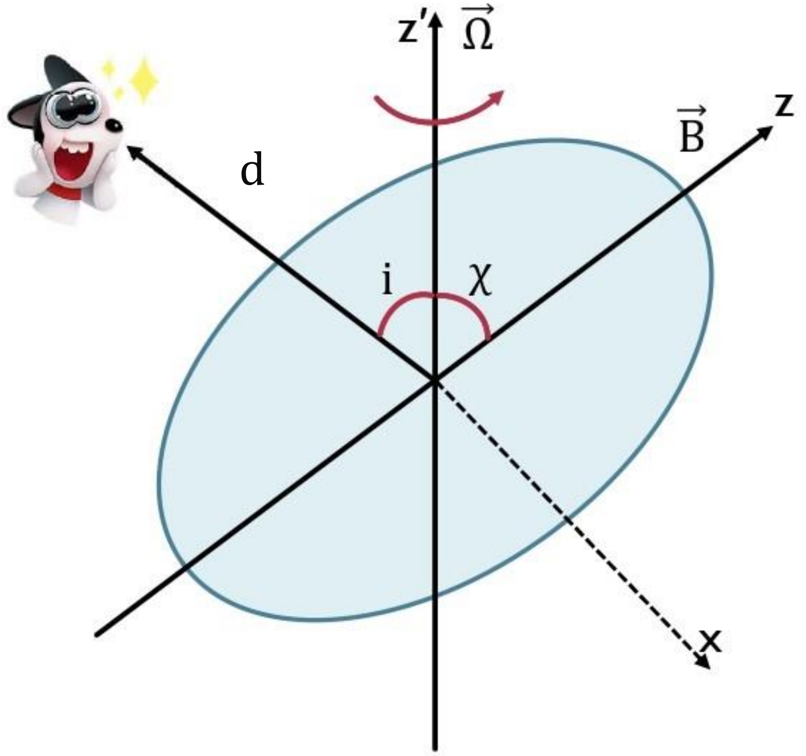
- We simulate CGW from isolated magnetized rotating NSs and try to understand observation plausibility.



2D cross-section of an asymmetric NS

Magnetic field

Rotation

→ **Deformation**

Deformation & Misalignment → Triaxial symmetry → Time varying, non zero quadrupole moment → **Continuous** GW

$$h_+ = h_0 \sin\chi \left[\frac{1}{2}\cos i \sin i \cos\chi \cos\Omega t - \frac{1+\cos^2 i}{2}\sin\chi \cos 2\Omega t\right]$$

$$h_\times = h_0 \sin\chi \left[\frac{1}{2}\sin i \cos\chi \sin\Omega t - \cos i \sin\chi \sin 2\Omega t\right]$$

$\chi \to 0 \qquad h_0 = \frac{4G}{c^4}\Omega^2 \epsilon \frac{I_{xx}}{d}, \qquad \epsilon = \frac{I_{zz}-I_{xx}}{I_{xx}}$

Bonazzola & Gourgoulhon 1996

MODELLING NS USING XNS Pili et al. 2014

Einstein's equation solver in GRMHD

XNS (A code to study magnetized NSs)

Einstein's equation (describes space-time metric)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$ds^2 = -\alpha^2\,dt^2 + \psi^4\left[dr^2 + r^2\,d\theta^2 + r^2\,sin^2\,\theta\left(d\emptyset + \beta^{\emptyset}\,dt\right)^2\right]$$

+

Magneto-Hydrostatic Equilibrium

(provides distribution of matter/energy)

$$T^{\mu\nu} = (e + p + b^2)u^{\mu}u^{\nu} - b^{\mu}b^{\nu} + \left(p + \frac{b^2}{2}\right)g^{\mu\nu}$$

→ TOV eq

Momentum-energy conservation: $\nabla_{\mu}T^{\mu\nu} = 0$

→

Axisymmetric equilibrium configuration (Uniformly/differentially rotating & Poloidal/Toroidal/mixed field)

of NSs in GRMHD

Input ➡

- EOS: polytropic law $P = k\rho^{1+\frac{1}{n}}$, adiabatic index γ = 1 + 1/n (We use K=100, γ=1.95).

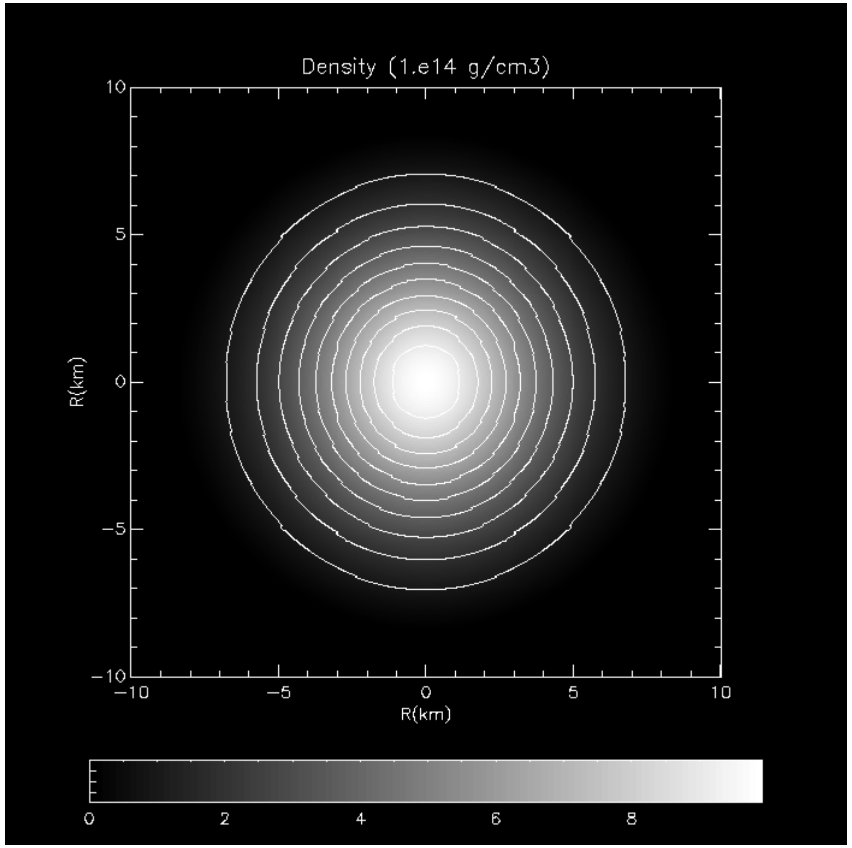
- Magnetic field:

  Toroidal: magnetic polytropic law $B^{\Phi} \sim r k_m \rho^m$, m =polytropic index, k_m= magnetization index.

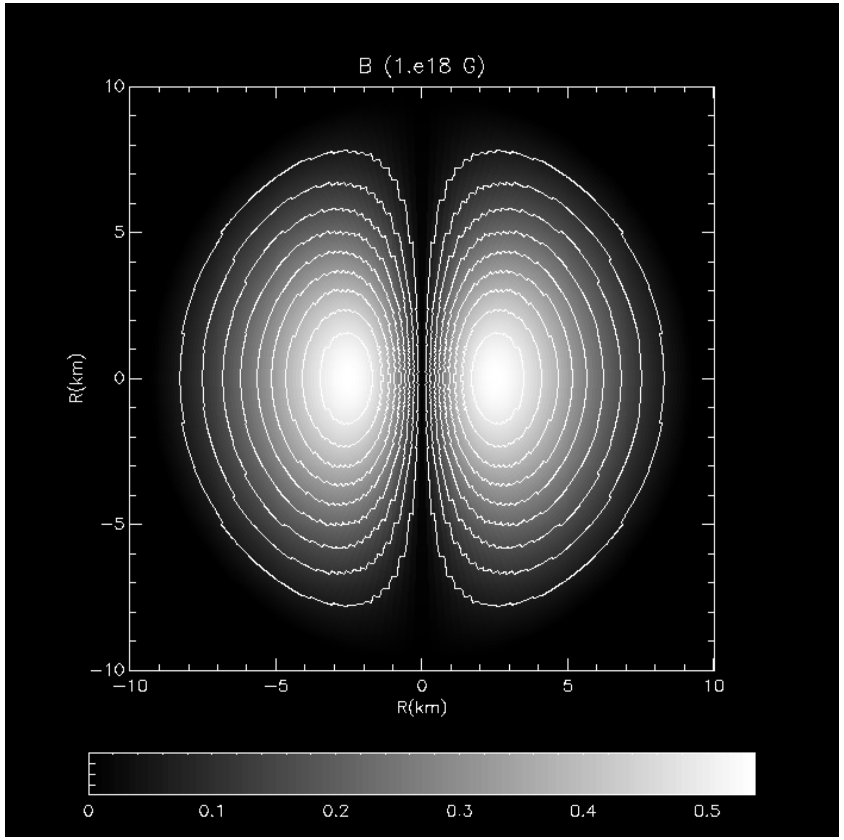
  Poloidal: Current $J^{\Phi} \sim r k_{pol} \rho$, currents are all confined within the star ➡ B^r, B^{Θ}

Output ➡ M, R, I_{xx}, I_{zz}

Toroidal Magnetic field ($\vec{B} = B_\phi \hat{\phi}$) with rotation



Density isocontour



Magnetic field isocontour

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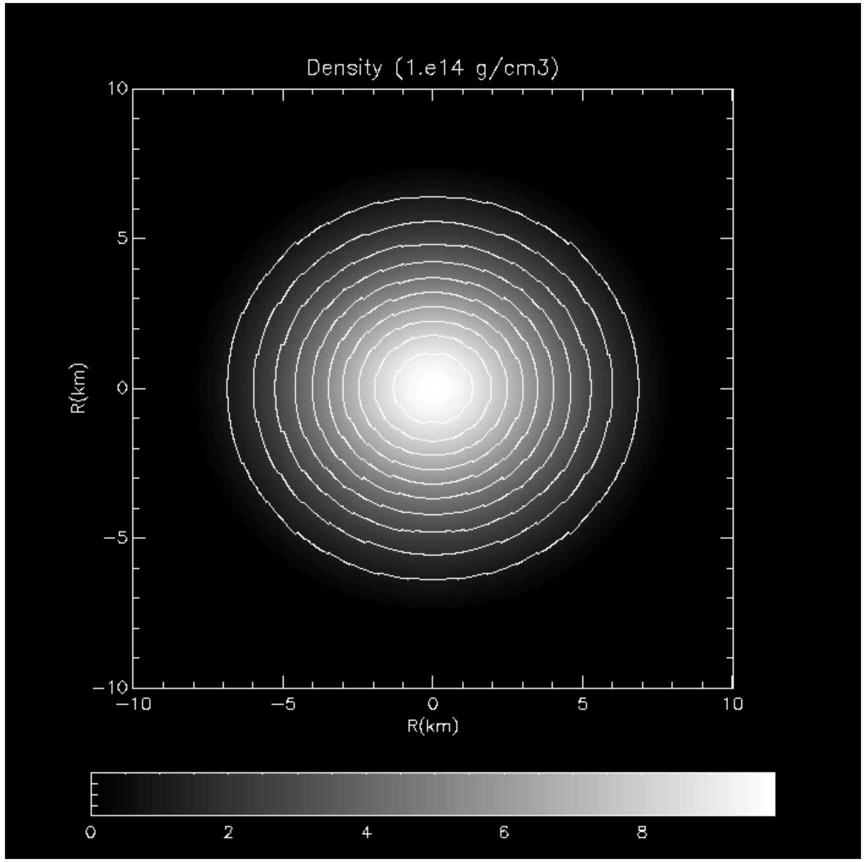
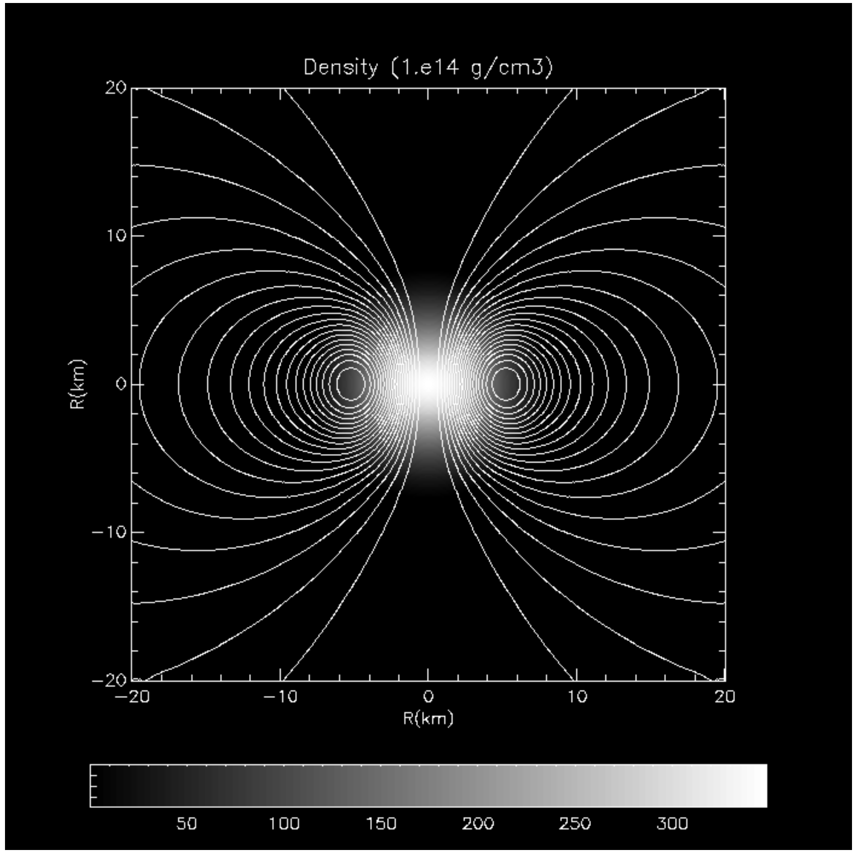
arXiv:2302.03706

$\rho_c = 10^{15} g/cc, B_{max} = 5 \times 10^{17} G, \nu = 500Hz$ ➡ $M = 2M_\odot, R_E = 14km, R_P/R_E = 0.92, ME/GE = 0.056$

Poloidal Magnetic field ($\vec{B} = B_r\hat{r} + B_\theta\hat{\theta}$) with rotation



Density isocontour



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$\rho_c = 10^{15} g/cc, B_{max} = 5\times10^{17} G, \nu = 50Hz,$ ➡ $M = 2M_\odot, R_E = 12km, R_P/R_E = 0.81, ME/GE = 0.02$

GRAVITATIONAL WAVE DETECTION

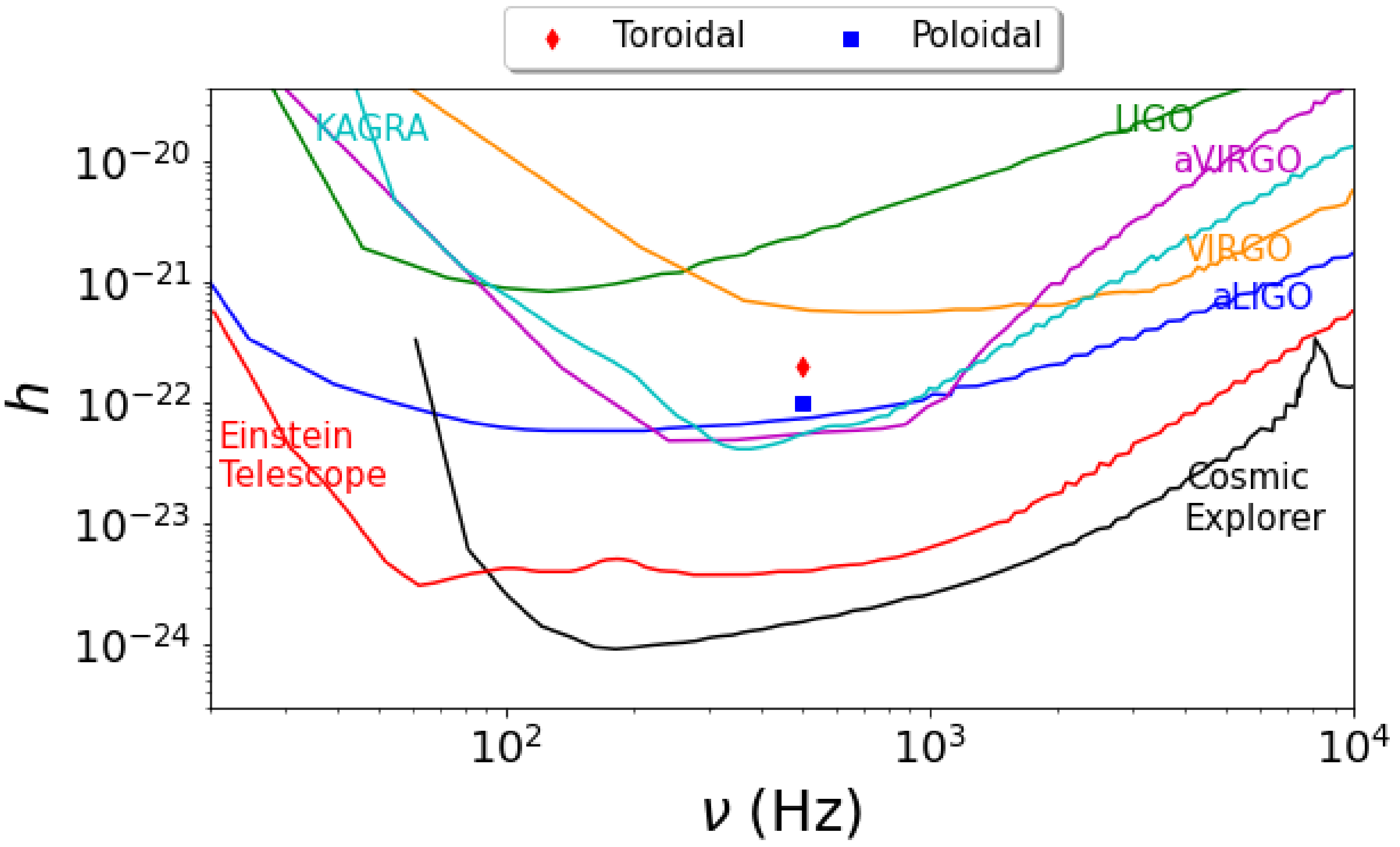
GW strain for various magnetic configuration (modelled from XNS) as a function of frequency along with the sensitivity curves of various detectors.

$h = 0.0110297h_0$ for $\chi = 3^\circ$,

d=10 kpc

$$B_{max} = 5 \times 10^{17} G,$$

$$\nu = 500 Hz$$



Actually not detected yet, REASON?

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GW strain (h) ∝ Rotation frequency (Ω), Obliquity angle (χ), Magnetic field strength (B) } Decays with time!

Then how long we can detect them?

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Then how long we can detect them?

Magnetic field decay:

Ohmic dissipation

$$t_{ohmic} \sim 2 \times 10^{11} \frac{L_5^2}{T_8^2} \left(\frac{\rho}{\rho_{nuc}} \right)^3 \text{ yr},$$

Ambipolar diffusion

$$t_{ambipolar} \sim \frac{5 \times 10^{15}}{T_8^6 B_{12}^2} \text{ yr} + t^s_{ambipolar},$$

where

$$t^s_{ambipolar} \sim 3 \times 10^9 \frac{L_5^2 T_8^2}{B_{12}^2} \text{ yr},$$

and

Hall drift

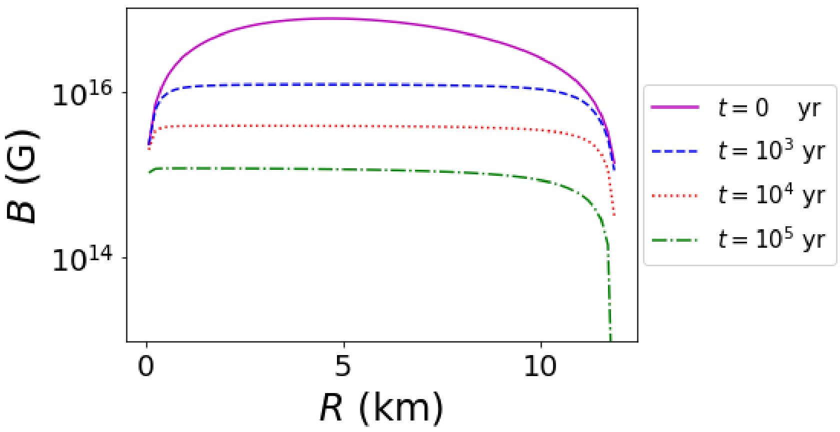
$$t_{Hall} \sim 5 \times 10^8 \frac{L_5^2 T_8^2}{B_{12}} \left(\frac{\rho}{\rho_{nuc}} \right) \text{ yr},$$

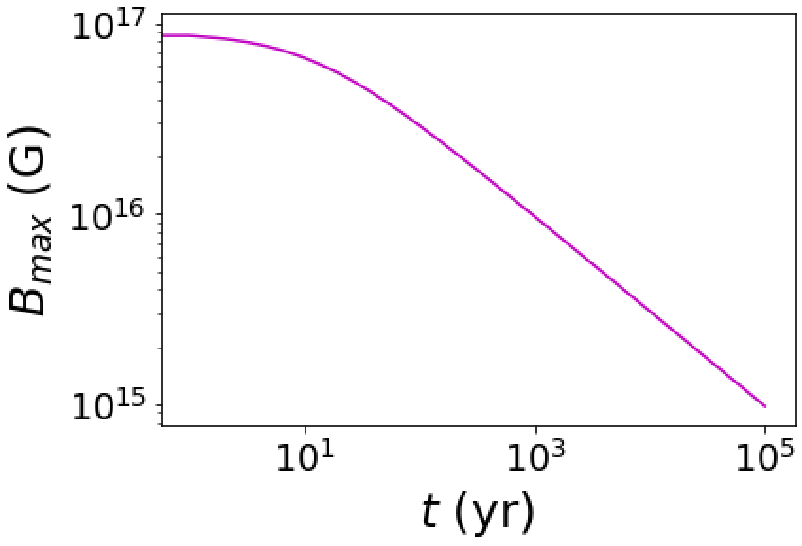
$$\frac{dB}{dt} = -B \left(\frac{1}{t_{ohmic}} + \frac{1}{t_{ambipolar}} + \frac{1}{t_{Hall}} \right)$$

Goldreich & Reisenegger 1992
Heyl & Kulkarni 1998

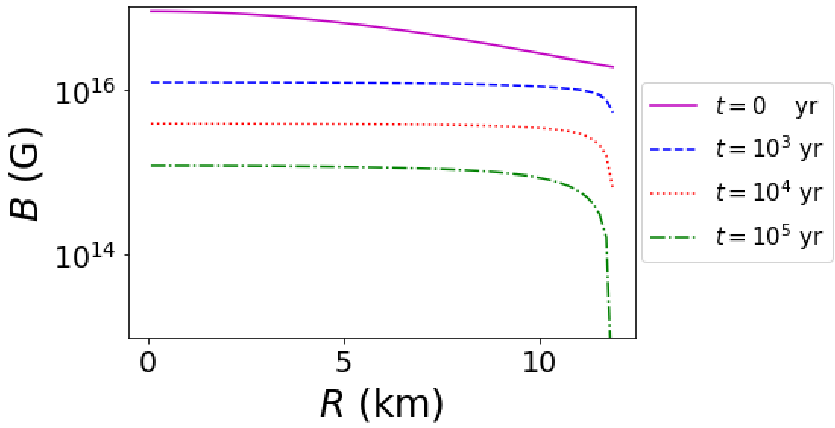
Evolution of B (ν, χ constant)

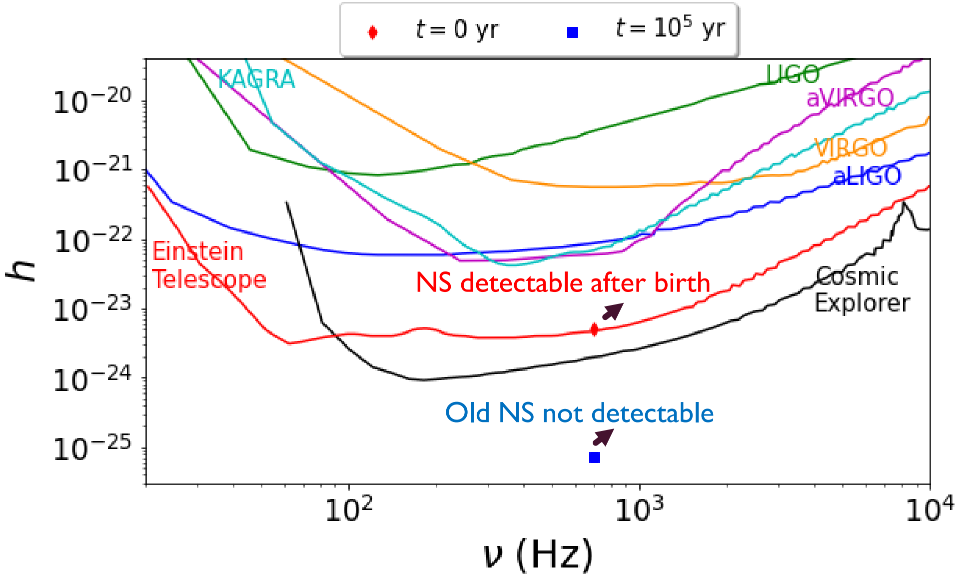
Toroidal

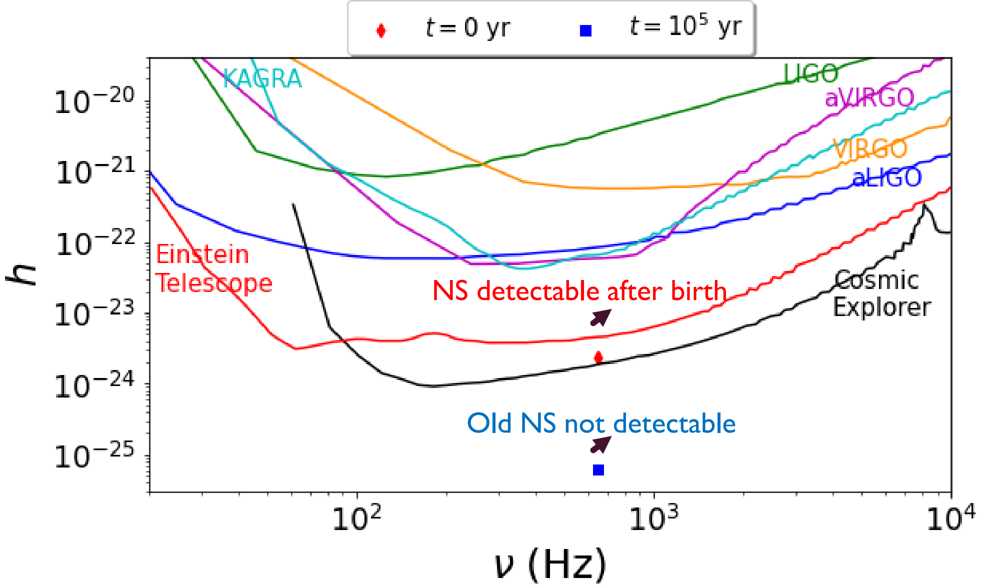




Poloidal







$$M = 2.02M_{\odot}, B_{max}^{Toroidal/poloidal}(initial) = 9 \times 10^{16}G$$

$\nu = 700Hz,$

$\nu = 650Hz$

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GW strain (h) ∝ Rotation frequency (Ω), Obliquity angle (χ), Magnetic field strength (B) } Decays with time!

Then how long we can detect them?

Ω and χ evolution:

GW (quadrupole) radiation & Electromagnetic (dipole) radiation: Ω, χ ⬇

$$\frac{d(\Omega I_{z'z'})}{dt} = -\frac{2G}{5c^5}(I_{zz}-I_{xx})^2\Omega^5\sin^2\chi(1+15\sin^2\chi) - \frac{B_p^2R_p^6\Omega^3}{2c^3}\sin^2\chi F(x_0)$$

$$I_{z'z'}\frac{d\chi}{dt} = -\frac{12G}{5c^5}(I_{zz}-I_{xx})^2\Omega^4\sin^3\chi\cos\chi - \frac{B_p^2R_p^6\Omega^2}{2c^3}\sin\chi\cos\chi F(x_0) + \zeta\epsilon_\Omega{}^2\epsilon R^3\frac{g(\chi)}{I_{zz}\sin\chi\cos\chi}$$

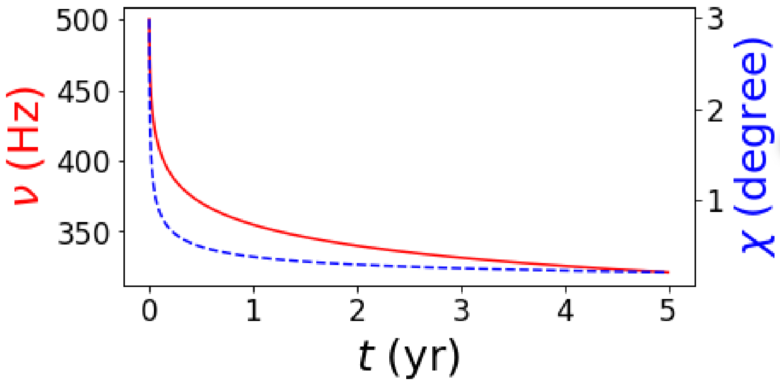
$$\zeta = \zeta(T), x_0 = x_0(\Omega), T(t) = \left(\frac{6N^s}{C}t + \frac{1}{T_0^6}\right)^{-1/6},$$

$$F(x_0) = \frac{x_0^4}{5(x_0^6 - 3x_0^4 + 36)} + \frac{1}{3(x_0^2+1)}.$$

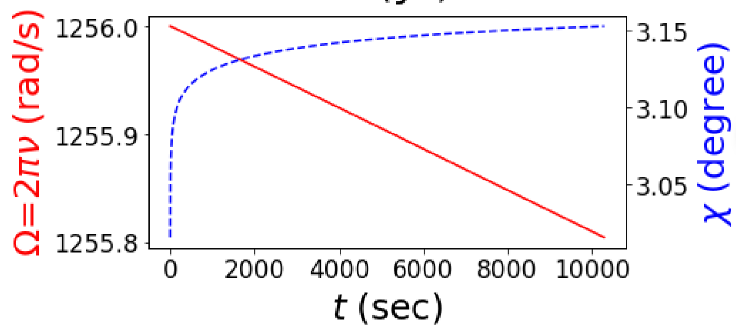
Viscous & thermal effect:
Negligible, in few hours χ increases upto 3% and saturates

Chau & Henriksen 1970
Lander & Jones 2018

Evolution of ν and χ (B constant): Toroidal magnetic field



Angular momentum extraction due to radiation: **Ω, χ decays (Timescale~years)**



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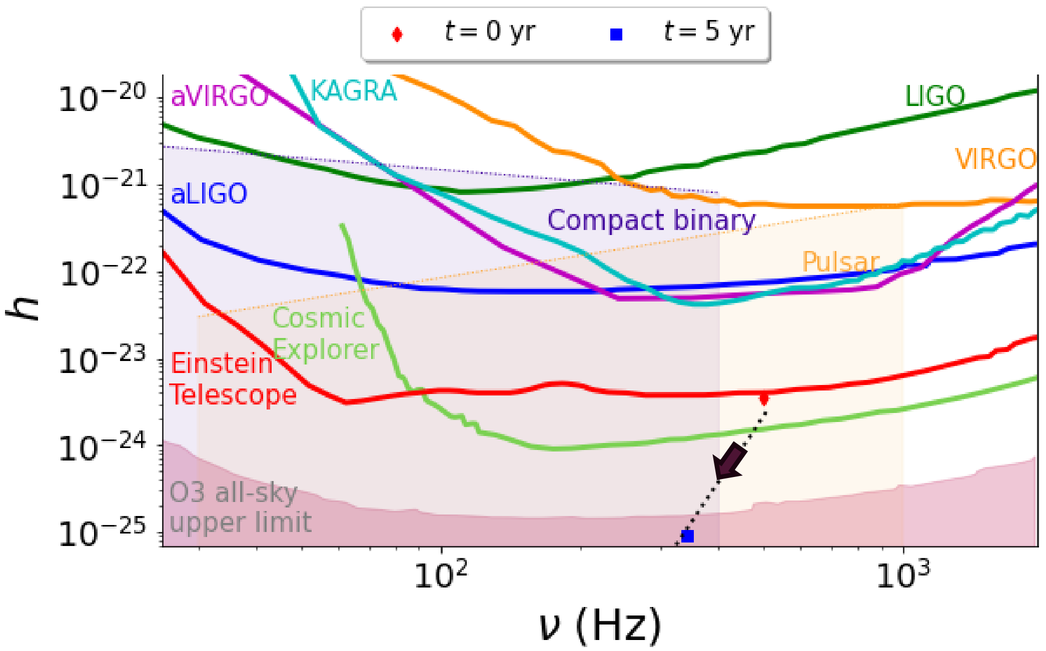

$$M = 2.02M_{\odot}, B_{max}^{Toroidal}(initial) = 1.4 \times 10^{17}G, \nu(initial) = 500Hz$$

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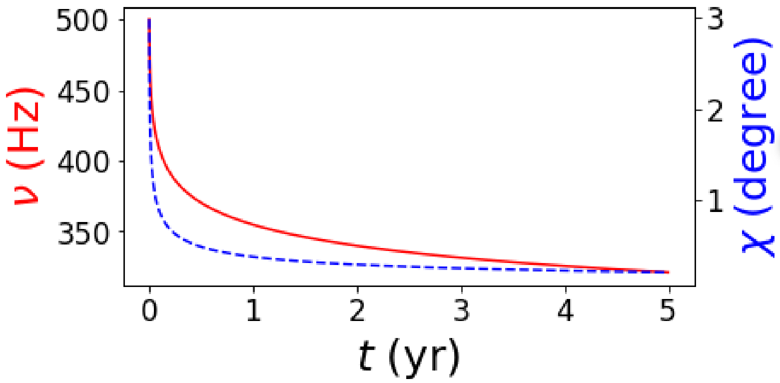

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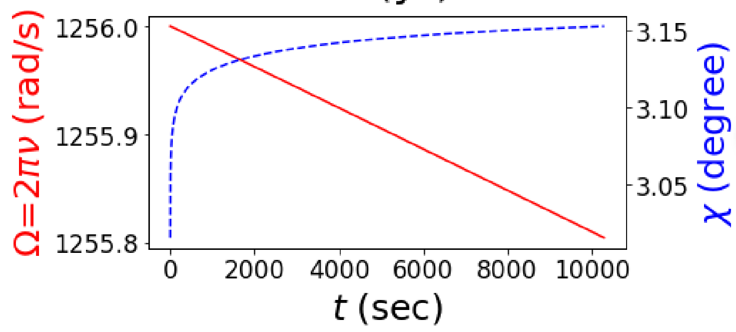

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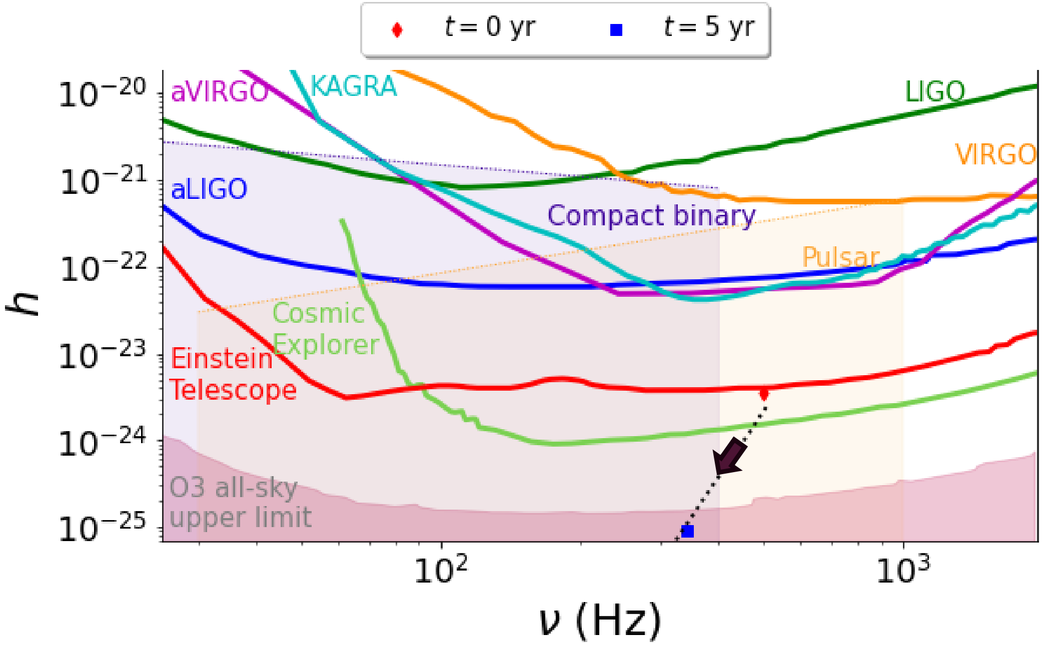

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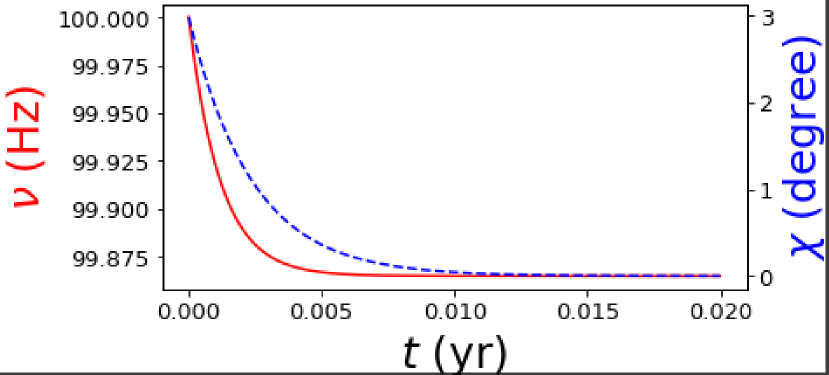
$$M = 2.02M_\odot \, , B_{max}^{Toroidal}(initial) = 1.4 \times 10^{17}G, \nu(initial) = 500Hz$$

However, for $\mathbf{B_{max}^{Poloidal}(initial) = 10^{15}G}$, Ω, χ decays in few days of time,

but stable NSs are actually toroidally dominated.

(Wickramasinghe et al. 2014)



Timescale for ν, χ decay $<<$ Timescale for B decay

Decay of ν, χ is more important to study GW amplitude decay

TO INCREASE GW DETECTION POSSIBILITY:

Signal to noise ratio (SNR) for 1 Year integration time

$$\langle S/N \rangle = \sqrt{\langle S/N_{\Omega}^2 \rangle + \langle S/N_{2\Omega}^2 \rangle},$$

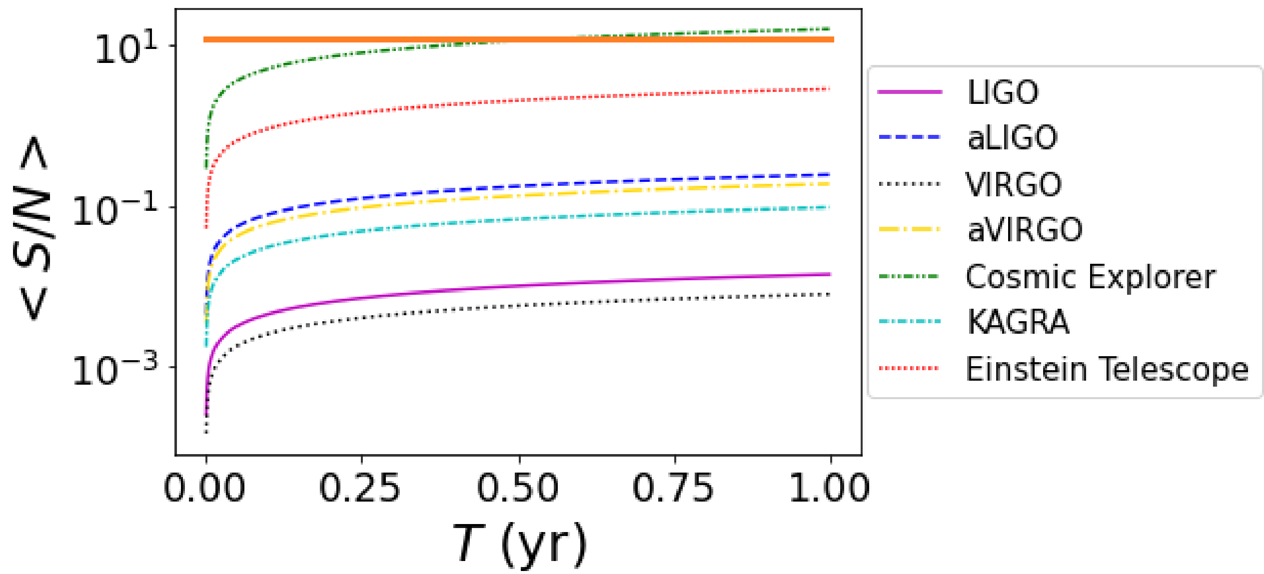
where

$$\langle S/N_{\Omega}^2 \rangle = \frac{\sin^2\zeta}{100} \frac{h_0^2 \sqrt{N} T_{stack} \sin^2 2\chi}{S_n(\nu)} = \frac{\sin^2\zeta}{100} \frac{h_0^2 T \sin^2 2\chi}{\sqrt{N} S_n(\nu)}$$

and

$$\langle S/N_{2\Omega}^2 \rangle = \frac{4\sin^2\zeta}{25} \frac{h_0^2 \sqrt{N} T_{stack} \sin^4 \chi}{S_n(2\nu)} = \frac{4\sin^2\zeta}{25} \frac{h_0^2 T \sin^4 \chi}{\sqrt{N} S_n(2\nu)}$$

Maggiore et al. 2020



$M = 2M_\odot, B_{max}^{Toroidal} = 9 \times 10^{16} G, \nu(initial) = 200 Hz$

$SNR(threshold) = 12$

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The signal is not detectable instantaneously.
After one month of integration time, it will be detectable!!

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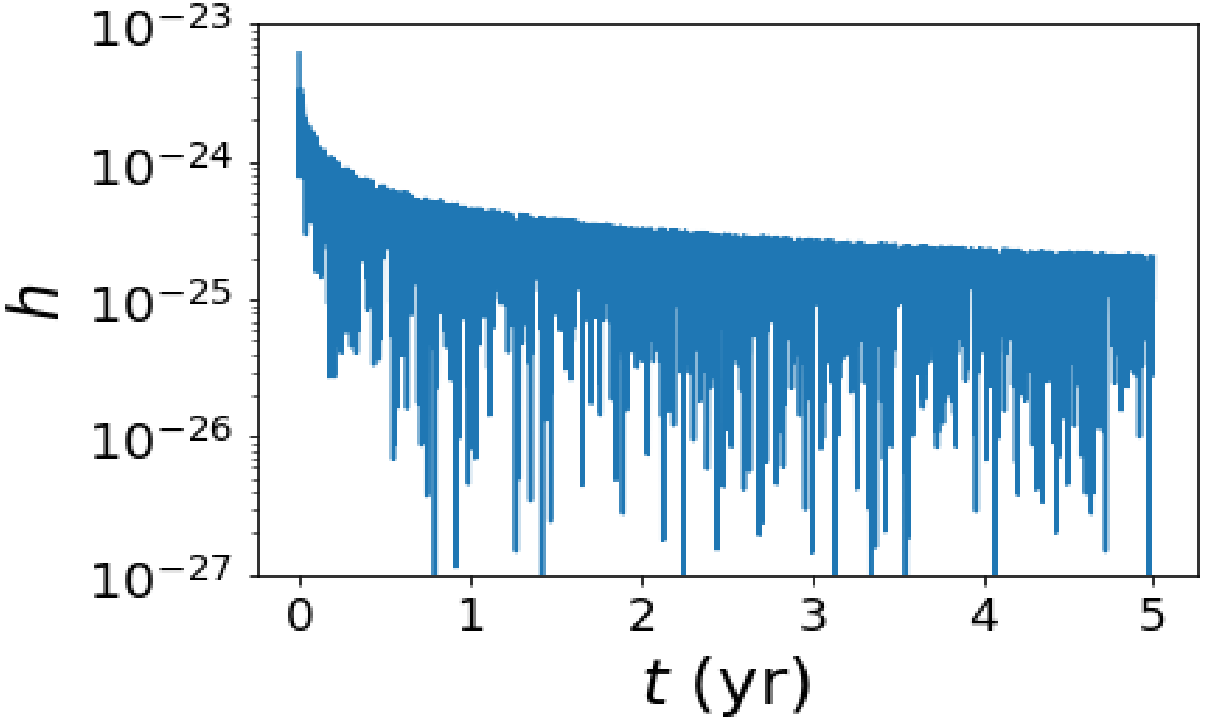

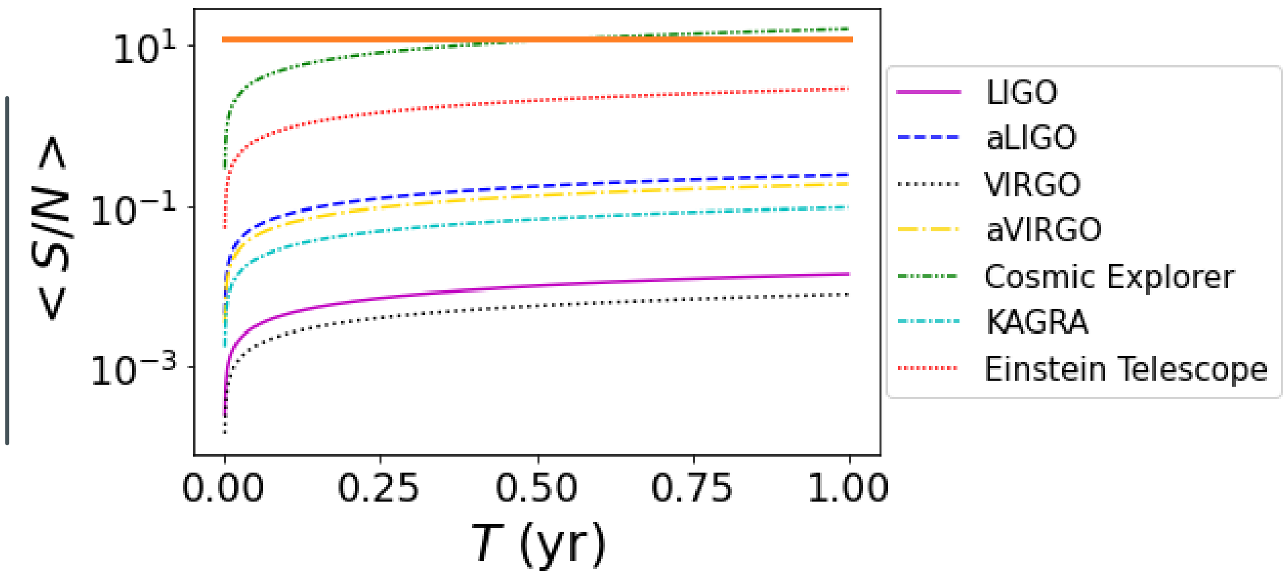

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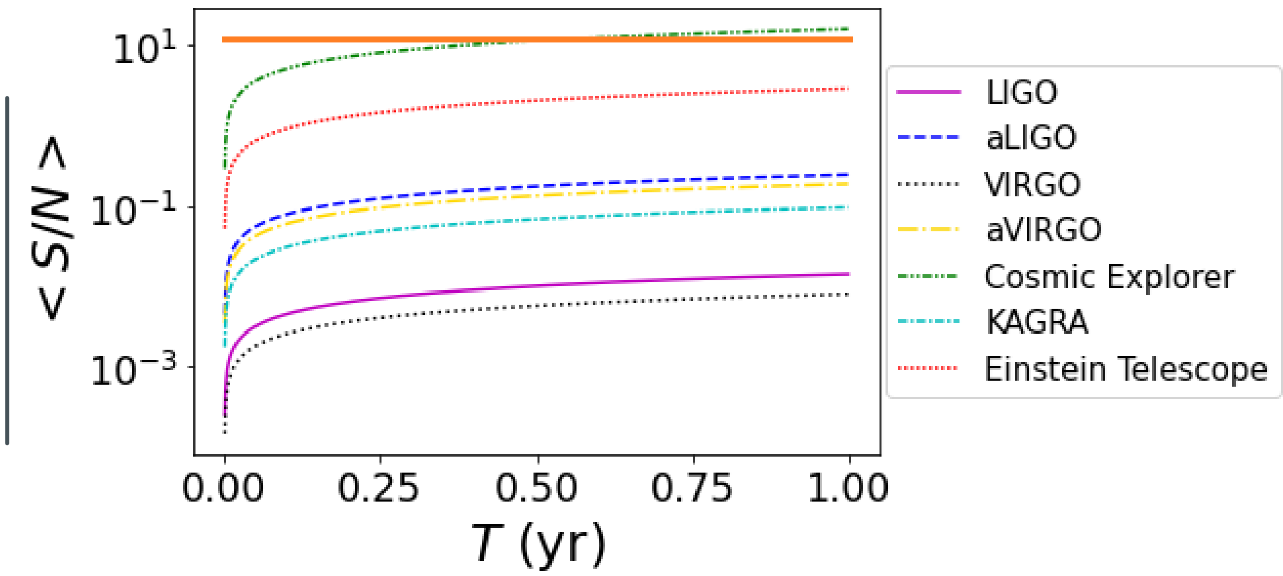

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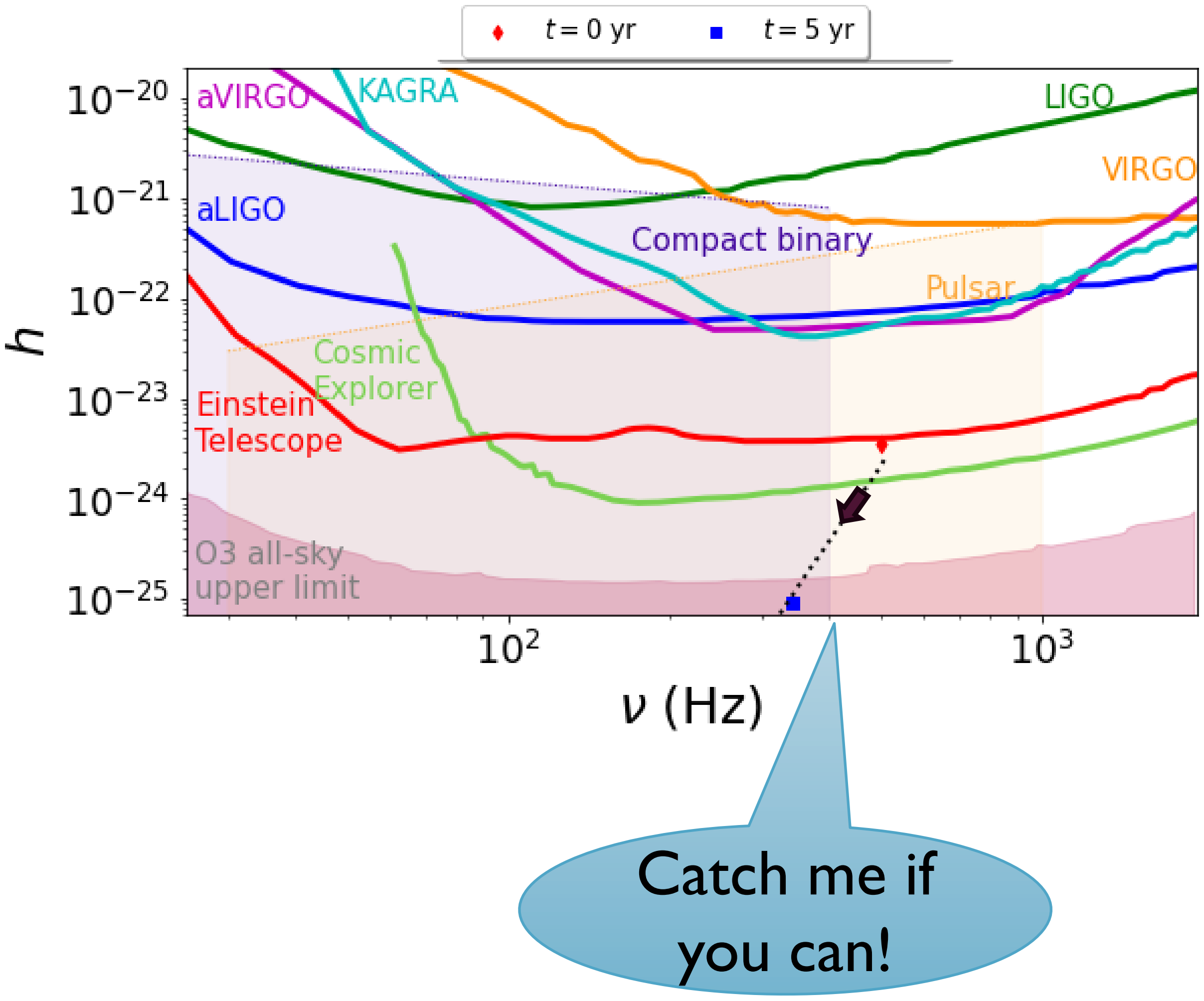
Number of NSs to be detected in observational timescale?



- Small probability of detection in this 5-10 year?
- Number of NSs to be detected (in 10 kpc^3 /galactic volume) in 10 yr observational timescale is < 0.02.

[Ns birthrate in galaxy 0.002/year: Beniamini et al. 2019]

CONCLUSION

Uniformly rotating massive NSs, which are detectable after birth, may not be detectable forever. Because its GW amplitude decays due to decay of B, Ω and χ. Perhaps this is why we have **not yet detected CGW** from NSs by aLIGO, aVIRGO, KAGRA.

We can try to **increase the detection** probability by calculating SNR over 1 year leading to direct detection of NSs (which cumulatively adds up the SNR, thus can have better probability to detect after some time).

Future gravitational wave missions with Einstein Telescope and Cosmic Explorer should be planned accordingly to detect such massive NSs, which, if successful can provide us an idea about its spin, magnetic field, as well as about the equation of state.

THANK YOU