Convolutional neural network for directed searches of continuous gravitational waves

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Continuous gravitational waves and neutron stars workshop, June 17-20, 2024 @ Hannover, Germany

Continuous gravitational waves

- Persistent, quasi-monochromatic signals
- Amplitude is very weak
- We need to accumulate O(yr) long data to detect CGWs
- The sensitivity of CGW search is limited by two difficulties.
    - Computational cost.
    - Instrumental lines (narrow spectral artifacts).

Topic of this talk

- This talk is based on two papers:
  - TSY & Tanaka, PRD103, 084049 (2020)
  - TSY, Miller, Sieniawska, and Tanaka, PRD106, 024025 (2022)

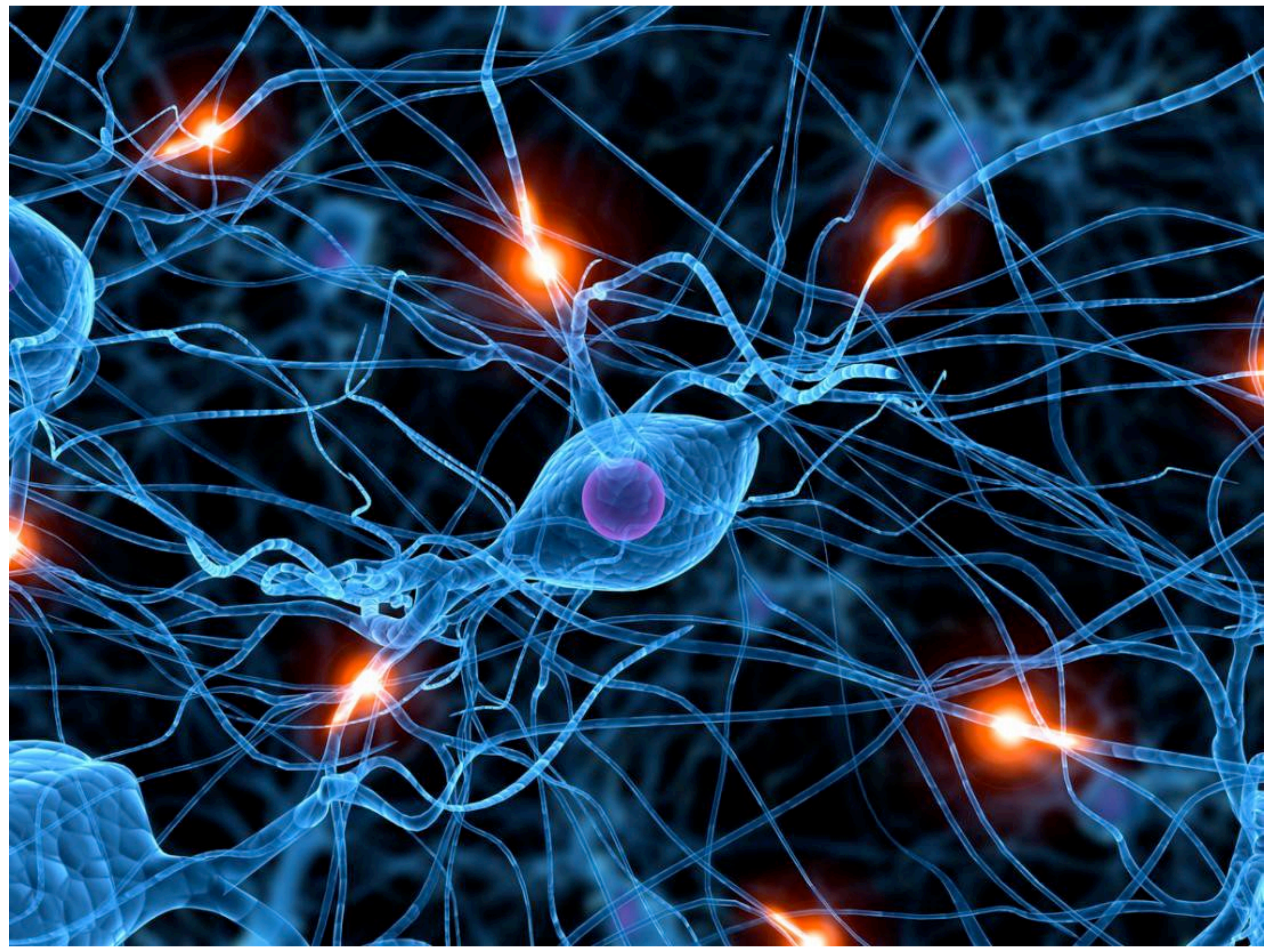
  in which we proposed new pipeline for **all-sky searches** (no information about source) with a convolutional neural network.

- I also talk about our ongoing work, in which we extend our work **for directed searches** (the source direction is known) with modified preprocessing algorithm.

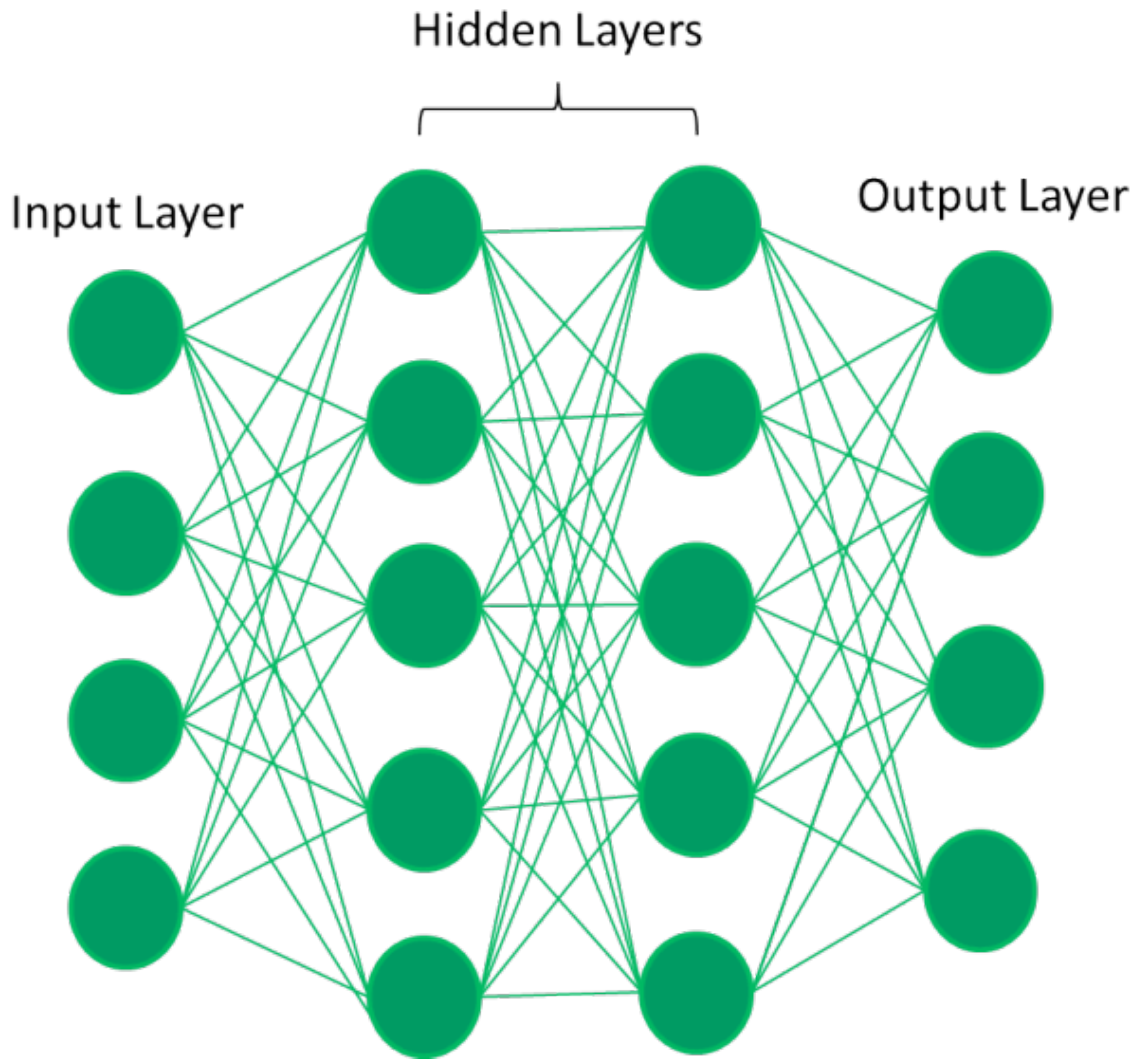
Deep learning

e.g. Goodfellow *et al.*, “Deep learning” as a textbook

- Neural network (NN) is inspired by the structure of a human brain, mimicking the way that biological neurons signal to one another.
- Highly non-linear function controlled by many parameters.
- NN’s parameters are optimized using a training dataset before we apply NN to test data or a real event.



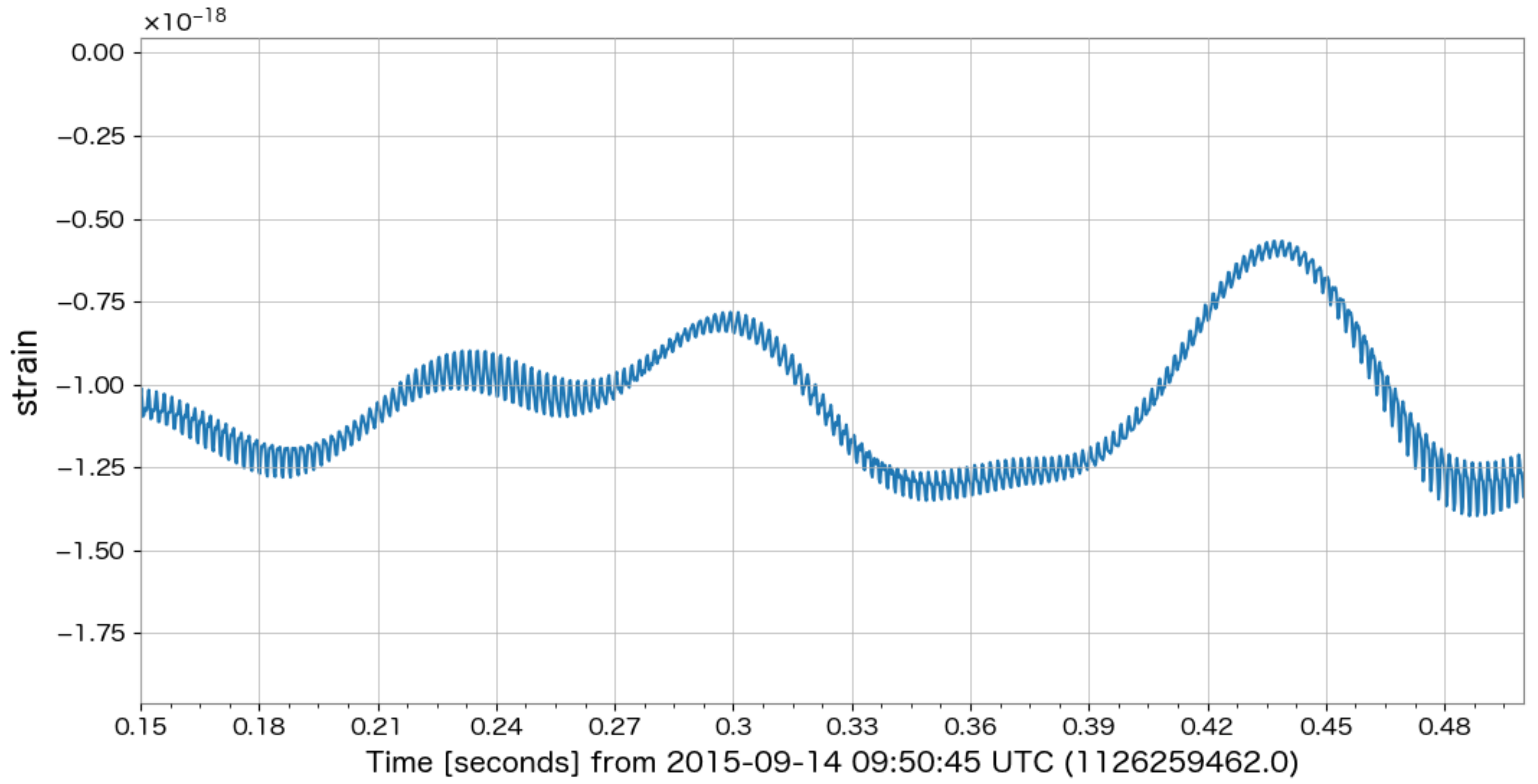
https://medium.com/predict/artificial-neural-networks-mapping-the-human-brain-2e0bd4a93160

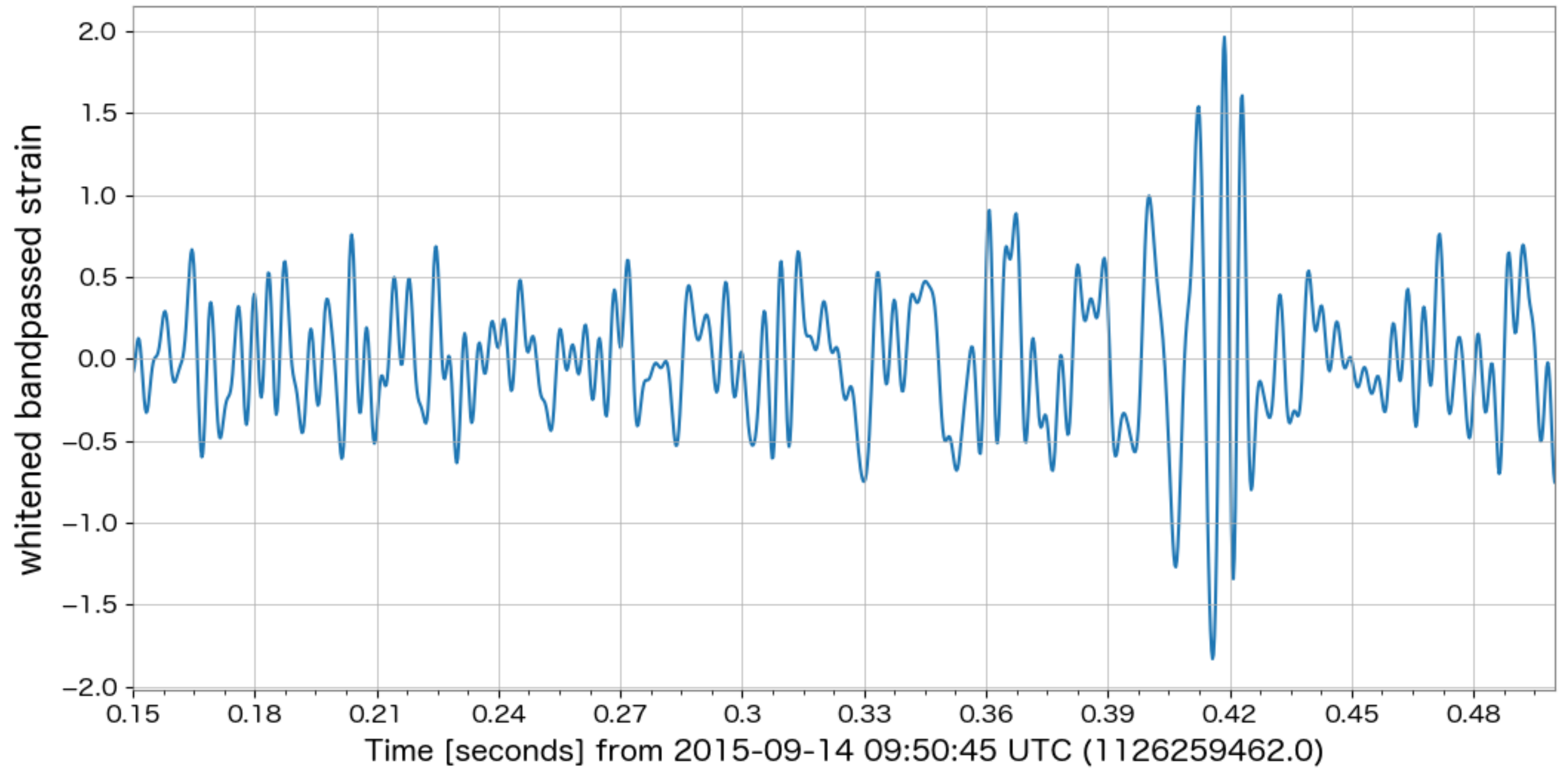


https://developer.nvidia.com/blog/digits-deep-learning-gpu-training-system/

Preprocess

- The preprocess transforms data into another type of data so that neural networks will be able to treat the data more easily.
- Example: whitening and bandpass for CBC signals
  - A raw strain data is dominated by detector noise. CBC signals can not be found by eye. After whitening and bandpass, CBC signals can be easily found by eye.





Proposed algorithm

```
Place the coarse grid points on the sky
For n_grid:
    Remove doppler modulation
    Make spectrogram
    For frequency bin:
        Perform Fourier transform over all times
        Give transformed data to neural network and get prediction
        If prediction = "CGW exists":
            Store {n_grid, frequency bin} as a candidate
```

Preprocess

Neural network

Waveform model of CGW & line

CGW

$$h_{\mathrm{obs}}(t) = h_0 \left[F_+(t) \frac{1+\cos^2\iota}{2} \cos \underline{\Phi(t)} + F_\times(t) \cos\iota \sin \underline{\Phi(t)} \right]$$

Phase

$$\underline{\Phi(t)} = 2\pi f_{\mathrm{gw}} \left(t + \underline{\frac{\boldsymbol{r}(t)\cdot\boldsymbol{n}}{c}} \right) + \pi \dot{f} \left(t + \underline{\frac{\boldsymbol{r}(t)\cdot\boldsymbol{n}}{c}} \right)^2 + \phi_0$$

Doppler modulation

$\mathbf{r}(t)$: detector position w.r.t. SSB, $\mathbf{n}$: source direction

Line

$$n_{\mathrm{line}}(t) = n_0 \cos(2\pi f_{\mathrm{line}} t + \phi_0)$$

(We assume a line has stable frequency and exists for entire observation period.)

Amplitude is measured by $$\hat{h}_0 = \frac{h_0}{\sqrt{S_n}}, \quad \hat{n}_0 = \frac{n_0}{\sqrt{S_n}}$$

Remove Doppler modulation

We use a coarse grid points on the sky. It should be enough dense to remove the Earth rotation (daily).

$$\Phi(t) \sim 2\pi f_{\mathrm{gw}} t + \Phi_{\oplus}(t) + \Phi_{\odot}(t)$$

On the frequency bin being closest to the GW frequency, SFT only has the residual phase originated from the Earth's orbital motion (annual).

$$\Phi(\tau) \sim 2\pi f_{\mathrm{gw}} \tau + \delta\Phi_{\odot}(\tau)$$

SSB time $\tau := t + \dfrac{\boldsymbol{r}(t) \cdot \boldsymbol{n}_{\mathrm{grid}}}{c}$

Residual phase $\delta\Phi_{\odot} := 2\pi f_{\mathrm{gw}} \dfrac{\boldsymbol{r}_{\odot}(\tau) \cdot \Delta\boldsymbol{n}}{c}$

$\Delta\boldsymbol{n} := \boldsymbol{n}_{\mathrm{source}} - \boldsymbol{n}_{\mathrm{grid}}$

* For explanation, we neglect the antenna pattern functions, the window function.
* Also, we neglect d*f*/d*t*.
* Lines can be distinguished from CGWs because of this process.

Perform Fourier transform after SFT

We pick up the frequency bin closest to f_{gw} .

$$h_{j,k}^{\mathrm{SFT}} \sim \exp[i\boldsymbol{r}(t)\cdot\Delta\boldsymbol{n}] \sim \exp[iR_{\mathrm{ES}}\Delta\theta\cos(\Omega_{\odot}t)]$$

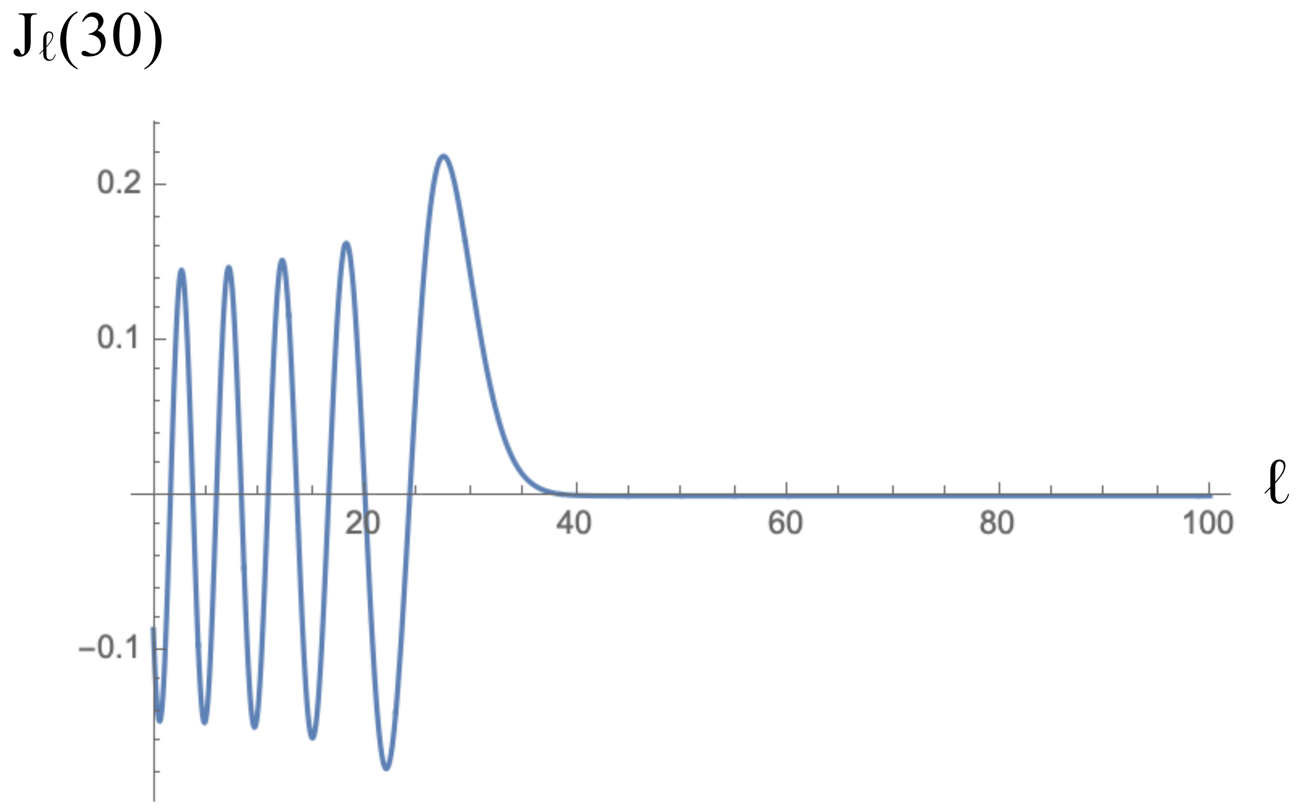
SFT data contains residual phase which can be modeled by cos function.

It can be rewritten by the Fourier transform of Bessel function by Jacobi-Anger expansion.

$$\exp[iR_{\mathrm{ES}}\Delta\theta\cos(\Omega_{\odot}t)] = \sum_{\ell=-\infty}^{\infty} i^{\ell} J_{\ell}(R_{\mathrm{ES}}\Delta\theta)e^{i\ell\Omega_{\odot}t}$$

By performing another Fourier transform, you can accumulate the signal power into small number of bins.

$$\mathrm{H}_{\ell,k} := \frac{1}{N}\sum_{j=0}^{N-1} h_{j,k}^{\mathrm{SFT}} e^{-2\pi i j\ell/N} \simeq J_{\ell}(R_{\mathrm{ES}}\Delta\theta)$$



Transformed data

TSY & Tanaka, PRD103, 084049 (2020)
TSY *et al.*, PRD106, 024025 (2022)

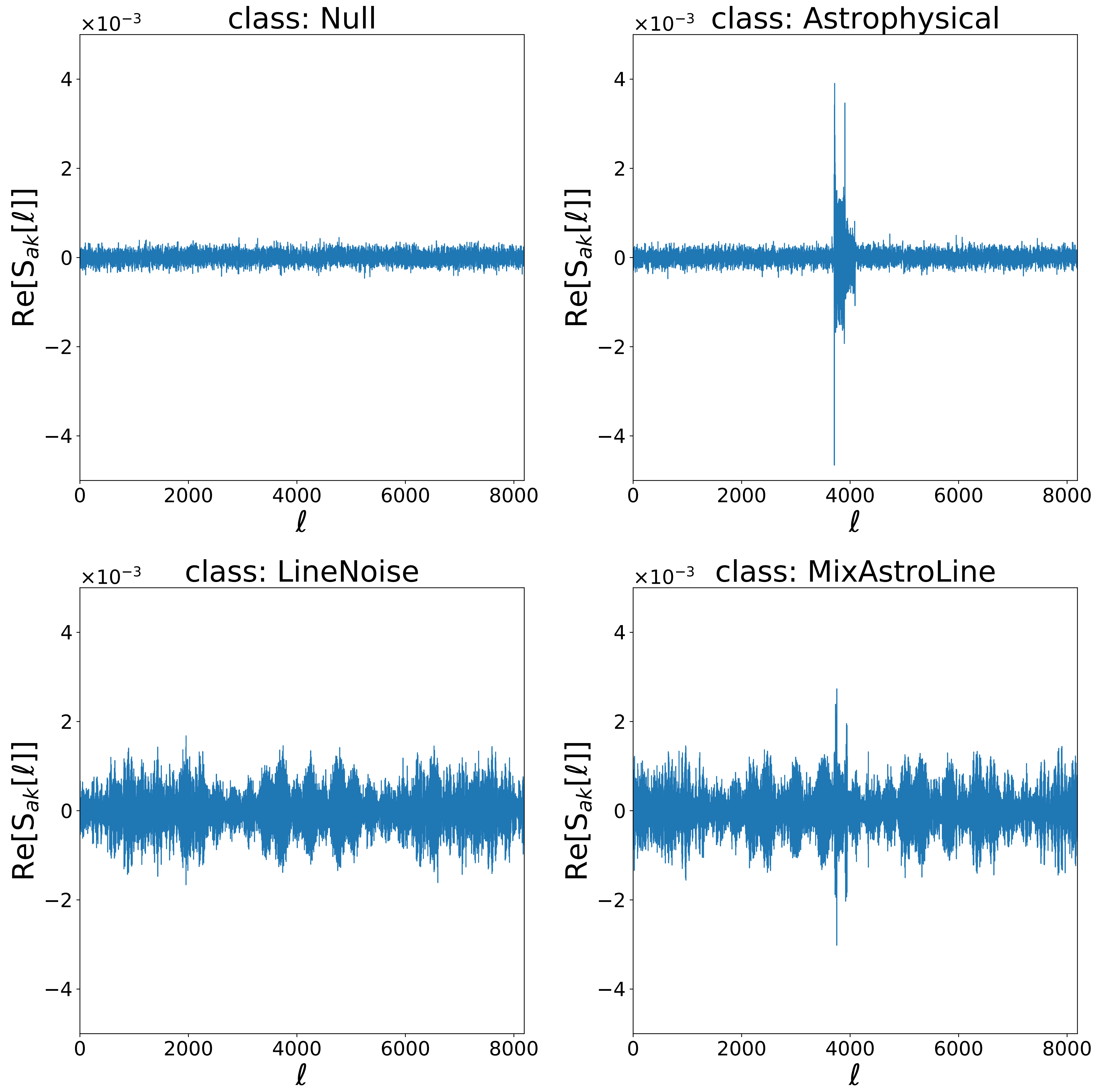
We assume four classes:

1. Null
2. Astrophysical (GW signal)
3. LineNoise
4. MixAstroLine (Line noise + GW signal)

(All classes contain Gaussian noises.)

CGW and lines can be distinguished by the differences in the response to the removal of Doppler modulation.

* In these figures, we inject a strong CGW signal for visualization.



Dataset settings

- $f_{\mathrm{gw}} = f_k + \beta T_{\mathrm{seg}}^{-1}$, where $f_k = 100$ Hz and $\beta \in [-0.5, 0.5]$
- Fix df/dt = 0 [Hz/sec] (but we tested CNN also for nonzero df/dt cases.)
- T_{seg} = 2048 sec
- T_{obs} = 16777216 sec (~ 0.5yr)
- Random inclination angle, polarization, initial phase
- Sky positions are also randomly chosen. For each waveform, we pick up the closest grid point to the source location.
- Amplitude of signal : 0.01 ~ 10.0
- Amplitude of line noise : 1.0 ~ 10.0
- Line noise is a sinusoidal waveform. $f_{\mathrm{line}} = f_k + \beta T_{\mathrm{seg}}^{-1}$, $\beta \in [-0.5, 0.5]$
- # of training data: Null = Astrophysical = LineNoise = Mix = 20,000

Neural network & training settings

- input size = 8192 (= T_{obs} / T_{seg})
- channels = 2 (real and imaginary parts of transformed signal)
- output size = 4, we interpret each of them as the probability of each class
- loss function: cross entropy loss
- batch size: 512
- total epoch: 300
- learning rate: 1.0e-4
- optimizer: Adam
- implemented with PyTorch, trained with a GPU GeForce 1080Ti

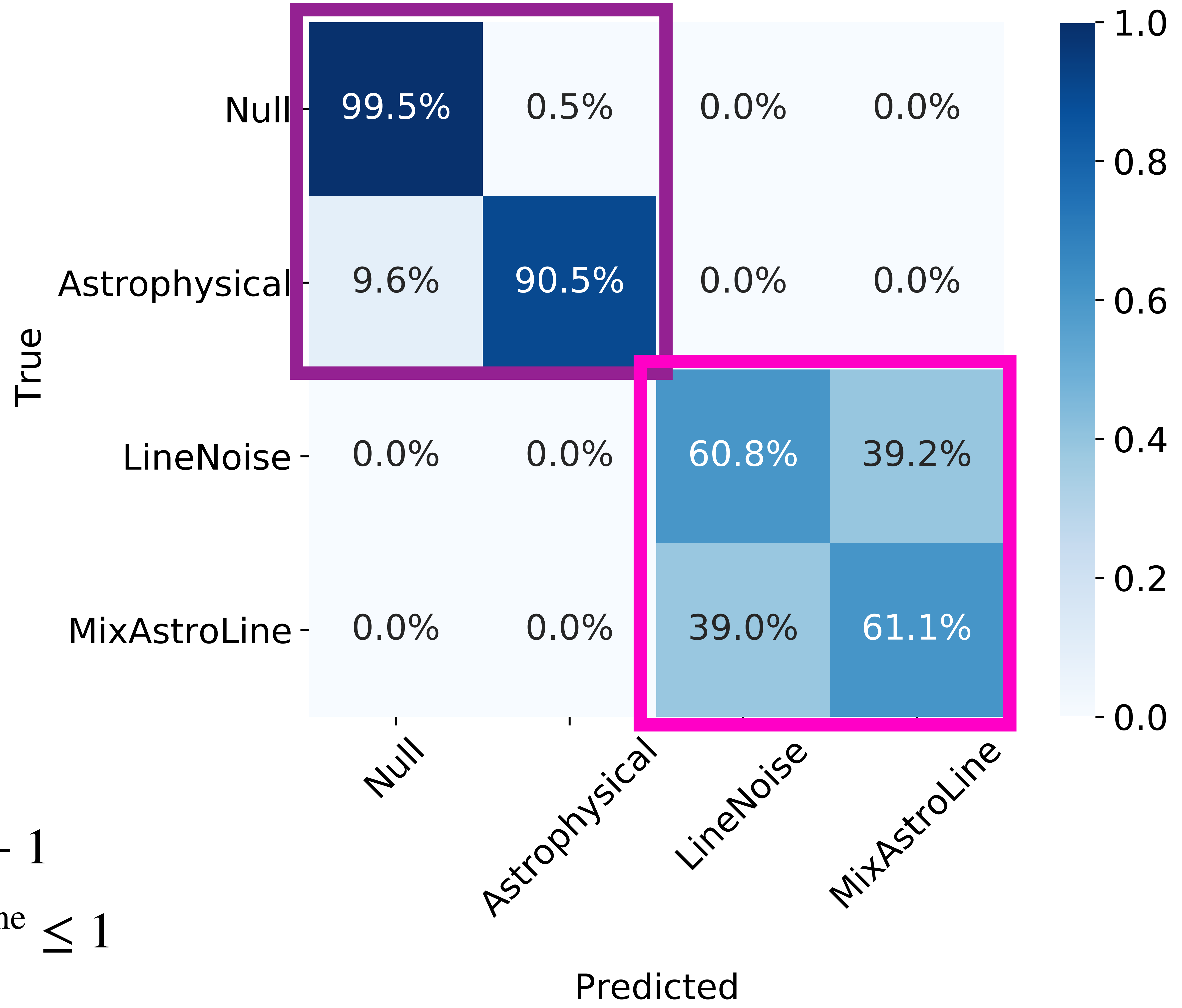
Layer	Output size	# of parameters
1D convolutional	(16, 8177)	528
ReLU	(16, 8177)	-
1D convolutional	(16, 8162)	4112
ReLU	(16, 8162)	-
Max pooling	(16, 2040)	-
1D convolutional	(32, 2033)	4128
ReLU	(32, 2033)	-
1D convolutional	(32, 2026)	8224
ReLU	(32, 2026)	-
Max pooling	(32, 506)	-
1D convolutional	(64, 503)	8256
ReLU	(64, 503)	-
1D convolutional	(64, 500)	16448
ReLU	(64, 500)	-
Max pooling	(64, 125)	-
Flattening	(8000,)	-
Fully-connected	(512,)	4096512
ReLU	(512,)	-
Fully-connected	(64,)	32832
ReLU	(64,)	-
Fully-connected	(4,)	130
Softmax	(4,)	-

Refs:
Kingma and Ba, arXiv: 1412.6980 (2014)
Paszke *et al.*, arXiv: 1912.01703 (2019)

Validity against to line noises

TSY & Tanaka, PRD103, 084049 (2020)
TSY *et al.*, PRD106, 024025 (2022)

- CNN can distinguish the **presence** and the **absence** of line noise.
- Null (only Gaussian noise) is misclassified as Astrophysical class with the probability of 0.5%.
- In the presence of line noise, it is hard to detect the signal.



- 2,000 data for each class
- GW amplitudes are uniformly sampled from $-2 \leq \log_{10} \hat{h}_0 \leq -1$
- Line noise amplitudes are uniformly sampled from $0 \leq \log_{10} \hat{h}_0^{\mathrm{line}} \leq 1$

Rough comparison

TSY & Tanaka, PRD103, 084049 (2020)
TSY *et al.*, PRD106, 024025 (2022)

Sensitivity

$$\mathcal{D}^{95\%} = \sqrt{S_n}/h_0^{95\%}$$

Method	Frequency band	$\mathcal{D}^{95\%}$
FrequencyHough	at 100 Hz	42$\sim$ 43
SkyHough	at 116.5 Hz	47.2
Time-domain $\mathcal{F}$-statistic	at 100 Hz	26$\sim$52
SOAP	on 40$\sim$500 Hz	9.9
Our method	$\lesssim$ 100 Hz	43.9

Computational cost (CPU)

Method	core-hour
FrequencyHough	9×10^6
SkyHough	2.5×10^6
Time-domain $\mathcal{F}$-statistic	2.4×10^7
SOAP	$1 - 2 \times 10^2$
Our method	1.4×10^5

Comparable or better sensitivity w/ O(10-100) speed up

- ✓ Intel E5-2670 8 operations/clock, 2.6GHz -> 20.8GFlops/core
- ✓ Simulated data for our method, observational result for other methods.
- ✓ The parameter region and the data duration are different depending on the method

Roughly estimated computational time for GPU $T_{\mathrm{CNN}} \simeq 1.02 \times 10^8$ [sec]

It can be expected to be decreased by

(i) the improvement of hardware, (ii) the use of multiple GPUs, (iii) optimize params.

Results for non-zero d*f*/d*t*

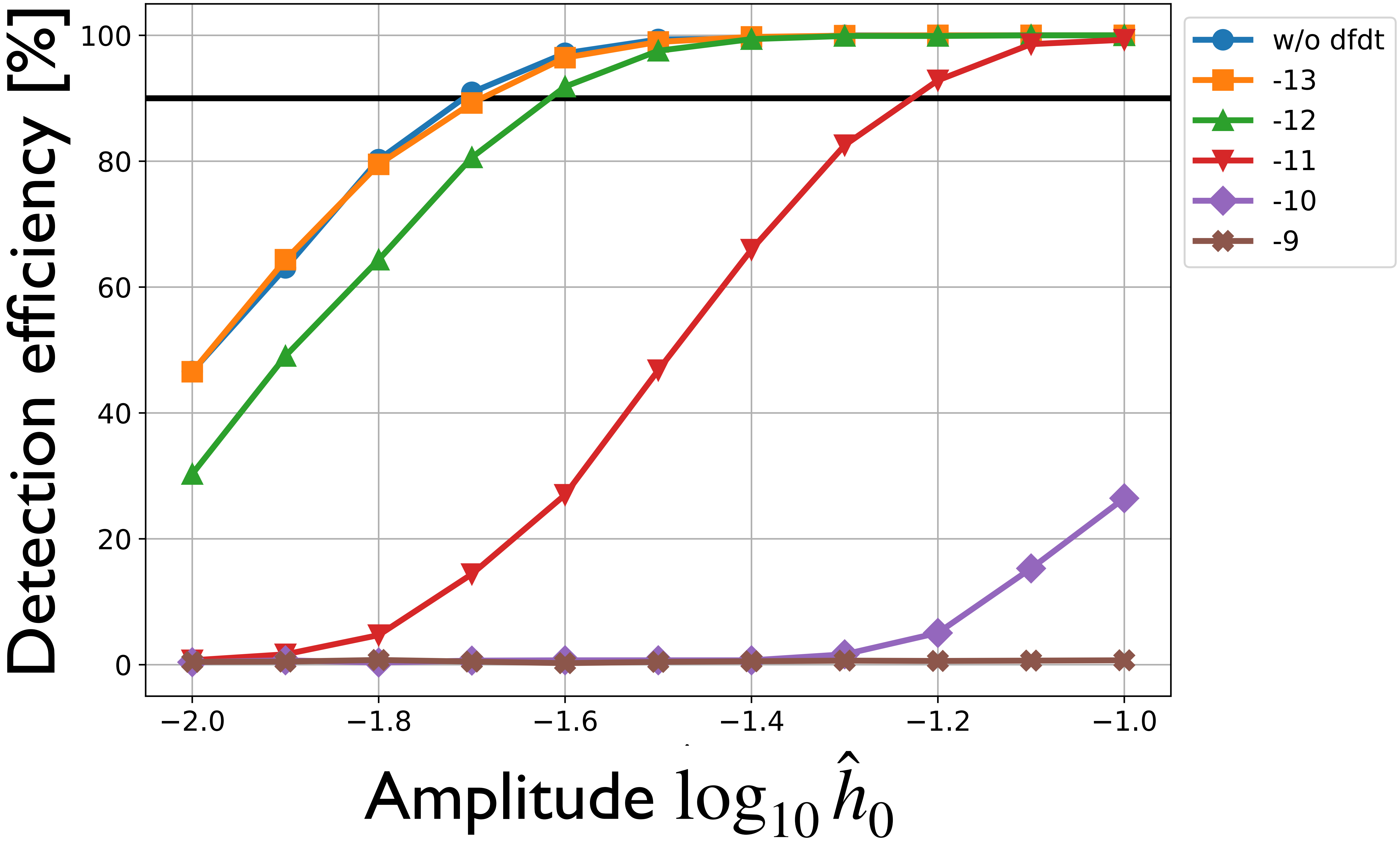
TSY & Tanaka, PRD103, 084049 (2020)
TSY *et al.*, PRD106, 024025 (2022)

We tested our neural network for the data with non-zero d*f*/d*t*, although the training data have no d*f*/d*t*.

The CNN seems good up to 10^{-12}Hz/sec, where

$(df/dt)(T_{\mathrm{obs}}) \sim (T_{\mathrm{seg}})^{-1}$

For $|df/dt| > 10^{-12}$Hz/sec, the signal cannot be contained into one frequency bin even after the preprocessing.



Extension to directed search

- We assume the grid point exactly matches with the source position.
  - In directed search, we can use the information about the source position.
  - However, the grid still can slightly deviate from the true source position.

- We use grids on df/dt to remove the effect of df/dt from the frequency evolution.
  - The grid width will be determined from the computational cost.

- We enable CNN to take multiple frequency bins as an input.
  - Input data become 2-dimensional image.
  - It may allow incomplete subtraction of the Doppler effects and/or the effect of df/dt.

Proposed algorithm

Place the coarse grid points on the sky and on df/dt

For $\{\boldsymbol{n}_{\mathrm{grid}}, (df/dt)_{\mathrm{grid}}\}$:

  Remove doppler modulation and frequency evolution by df/dt

  Make spectrogram

  For frequency band:

    Perform Fourier transform over all time for each frequency bin

    Give transformed data to neural network and get prediction

    If prediction = "CGW exists":

      Store $\{\boldsymbol{n}_{\mathrm{grid}}, (df/dt)_{\mathrm{grid}}$, frequency bin$\}$ as a candidate

Preprocess

Neural network

Dataset settings (diff)

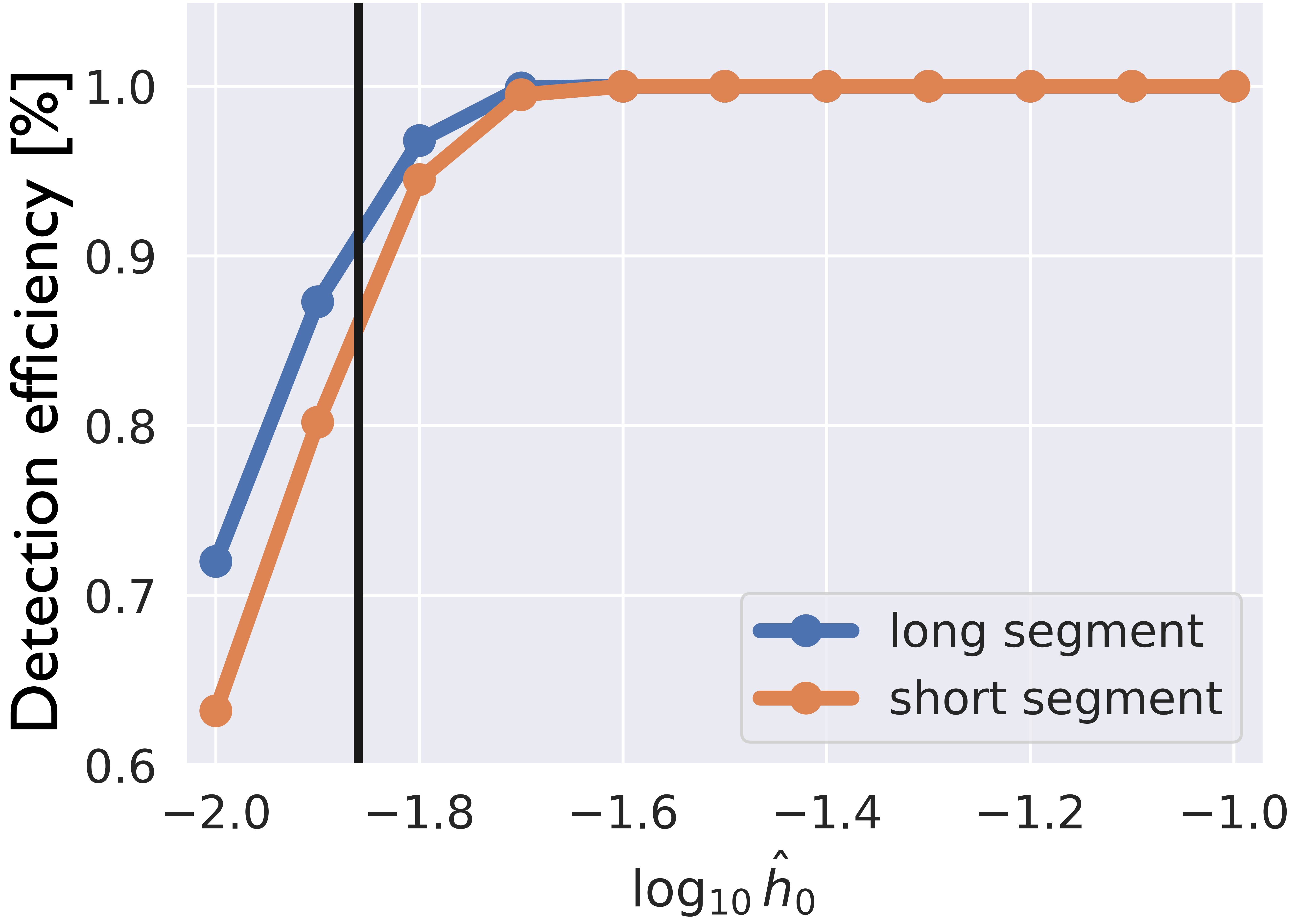
- f_{gw} is sampled from uniform distribution on [10, 1000] Hz.
- df/dt is sampled from log-uniform distribution on [-10^{-8}, 10^{-8}] Hz/sec.
- The source direction and the grid direction coincide (CasA).
- T_{seg} = 65536 sec or 262144 sec
- Dataset consists of 2 classes, {Null, Astrophysical}
- Input is two dimensional image: size (height, width) = (200, 512) or (1200, 128)

- Random inclination angle, polarization, initial phase
- $\log_{10} \hat{h}_0 \in [-2.0, 1.0]$ (→ amplitude of signal : 0.01 ~ 10.0)
- # of data, training : validate = 10000 : 1000. The dataset is augmented by generating random Gaussian noise for every iteration.

Detection efficiency

- Grid width on df/dt = 10^{-11} Hz/sec
- Orange line: T_{seg} = 65536 sec
- Blue line: T_{seg} = 262144 sec
- Black line: $\log_{10} \hat{h}_0$ = -1.86=95% upper limit of CasA at 500Hz with early O3 data (ref: LIGO&Virgo, PRD105, 082005 [2022])

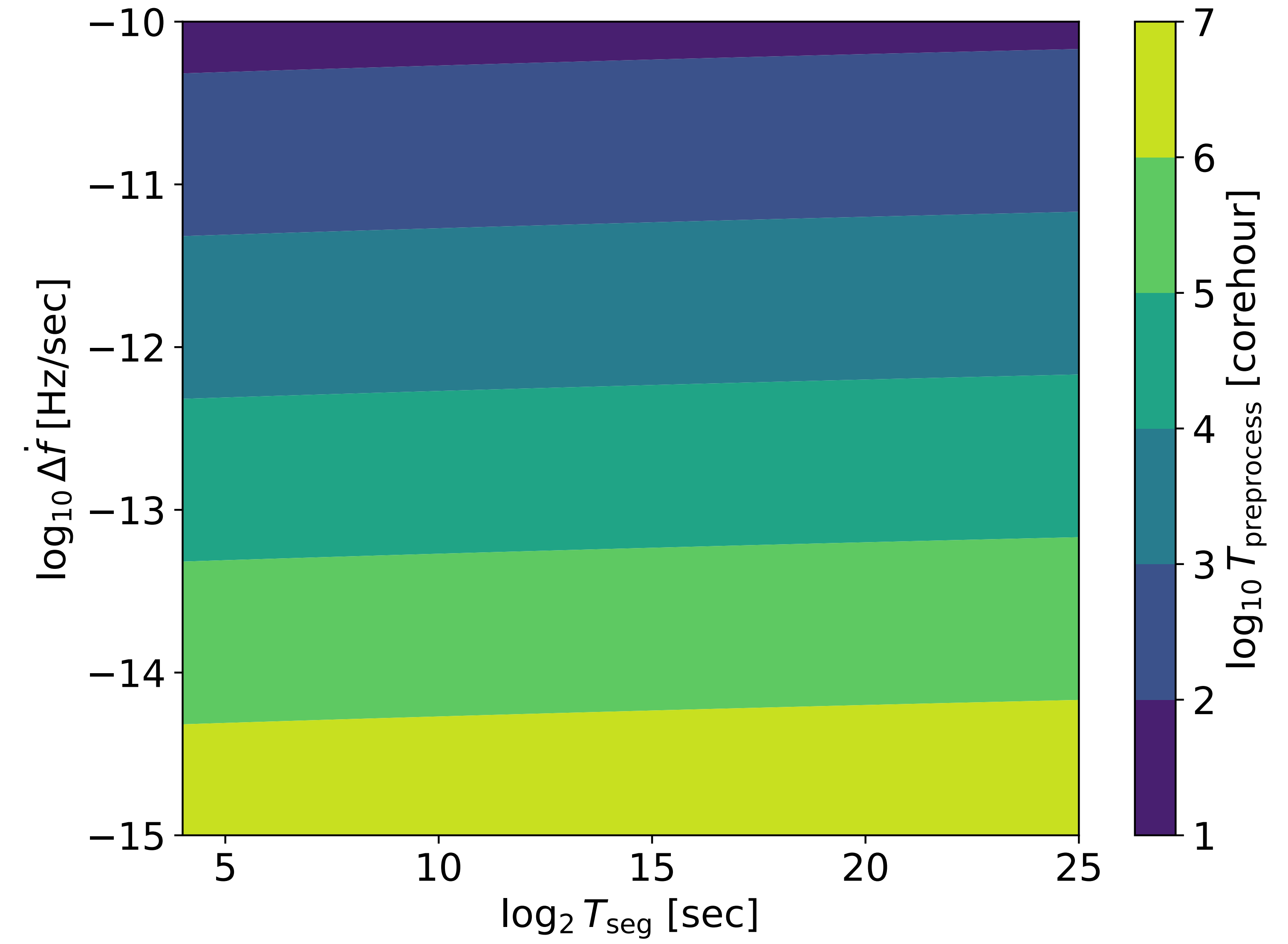
Preliminary



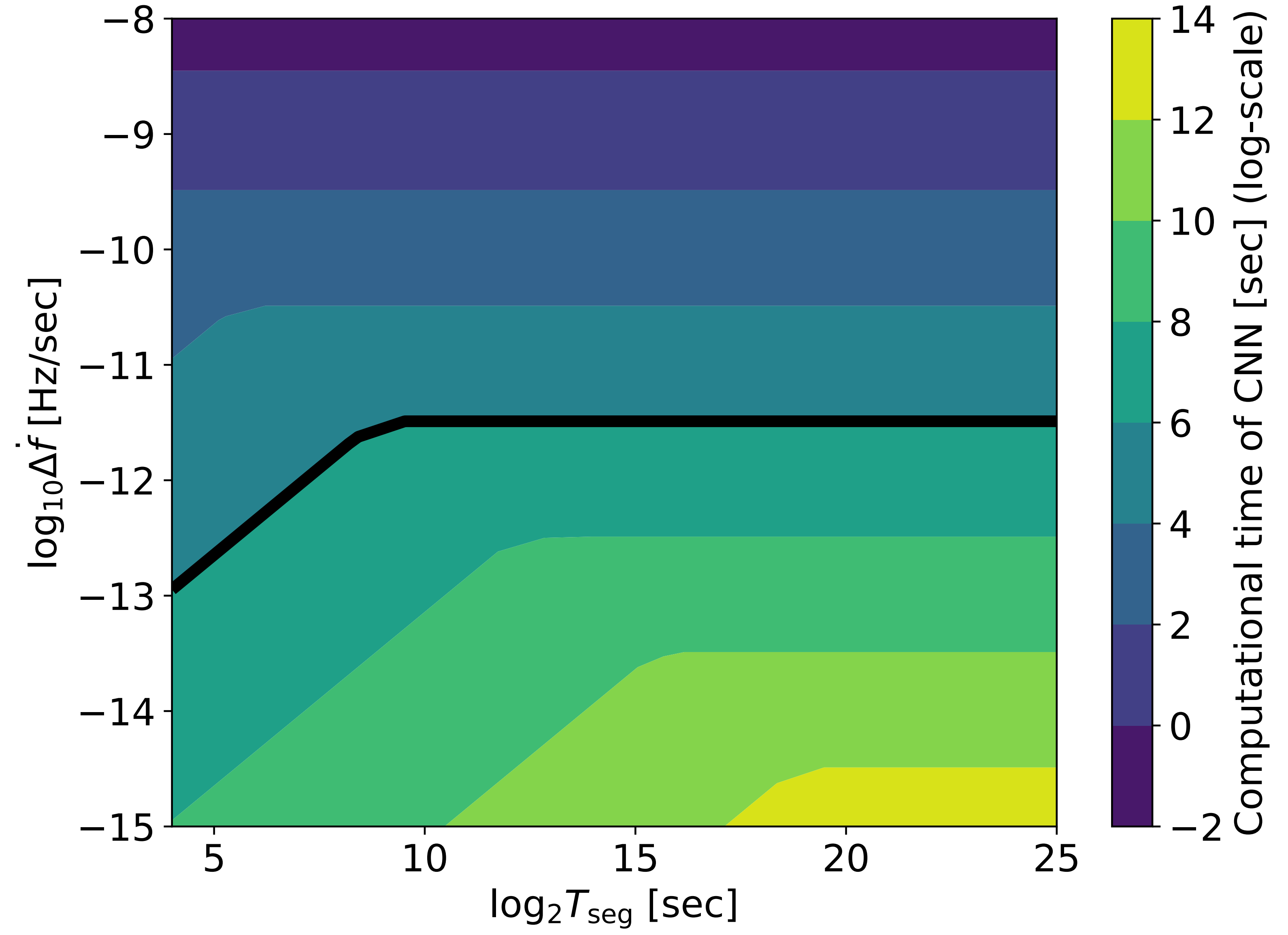
Computational cost

Preliminary

The algorithm is controlled by two parameters: SFT segment duration and grid width on d*f*/d*t*.



Upper: Computational time for preprocess



Lower: Computational time for CNN

Assuming 8.8742 x 10^{-5} sec/image which is used in our previous work

Summary

- We proposed the deep learning method for all-sky search with double Fourier transform. It has the comparable sensitivity to other pipelines with cheaper computational cost though the comparison is very rough.
- We are going to extend our algorithm for the directed search. Based on the preliminary test in which a signal is injected into simulated Gaussian noise, our method can have a comparable sensitivity to the current pipelines.
- To be implemented: data gaps, realistic line models (e.g., fluctuating frequency), non-stationarity of PSDs, multiple detectors

Appendix

Maybe too technical

Classification problem

NN returns four-dimensional vectors in which each component takes value from 0 to 1 and their sum equals to unity.

$$\boldsymbol{p}_{\mathrm{pred}} = (p_1, p_2, p_3, p_4)\,, \qquad \sum_{i=1}^{4} p_i = 1$$

$$(\text{predicted class}) = \mathrm{argmax}_{i=1,2,3,4} p_i$$

1-of-K representation: standard way to label the data for classification problem

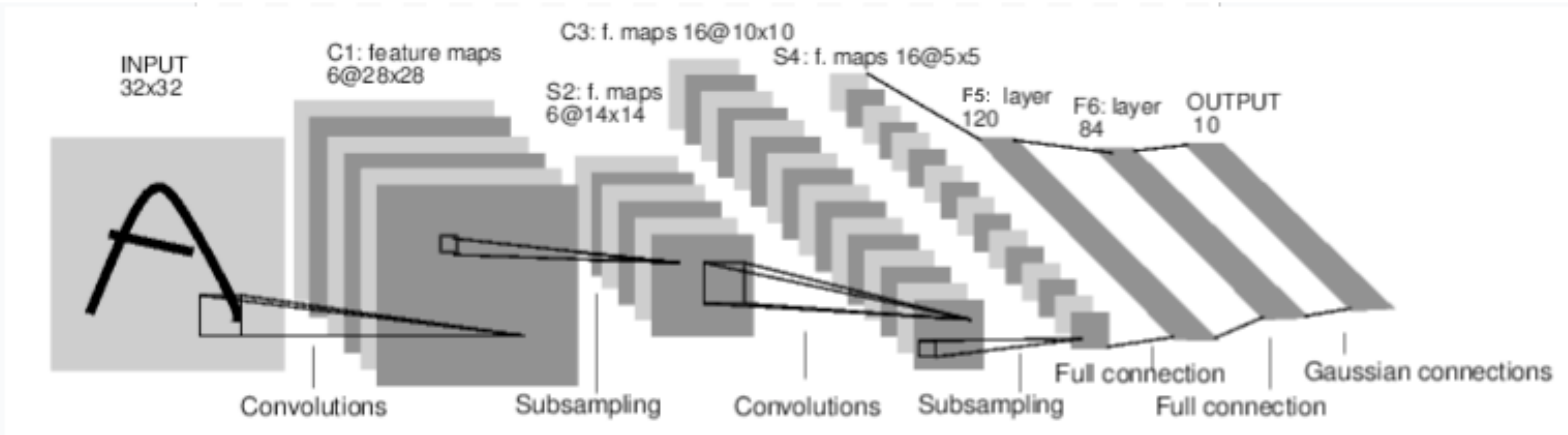
$$t_i = \begin{cases} 1 & \text{if } i\text{th class is true} \\ 0 & \text{otherwise} \end{cases}$$

Loss function: measure the difference between the answer and the NN prediction.

$$L(\boldsymbol{p}, \boldsymbol{t}) = -\sum_{i=1}^{4} t_i \ln p_i$$

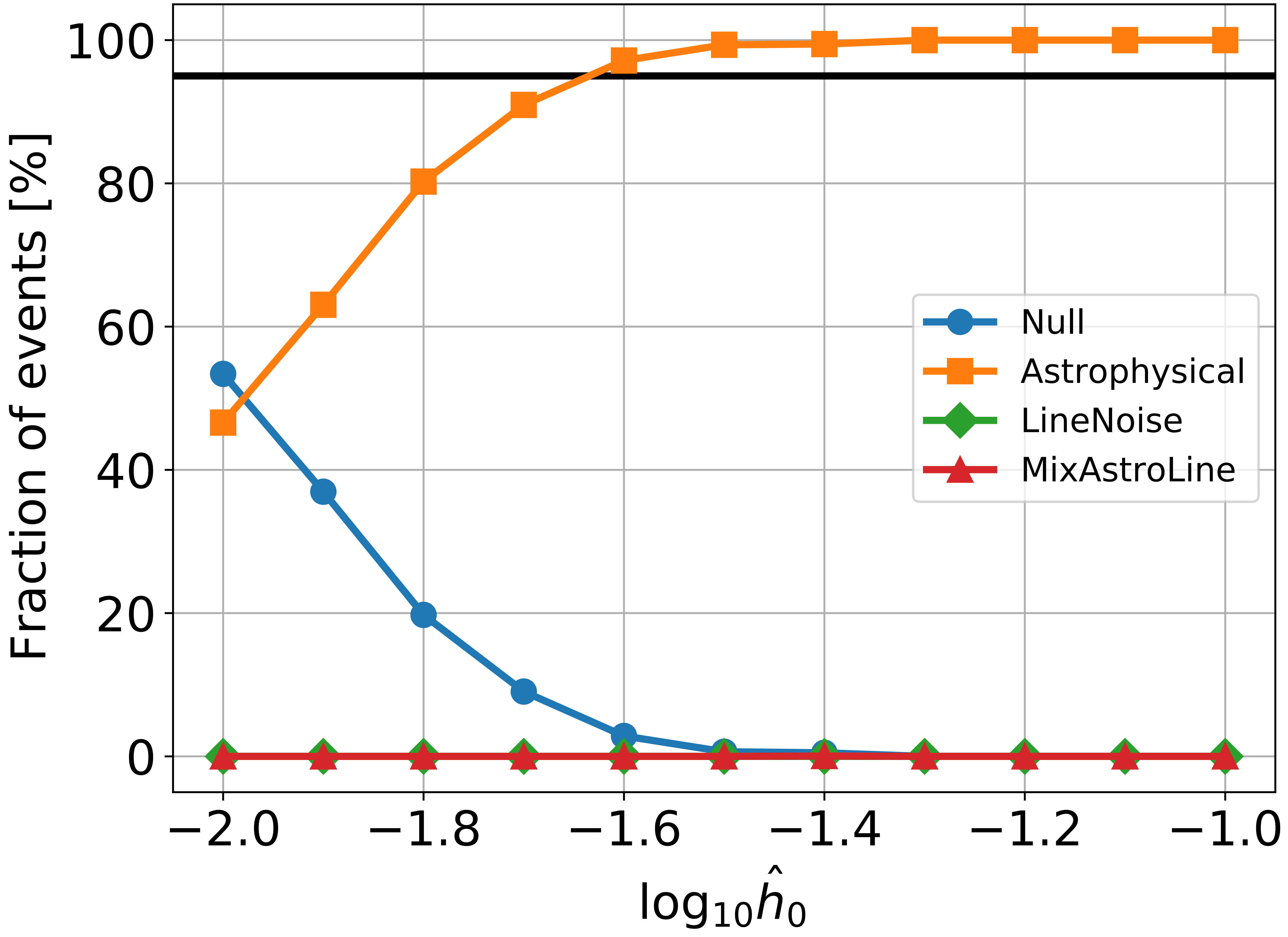
Convolutional neural network

- Convolutional neural network is advantageous for extracting local patterns.
- Pick up a small patch from an image and convolute it with filters.
- Repeating this process for many different patches, we get feature maps.
- e.g., colored image = 2-dimansional pixels with 3channels (RGB)



Results

For Astrophysical class



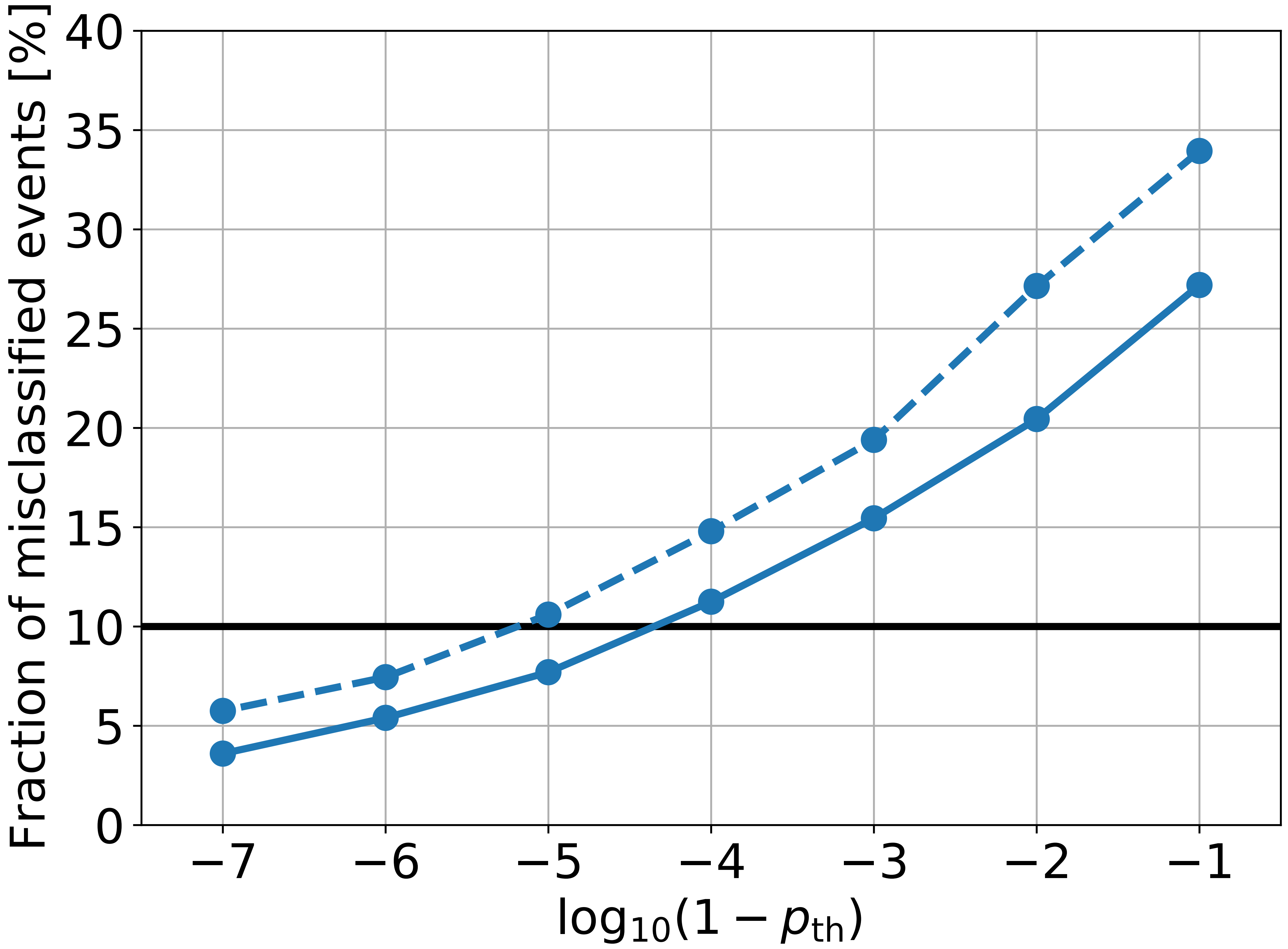
$$\text{Sensitivity depth} = \frac{1}{\hat{h}_0^{95\%}} \simeq 43.9$$

Results

Line noise class

The detection threshold can be changed.

If $p_{\mathrm{th}} \leq p_{\mathrm{Mix}}$, CNN classify the event as Mix class.



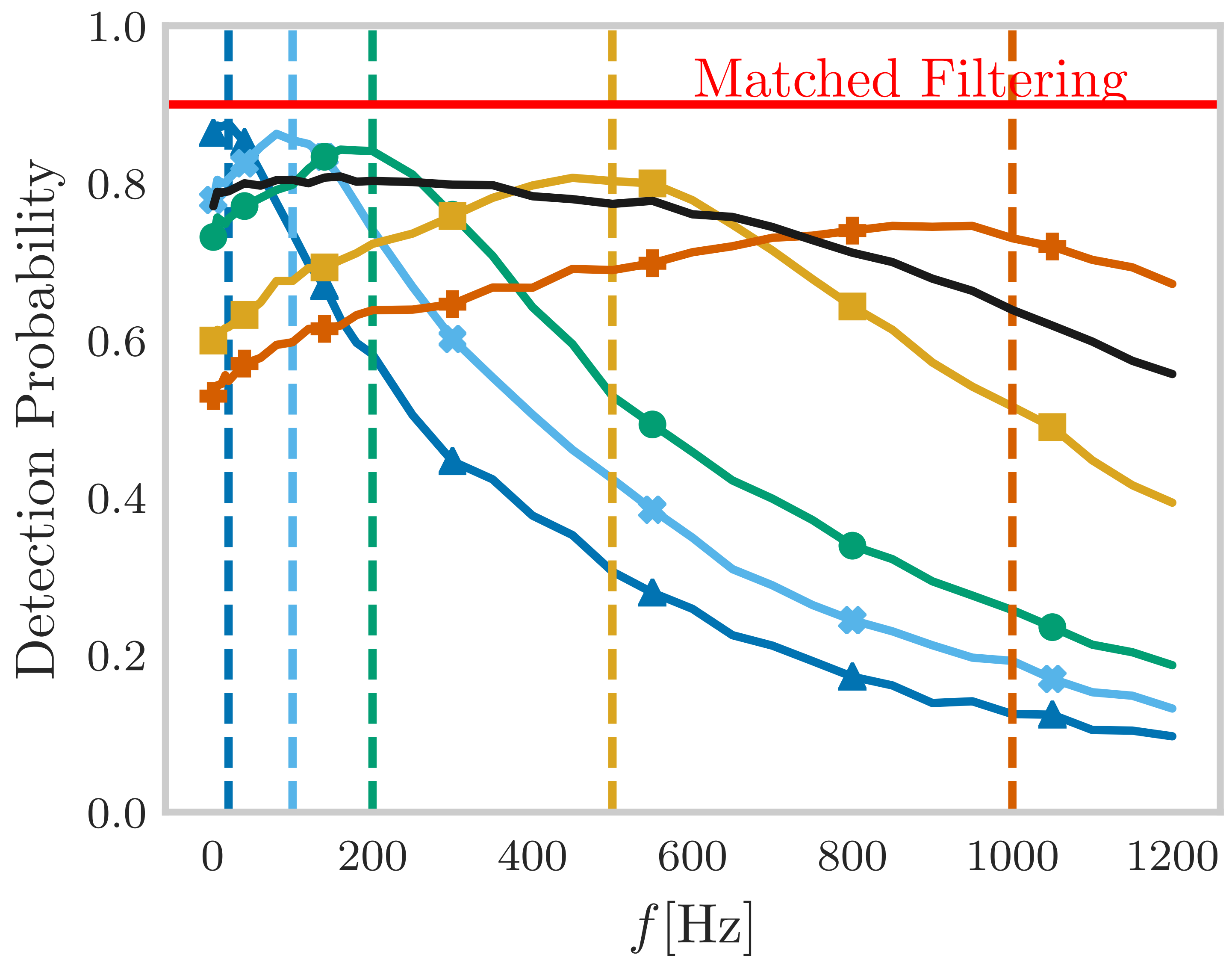
Solid : $\log_{10} \hat{h}_0^{\mathrm{line}} = 0.0$

Dashed : $\log_{10} \hat{h}_0^{\mathrm{line}} = 1.0$

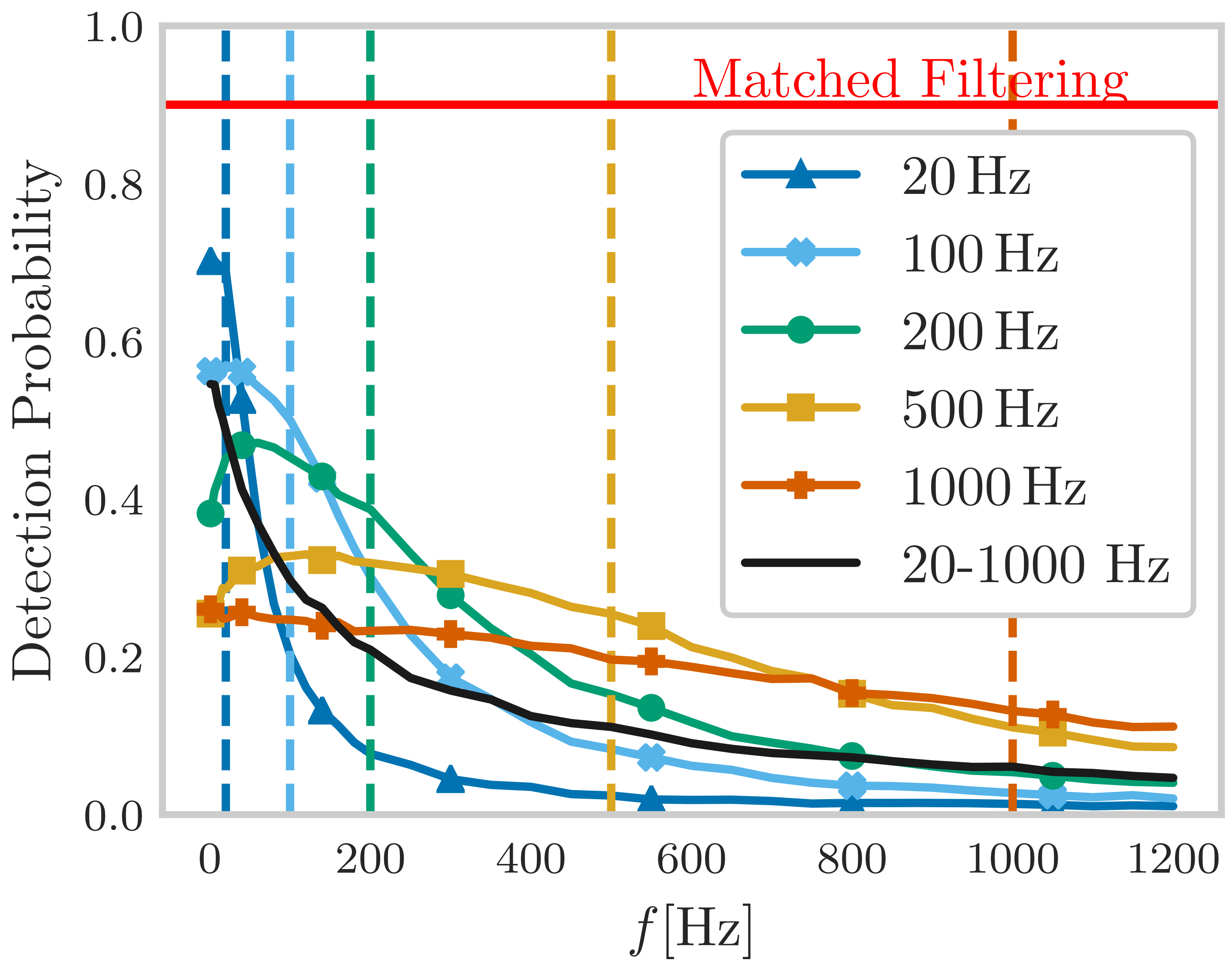
← FAP = 10%

Deep learning application

ref
Dreissigacker *et al.*, PRD100, 044009 (2019)
Dreissigacker & Prix, PRD102, 022005 (2020)



T=10⁵sec



T=10⁶sec

TABLE II. WEAVE parameters and characteristics for the two searches.

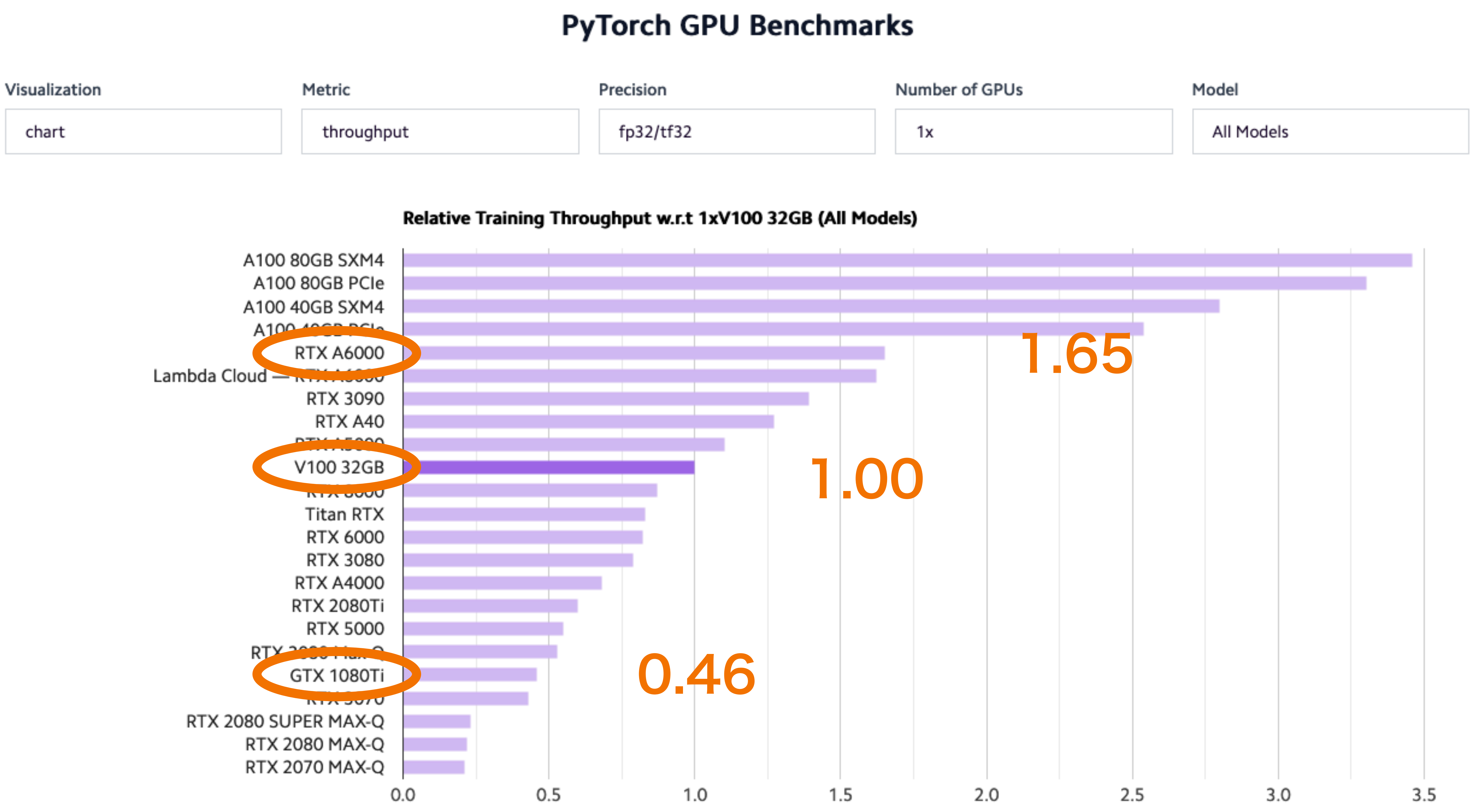
Name	$T = 10^5$ s	$T = 10^6$ s
Mismatch parameter m	0.1	0.2
Average SNR loss $\langle\mu\rangle$	5%	11%
Number of templates $\mathcal{N}$	4×10^{11}	3×10^{14}
Search time [single CPU core]	6.7×10^6 s	3.9×10^9 s

TABLE VII. DNN computing cost (in seconds) for training, search and follow-up (using matched filtering). The respective matched-filtering cost can be found in Table II.

Cost [s]	Training	Search	Follow-up	Total
$T = 10^5$ s	4.3×10^5	58.8	2.2×10^4	4.5×10^5
$T = 10^6$ s	4.3×10^6	196	6.5×10^7	6.9×10^7

Input : the Fourier transform of strain data.

GPU Benchmark



Speed up factor

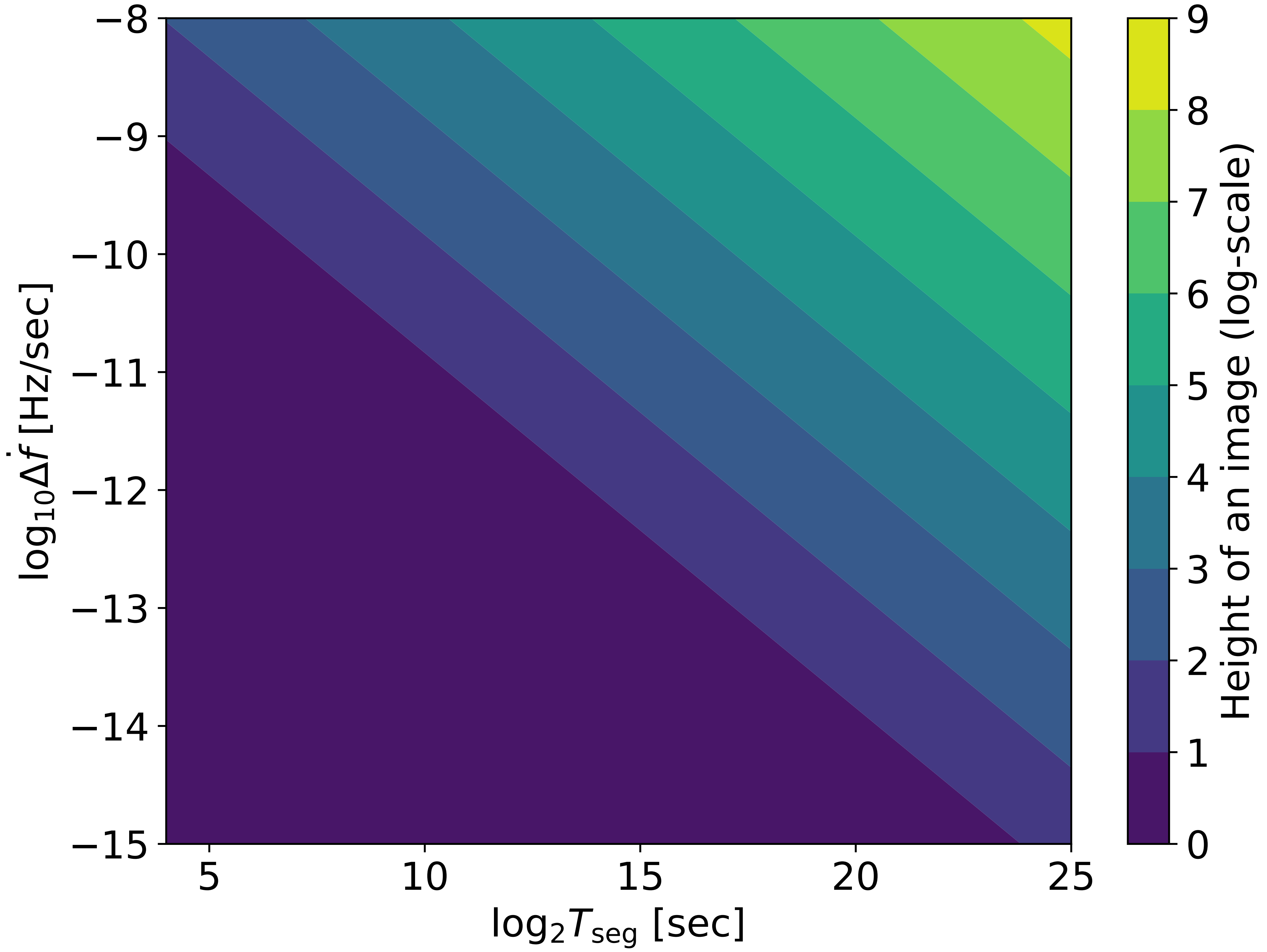
https://lambdalabs.com/gpu-benchmarks

Height of an image

Image size = (Height, Width)

Width = Nseg = Tobs / Tseg

Height = $20 \cdot \dfrac{[\Delta \dot{f}] T_{\mathrm{obs}}}{T_{\mathrm{seg}}^{-1}}$

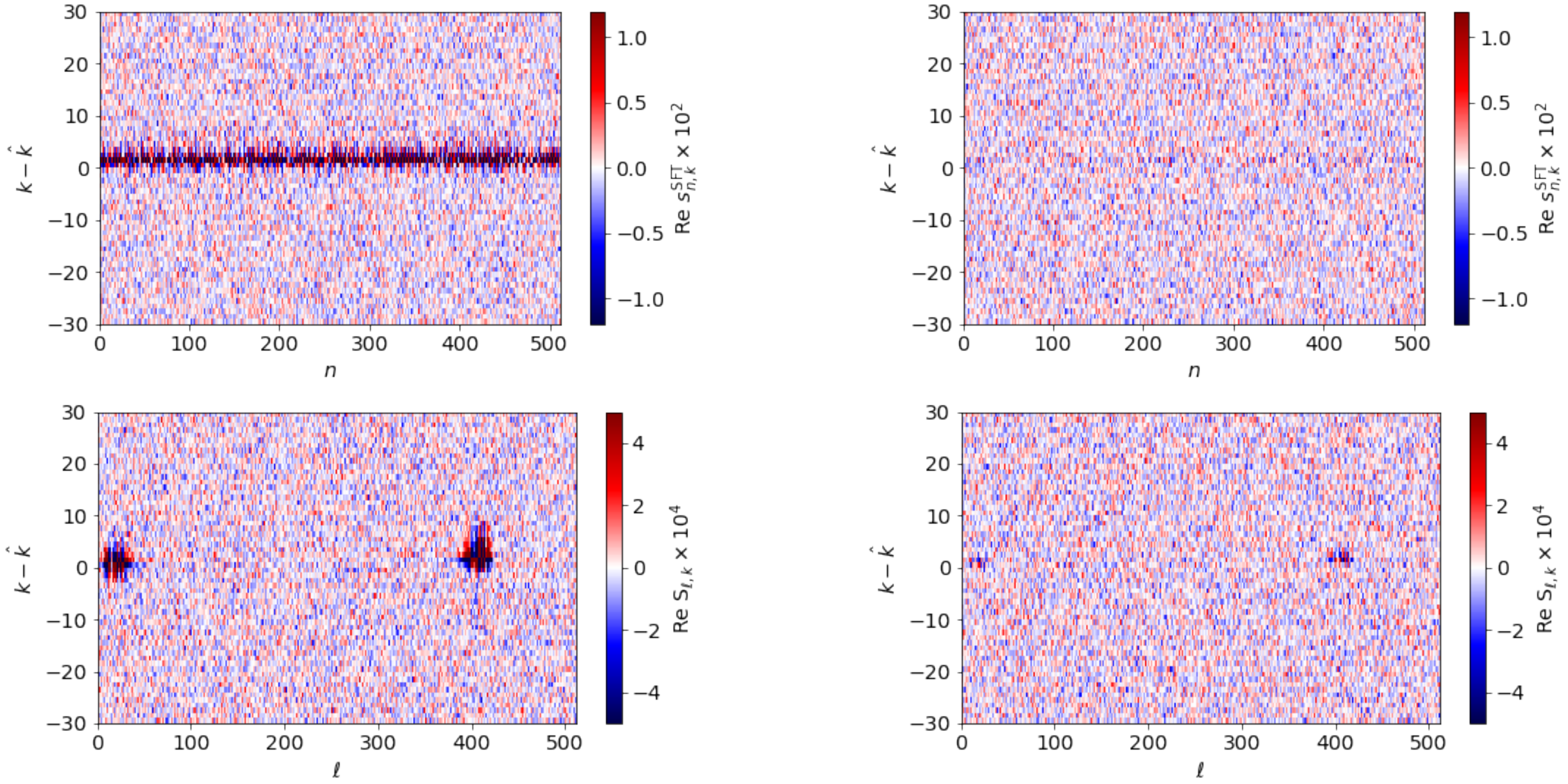


Example of an image

Fgw = 489.08Hz, residual df/dt = 0.0

SFT image

SFT + double FT image



$\log_{10}\hat{h}_0 = -1.0$

$\log_{10}\hat{h}_0 = -2.0$