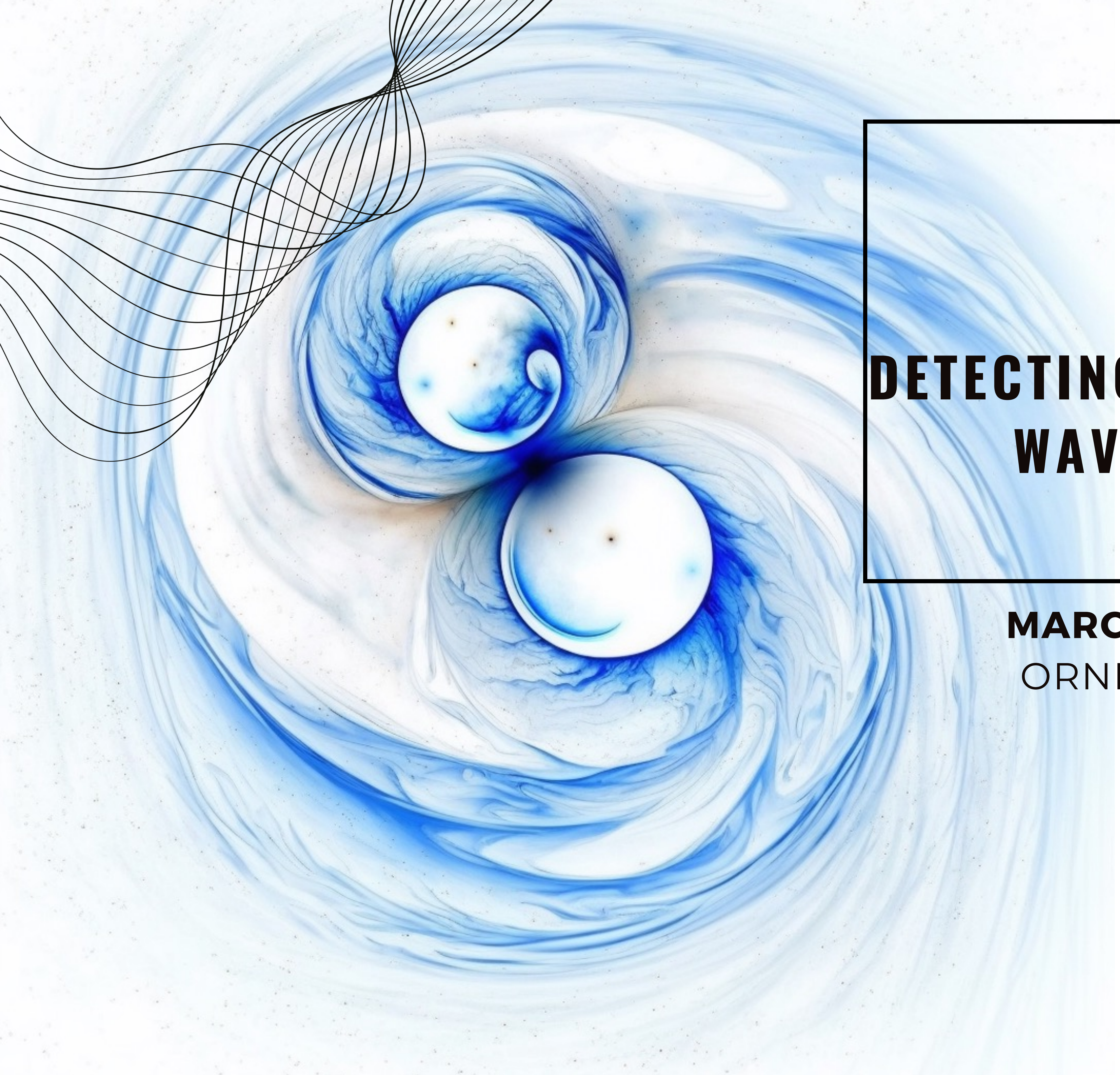


BSD-COBI

DETECTING CONTINUOUS GRAVITATIONAL WAVES FROM LIGHT BINARIES

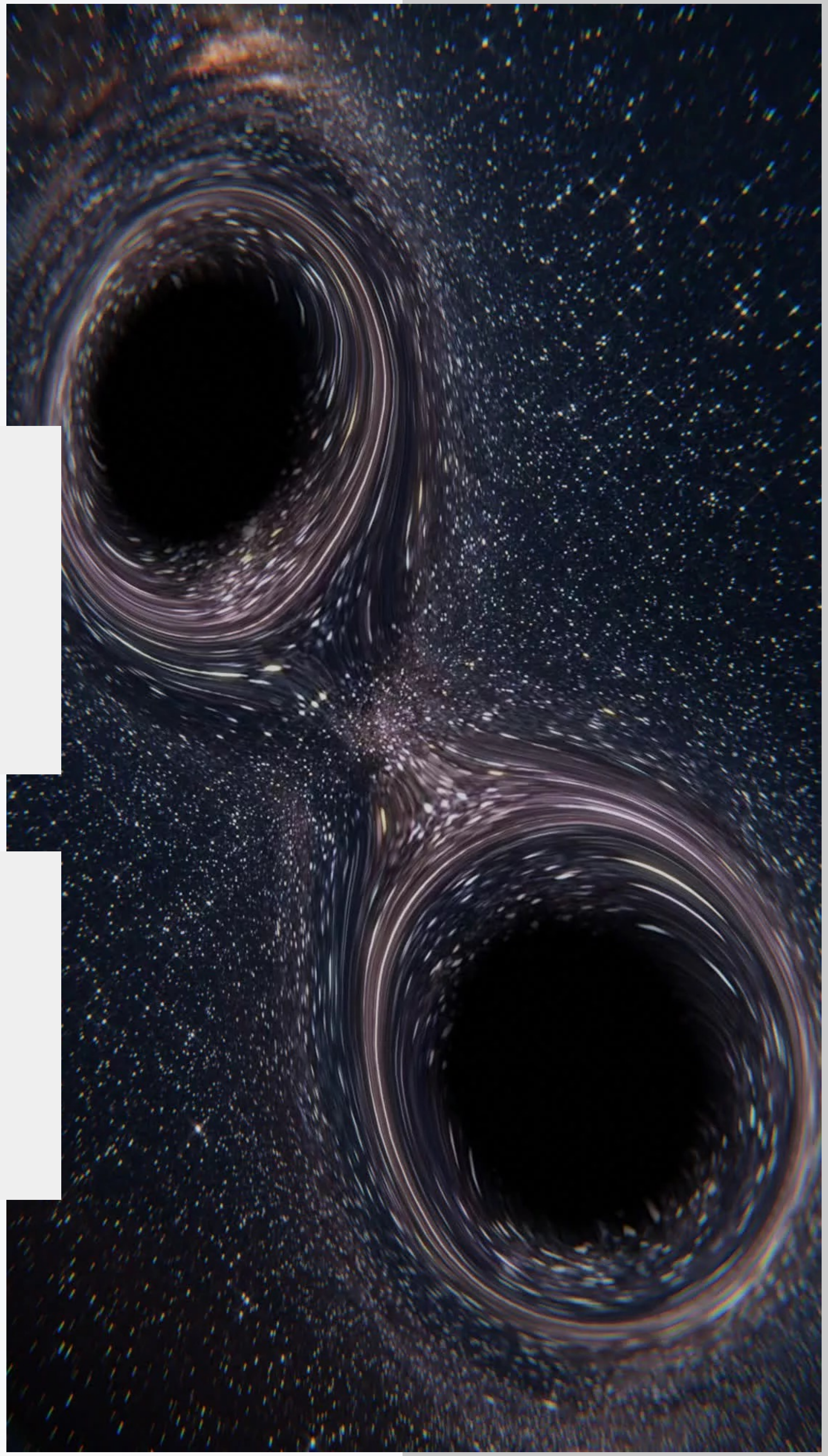
MARC ANDRÉS-CARCASONA (IFAE)
ORNELLA PICCINNI (OzGrav/ANU)

CONTENT

INTRODUCTION

Objects with very light masses (10^{-6}-10^{-3} Msun) can emit continuous gravitational waves during the inspiral phase in the frequency range of LIGO-Virgo-KAGRA.

Such objects are interesting as they can potentially be dark matter. One example, would be primordial black holes.



GOALS AND OBJECTIVES

Objective n° 1

Correctly model the signal in order to apply the heterodyne correction method

Objective n° 2

Reduce the computing cost of the search by efficiently constructing a grid that exploits the particularities of this signal

Objective n° 3

Test the employed method with real O3 data from LVK

SIGNAL MODELING

The first step to search for these signals is to correctly model them as the heterodyne method requires it

SIGNAL

Assuming the small mass approximation and taking only the 0PN order, the spin-up of the signal can be modelled as:

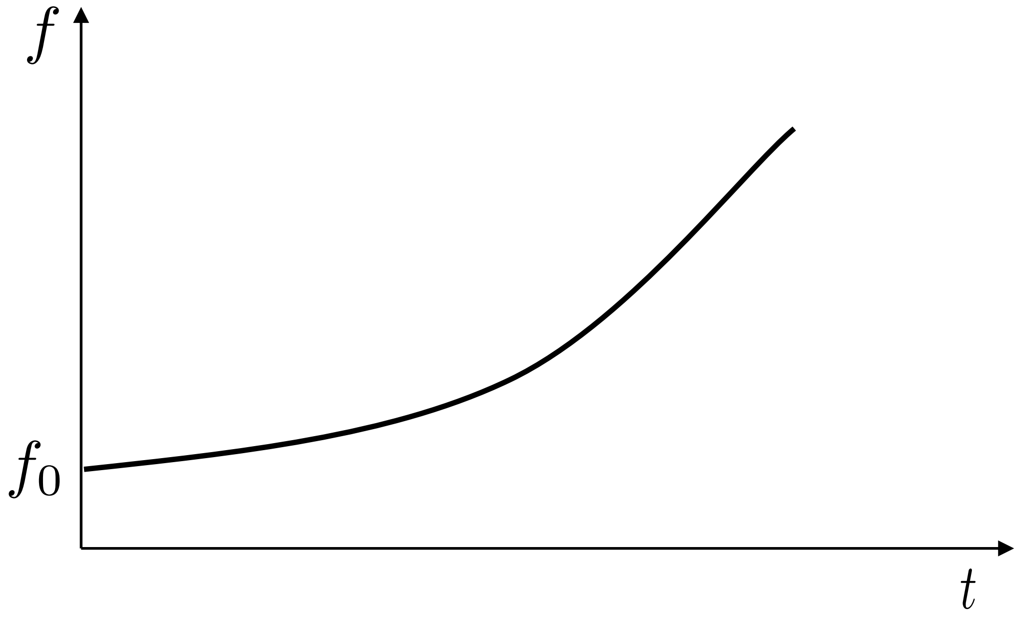
$$\frac{df}{dt} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f^{11/3} = kf^n$$

Integrating it leads to an expression of the form of a power law

$$f(t) = f_0\left(1 + \frac{t}{\tau}\right)^{\alpha}$$

Additionally, the signal is subject to the Doppler modulation. This modifies the frequency as

$$f_{det}(t) = f(t)\left(1 + \frac{\vec{v}\cdot\hat{n}}{c}\right)$$



HETERODYNING

The core of the method

HETERODYNING

The data measured in one detector can be assumed to be $h(t) = s(t) + n(t)$, where $n(t)$ represents the noise and is characterized by a power spectral density (PSD) and the signal can be modeled by $s(t) = A(t)e^{i\Phi(t)}$.

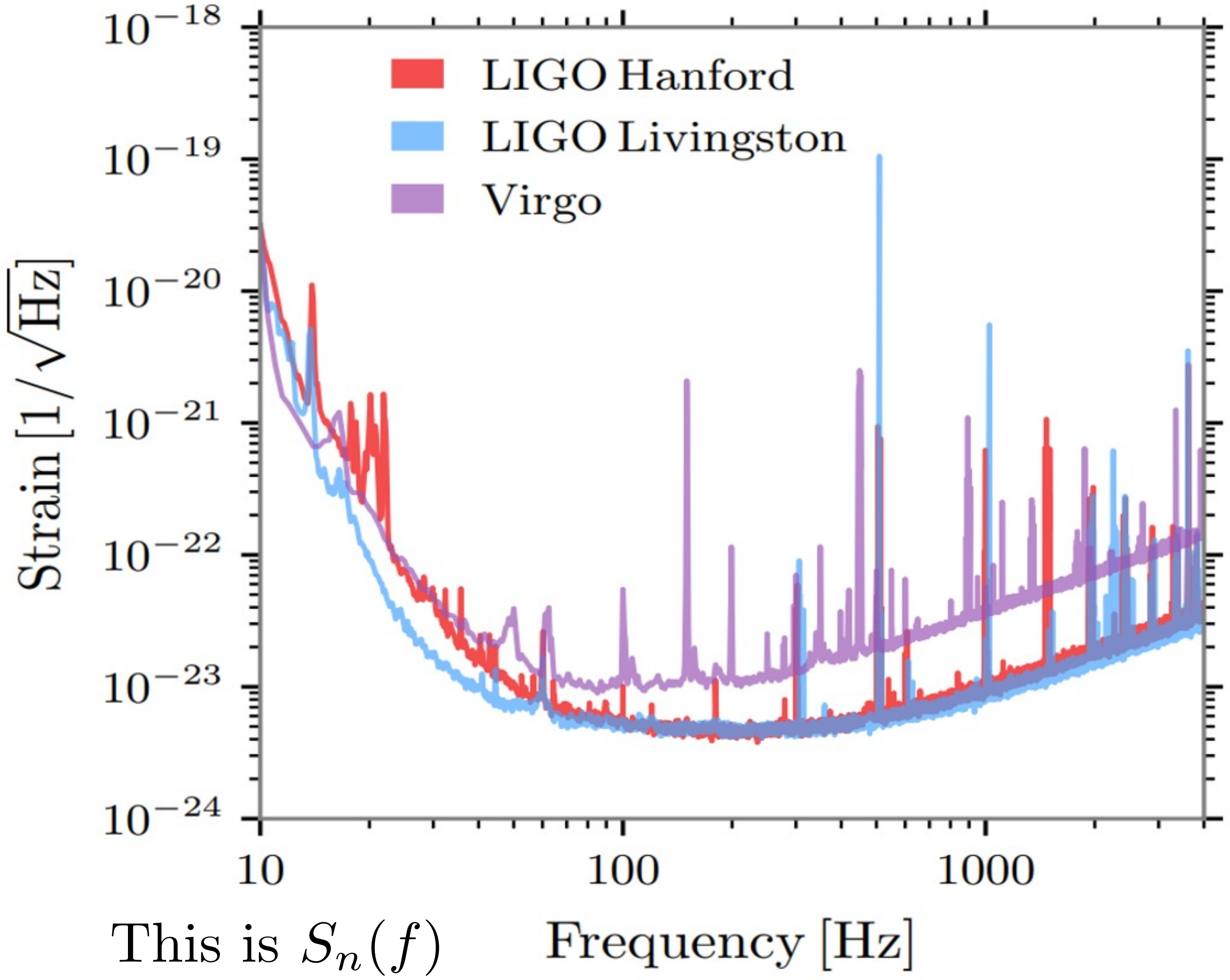
The heterodyne correction is based on applying the following operation on the data:

$$h_{corr}(t) = [s(t) + n(t)]e^{-i\Phi_{corr}(t)}$$

O.J. Piccinni et al. (2018)

With all this information we can compute the correction phase as $\Phi_{corr}(t) = \Phi_{dopp}(t) + \Phi_{sig}(t)$ where each individual phase can be computed from the relation

$$f(t) = \frac{1}{2\pi}\frac{d\Phi(t)}{dt}$$



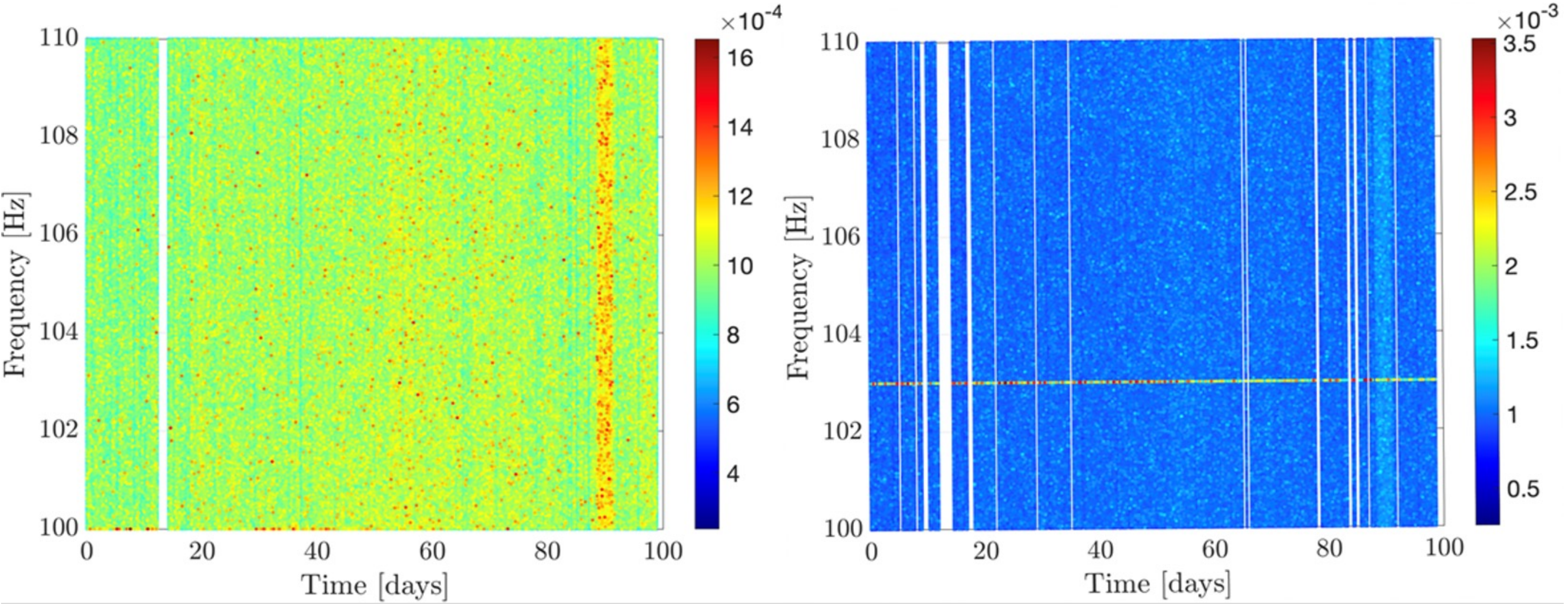
This is $S_n(f)$

LVK (2021)

PEAKMAP

The peakmap is a collection of peaks in time and frequency, created by computing the periodogram of a set of fast Fourier transforms (FFTs) of length Tcoh of the data, normalized by an average spectrum, and selecting only the peaks above a given threshold.

P. Astone et al. (2014)



Peakmap of an injected signal with a chirp mass of 10^{-4} Msun at 1 kpc and with a reference frequency of 103 Hz with no correction (left) and after the heterodyne correction (right).

This method is based in seeing a power excess corresponding to the CW signal. By heterodyning we make the signal visible in the peakmap, increasing the SNR. We then project the peaks in the frequency axis and use the Critical Ratio as our statistic

$$CR = \frac{n - \mu}{\sigma}$$

NOTE: The frequency bin width is 1/Tcoh.

SENSITIVITY

The minimum detectable strain at a given confidence level (denoted by the gamma) can be computed as a function of the noise, the length of FFT, the observing time and some factor dependent on the antenna.

$$h_{min}(f) = \frac{\mathcal{B}}{(T_{obs}/T_{coh})^{1/4}} \sqrt{\frac{S_n(f)}{T_{coh}}} \sqrt{\rho_{CR} - \sqrt{2}\mathrm{erfc}^{-1}(2\Gamma)}$$

P. Astone et al. (2014)

At the same time, an estimation of the strain of the signal can be obtained by evaluating the strain at the initial time. This is,

$$h = \frac{4}{d} \left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3} \left(\frac{\pi f_0}{c}\right)^{3/2}$$

which leads to a maximum reachable distance of

$$d_{max} = \frac{4}{\mathcal{B}} \left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3} \left(\frac{\pi f_0}{c}\right)^{3/2} \left(\frac{T_{obs}}{T_{coh}}\right)^{1/4} \sqrt{\frac{T_{coh}}{S_n(f)}} \left[\rho_{CR} - \sqrt{2}\mathrm{erfc}^{-1}(2\Gamma)\right]^{-1/2}$$

GRID CONSTRUCTION

The core of the new contributions!

PROBLEM SOLVED?

01

PARAMETERS

Select the parameters that fully define the signal

02

SIGNAL

With the parameters set, the signal is completely defined, and the frequency evolution can be computed

03

HETERODYNE

With the signal model, the heterodyne correction can be applied to the data

04

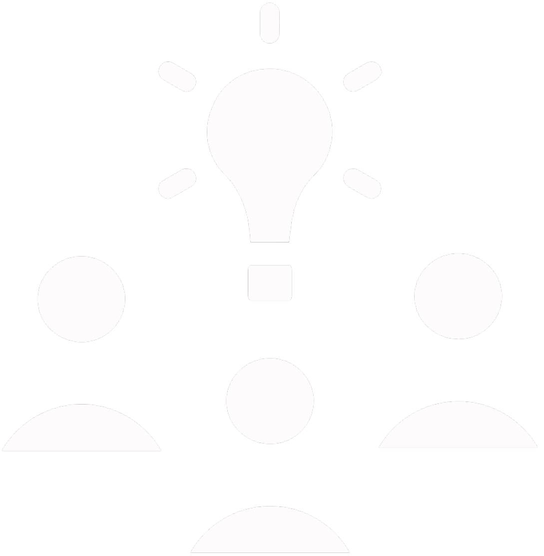
PEAKMAP

With the heterodyned data, the peakmap can be computed and if a signal is present, identified

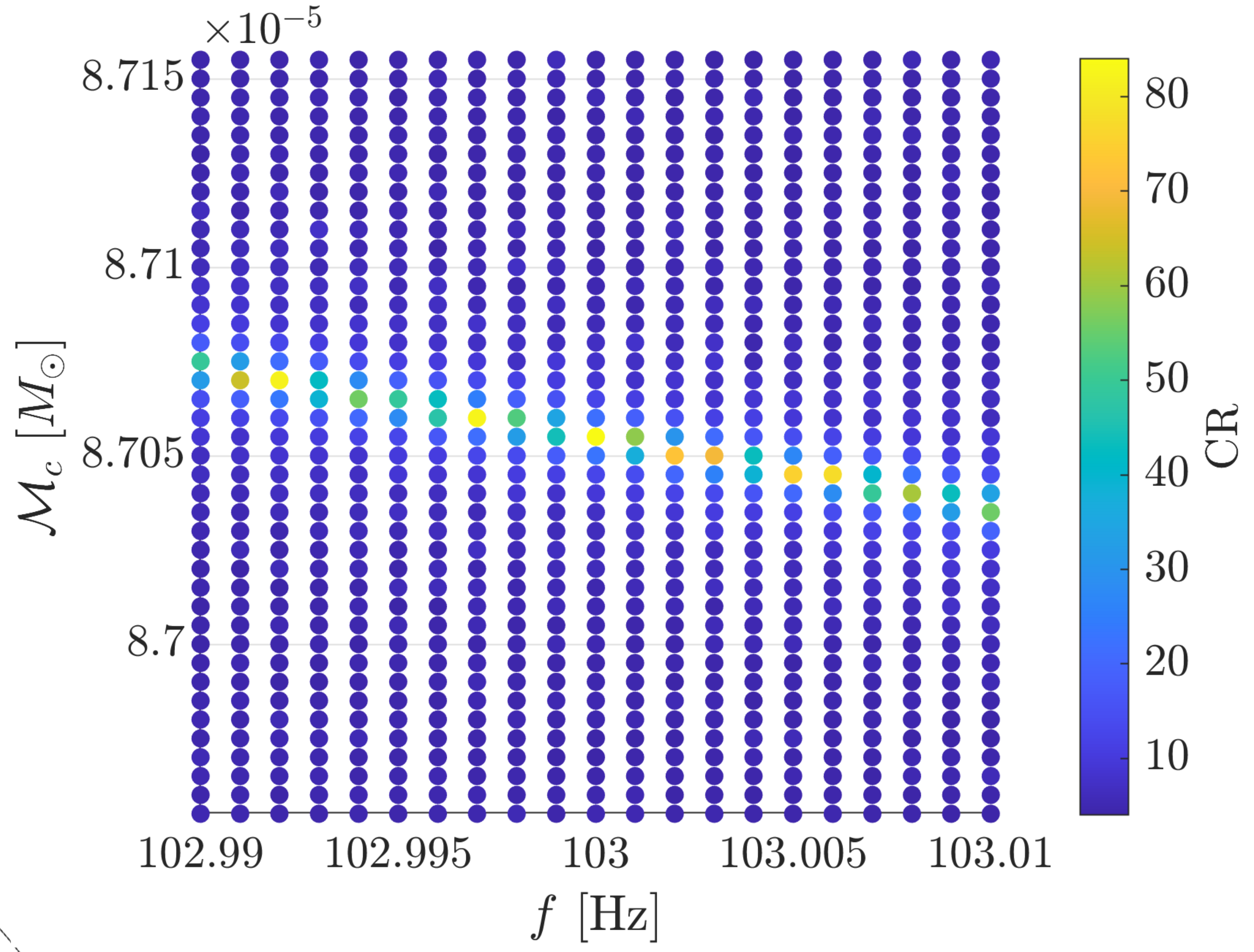
CAN WE BE SMART?

Is there a way in which we can reduce the parameter space to be searched?

In fact, is there a special feature of the signal that can be exploited?



CAN WE BE SMART?



PARAMETER DEGENERACY

Inspecting the form of the signal's frequency evolution we note a potential degeneracy of the parameters. In other words, various combinations of both parameters can lead to similar signals.

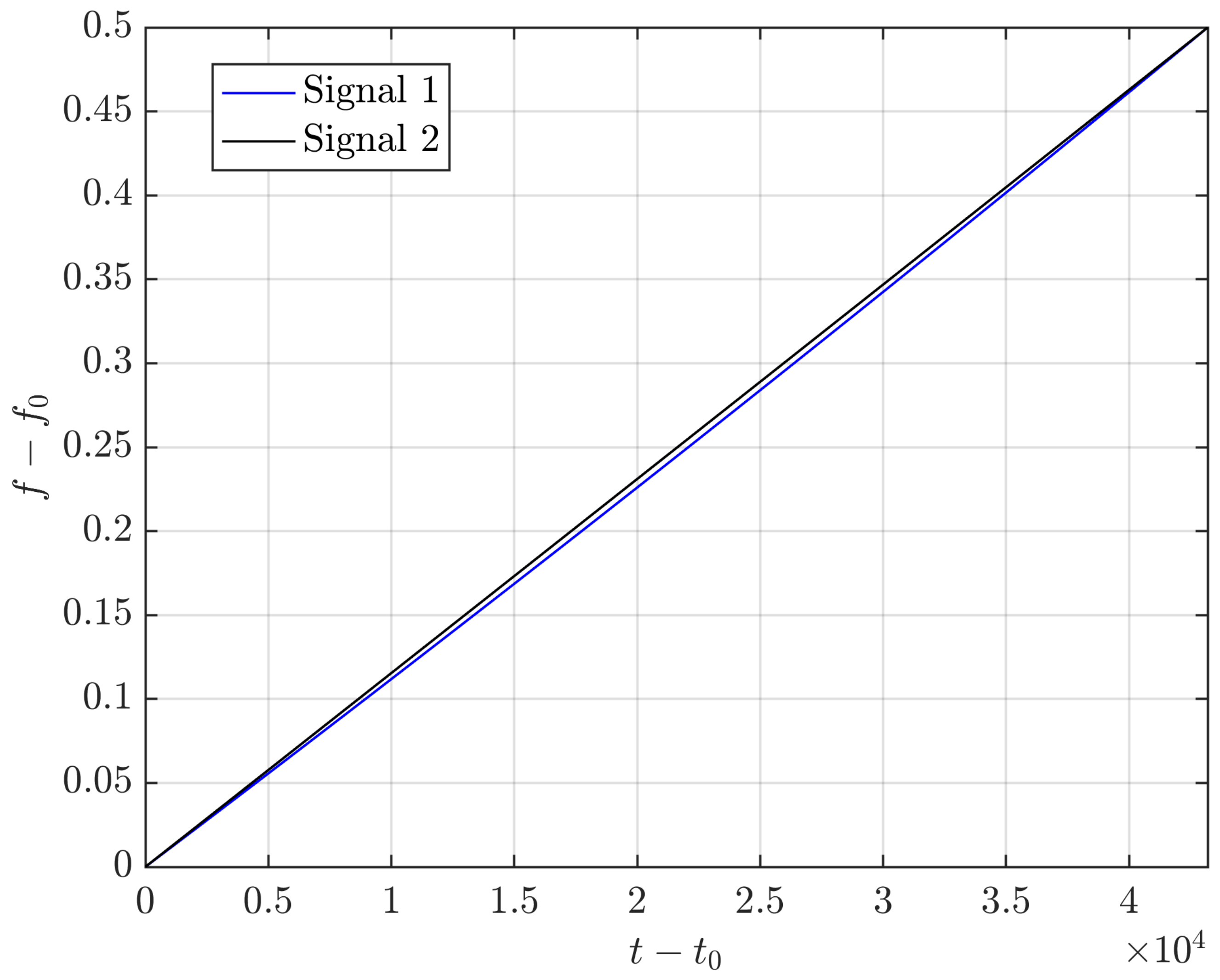
Let's create a new variable that quantifies the overall frequency shift of a signal in a period of time Tobs as

$$\xi(f_0, \mathcal{M}_c, T_{\text{obs}}) = \left[f_0^{1/\alpha} + (1-n)kT_{\text{obs}}\right]^{\alpha} - f_0$$

For a fixed observing time, this expression maps the two-dimensional space to a single dimensional one. The question is, how do signals with the same value of this new variable look like?

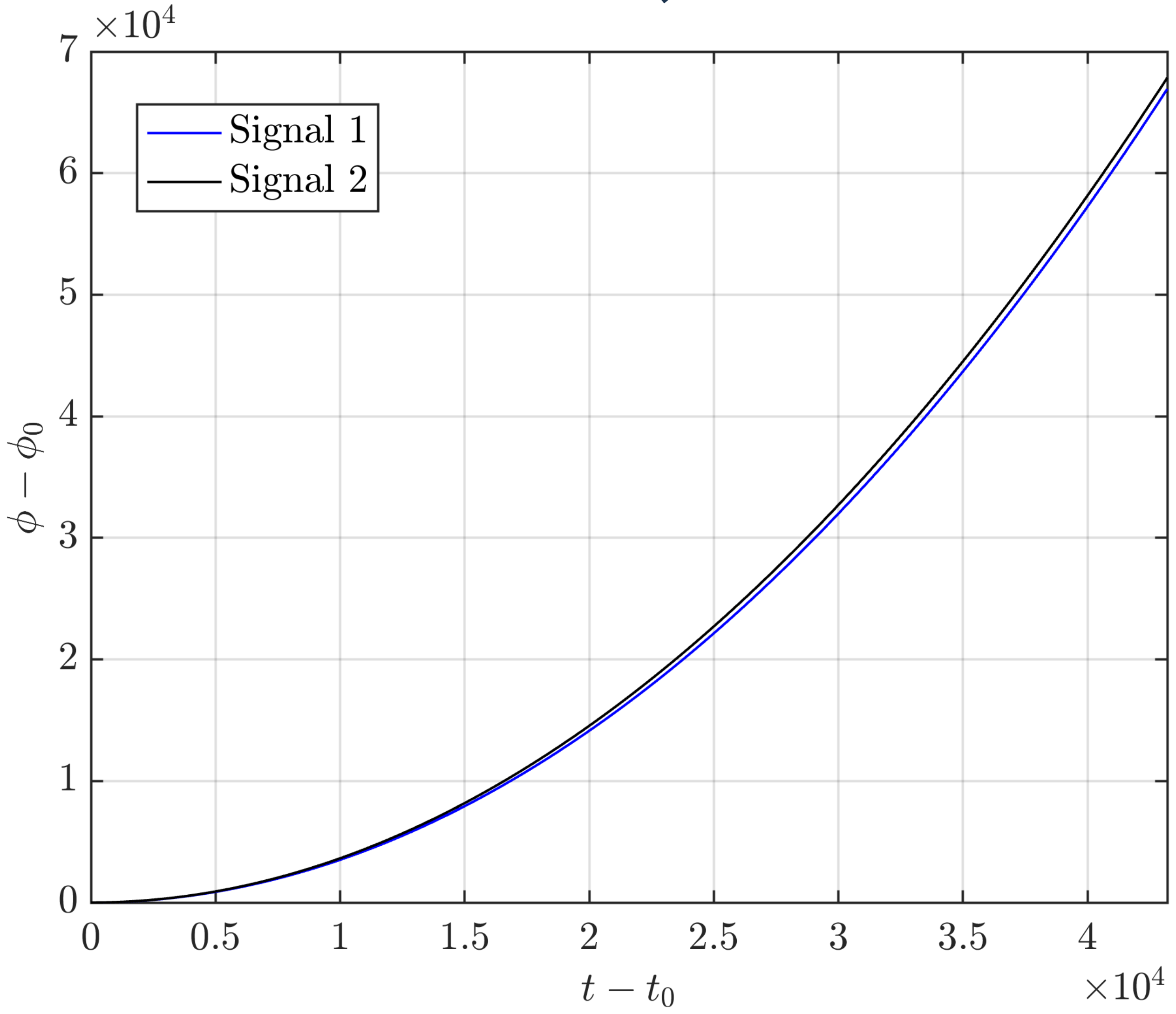
Example of

Signal 1: $f_0 = 20.365Hz$, $\mathcal{M}_c = 0.0077M_\odot$, $\xi = 0.5Hz$
Signal 2: $f_0 = 254.8745Hz$, $\mathcal{M}_c = 3.1 \times 10^{-5}M_\odot$, $\xi = 0.5Hz$

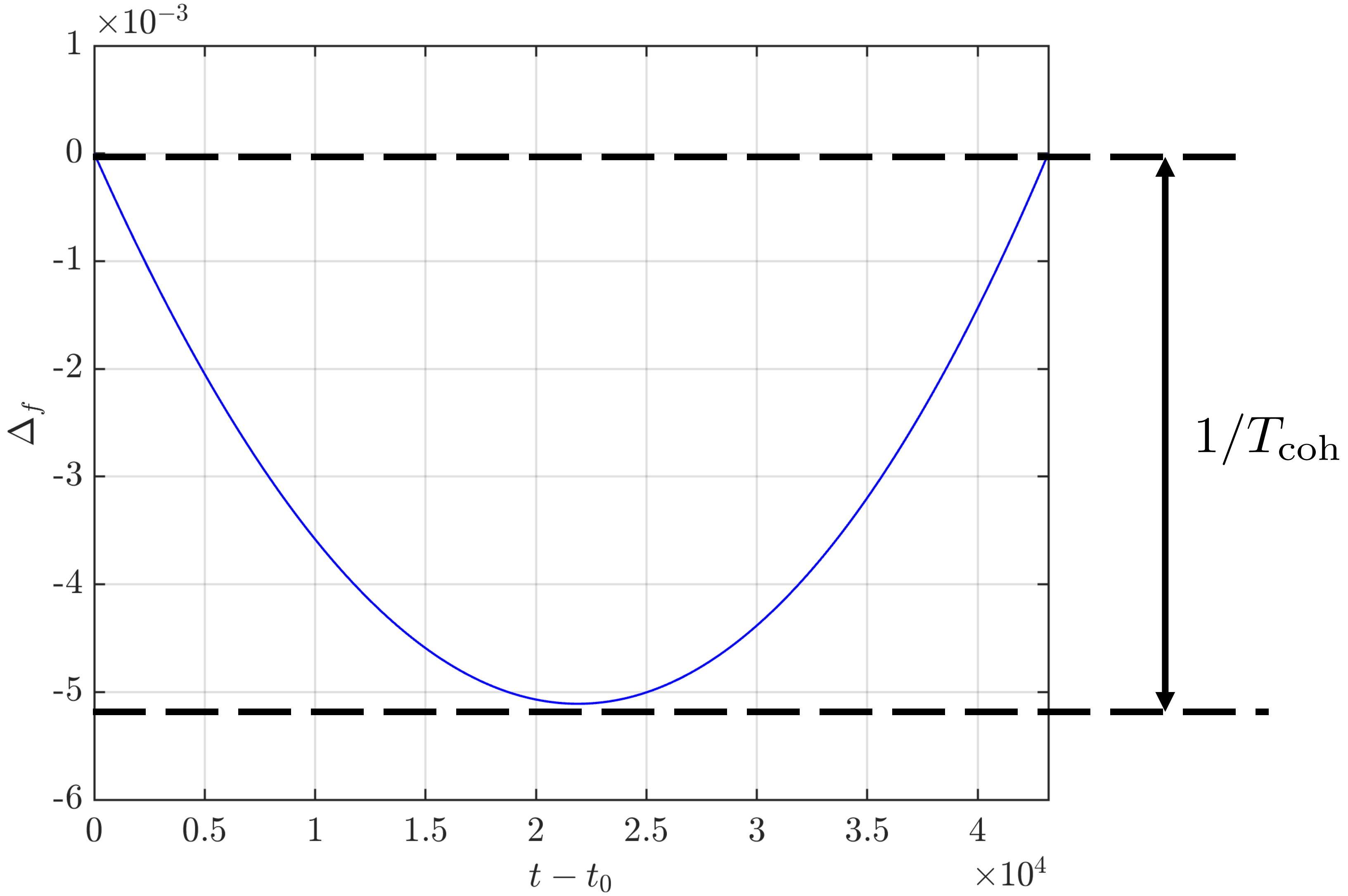


PARAMETER DEGENERACY

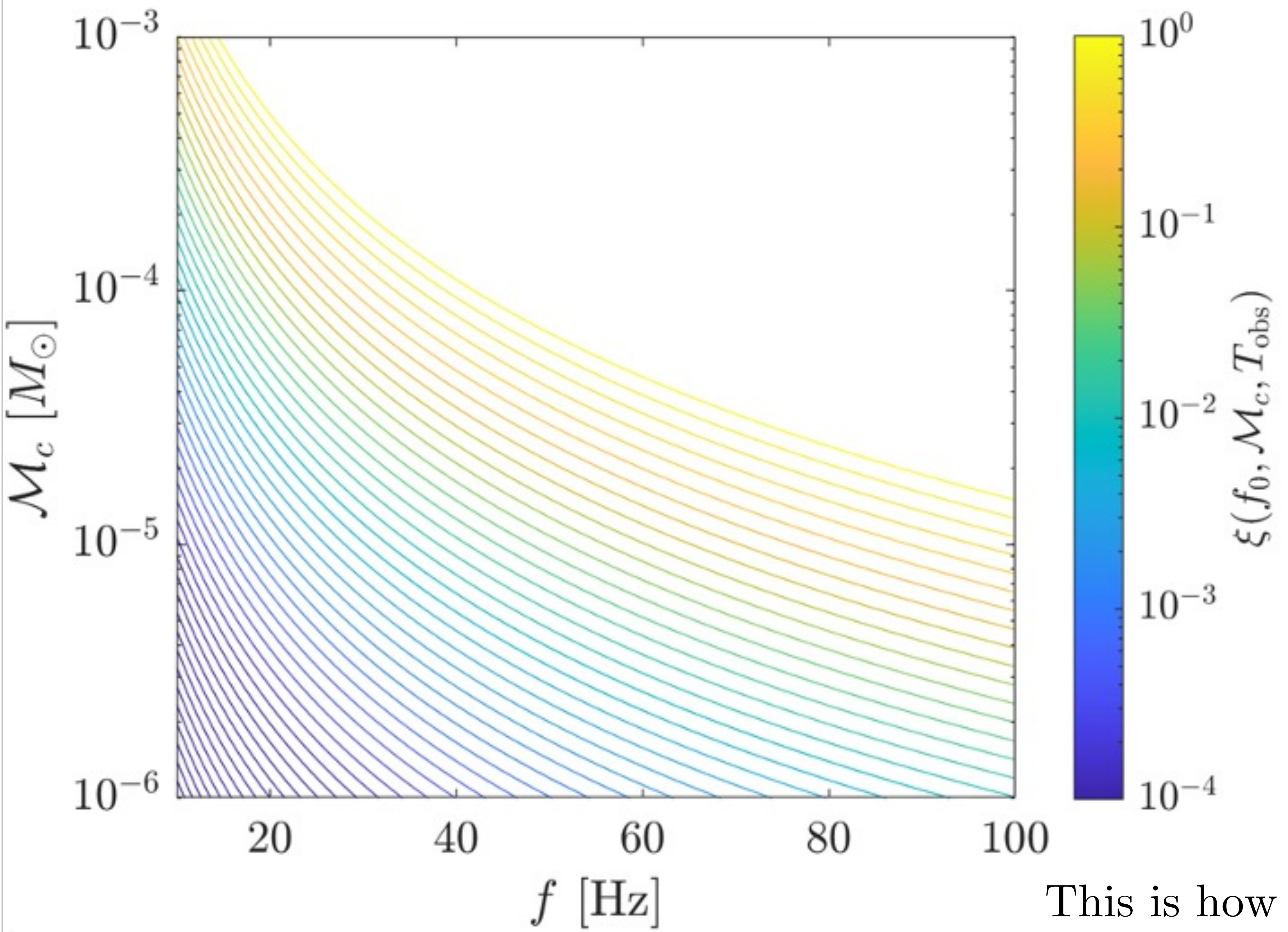
This similar behavior is observed in the phase evolution and hence it can be used for the heterodyne correction



There is still a residual in the frequency... BUUT we can use the Tcoh to absorb this modulation in a single frequency bin! And even better, the Doppler shift can also be contained inside a frequency bin if Tcoh is not too long



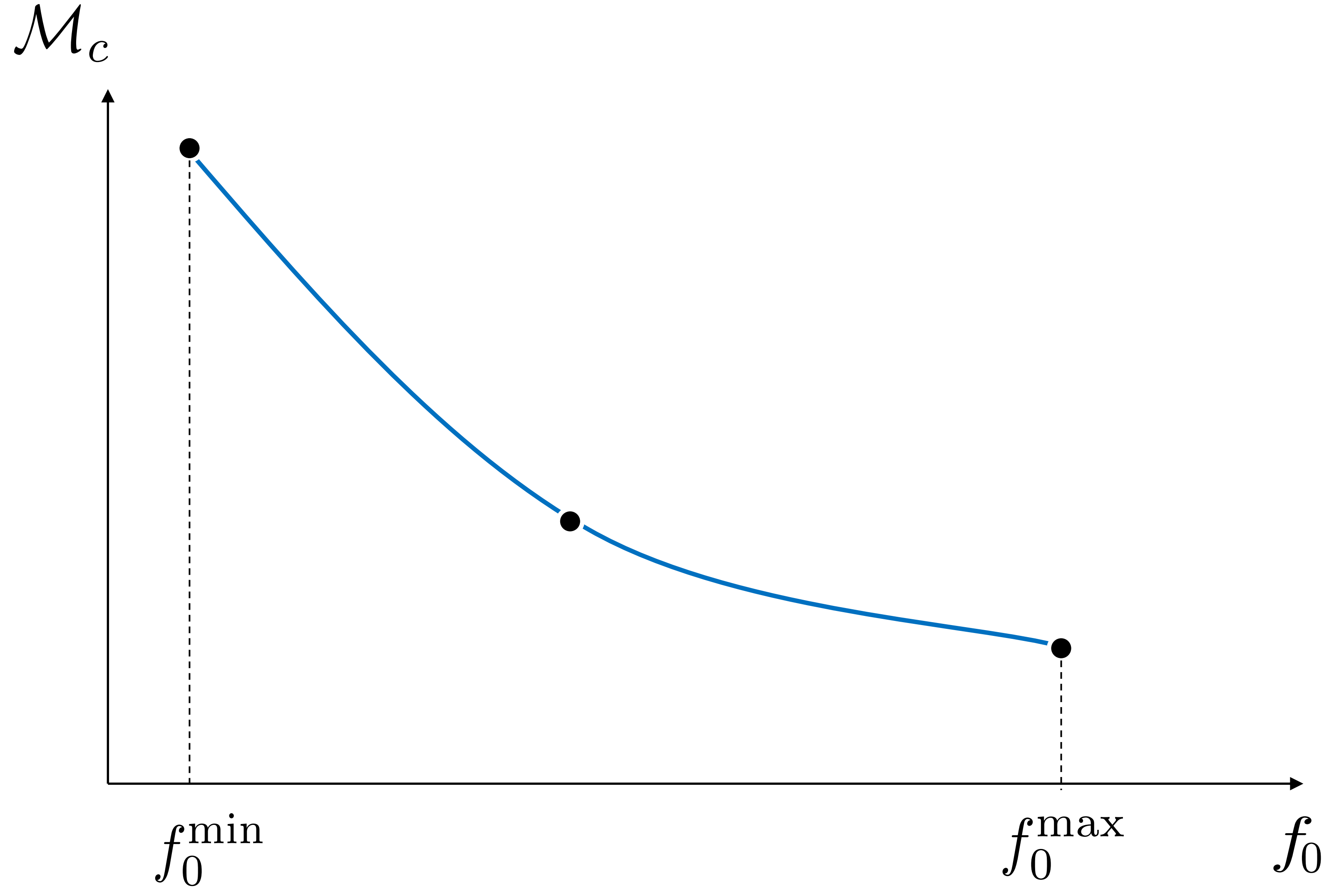
NEW PARAMETER SPACE



Theoretically, now we can construct a grid by defining points in the xi-parameter space which is considerably less expensive than covering the two-dimensional parameter space of frequency and chirp mass.

This is how the iso-xi lines look like for Tobs = 0.5 days

GRID 1: VARIABLE Tcoh



1. Define the limits $\xi_{\min}, \xi_{\max}$, $f_0^{\min}$, $f_0^{\max}$. Set $\xi_0 = \xi_{\min}$.
2. Solve the following optimization problem for a given ξ_i

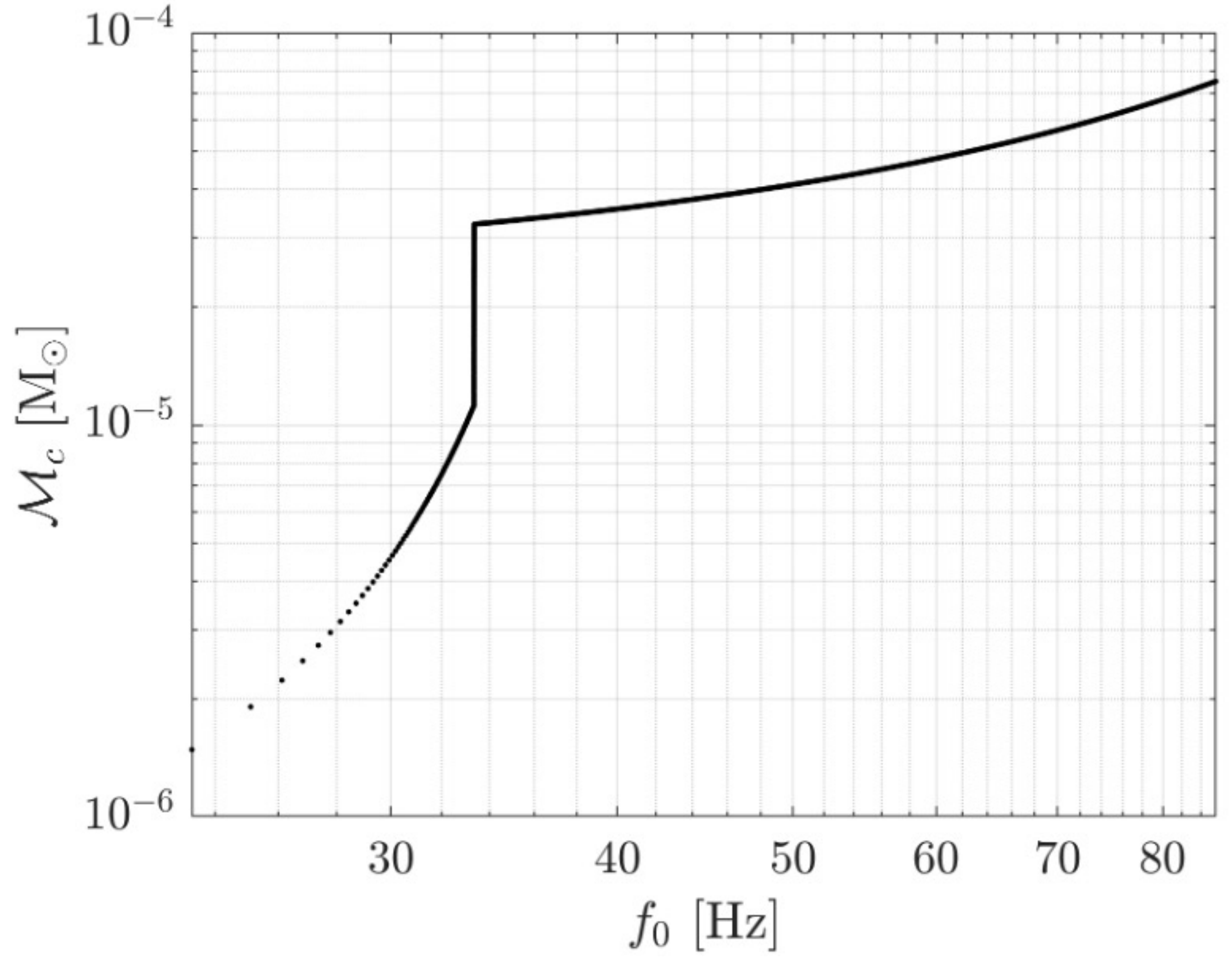
$$\min_{f_0} \quad \max(\mathcal{G}(\xi, f_0, f_0^{\min}), \mathcal{G}(\xi, f_0, f_0^{\max}))$$

$$\text{s.t.} f_0 \in [f_0^{\min}, f_0^{\max}]$$

3. Set $T_{\text{coh}} = 1/\mathcal{G}_{\max}$
4. Set $i = i + 1$ and $\xi_i = \xi_{i-1} + 1/T_{fft}$.
5. If $\xi_i < \xi_{\max}$ go to step 2.

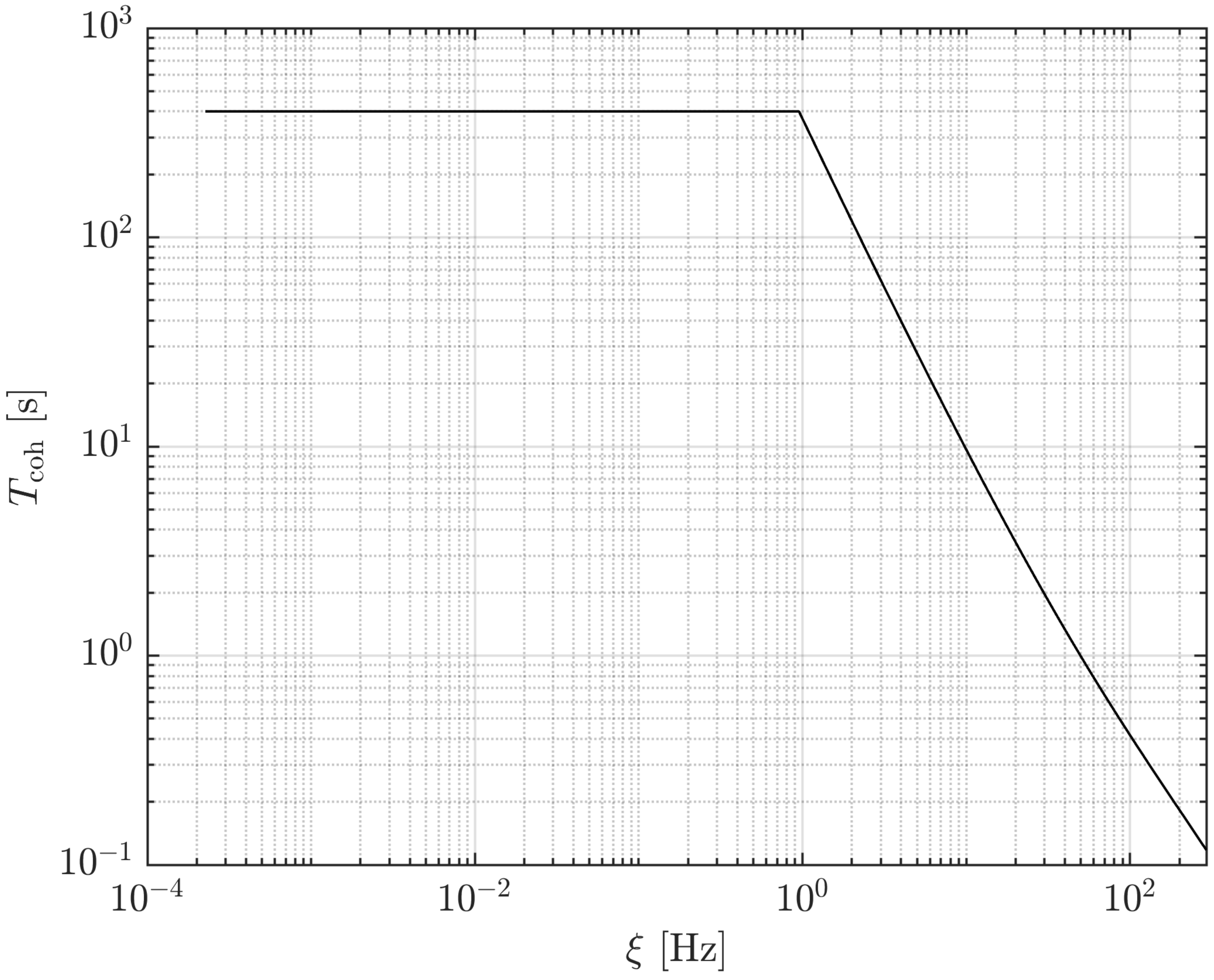
$$\mathcal{G}(\xi, f_0, f_0') = \max_t |f(t) - f'(t) - f_0 + f_0'|$$

GRID 1: VARIABLE Tcoh



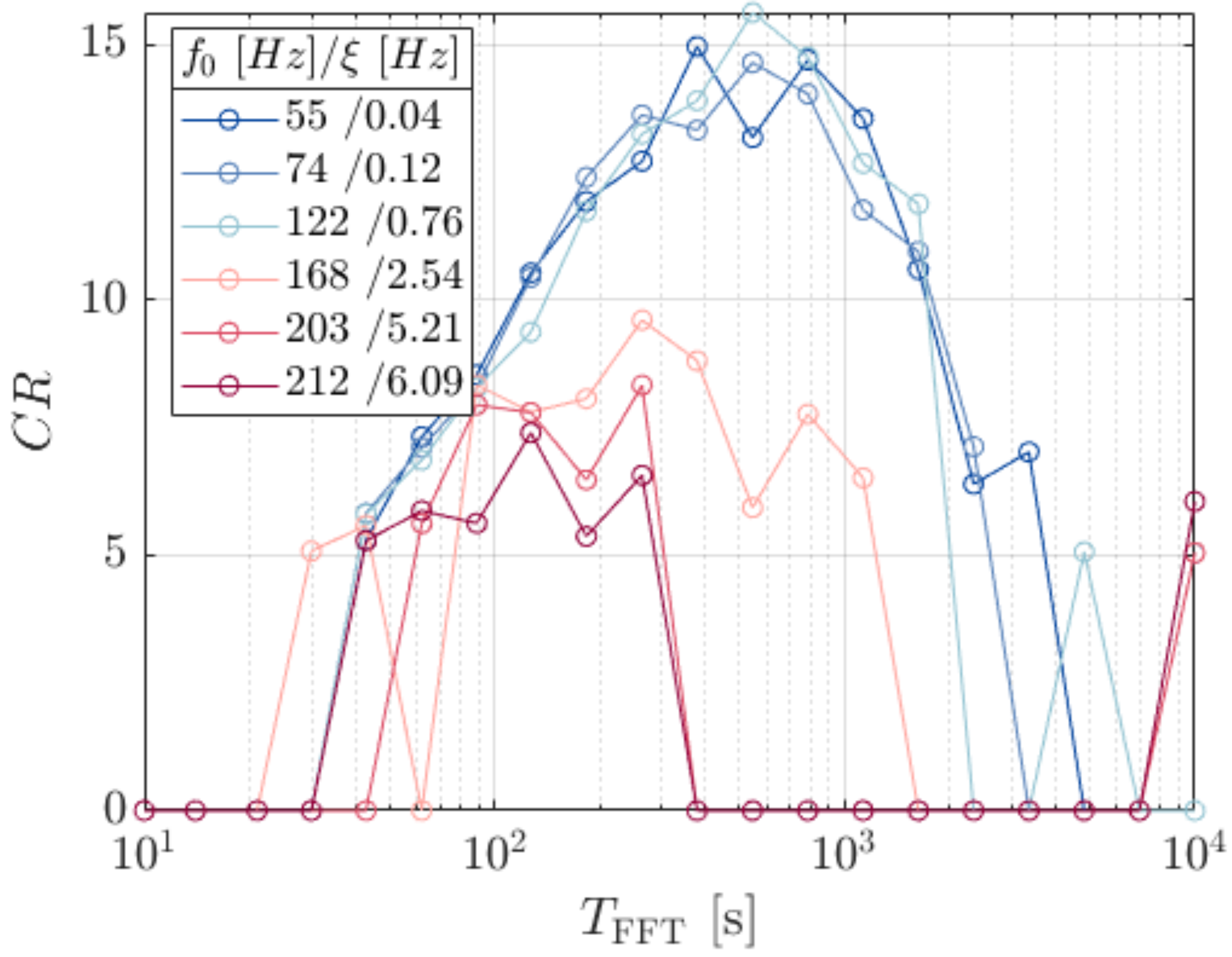
$\mathcal{O}(10^3)$ number of points

We must set a maximum coherence time.



Unfortunately, the Tcoh gets very small values, reducing the sensitivity of the search

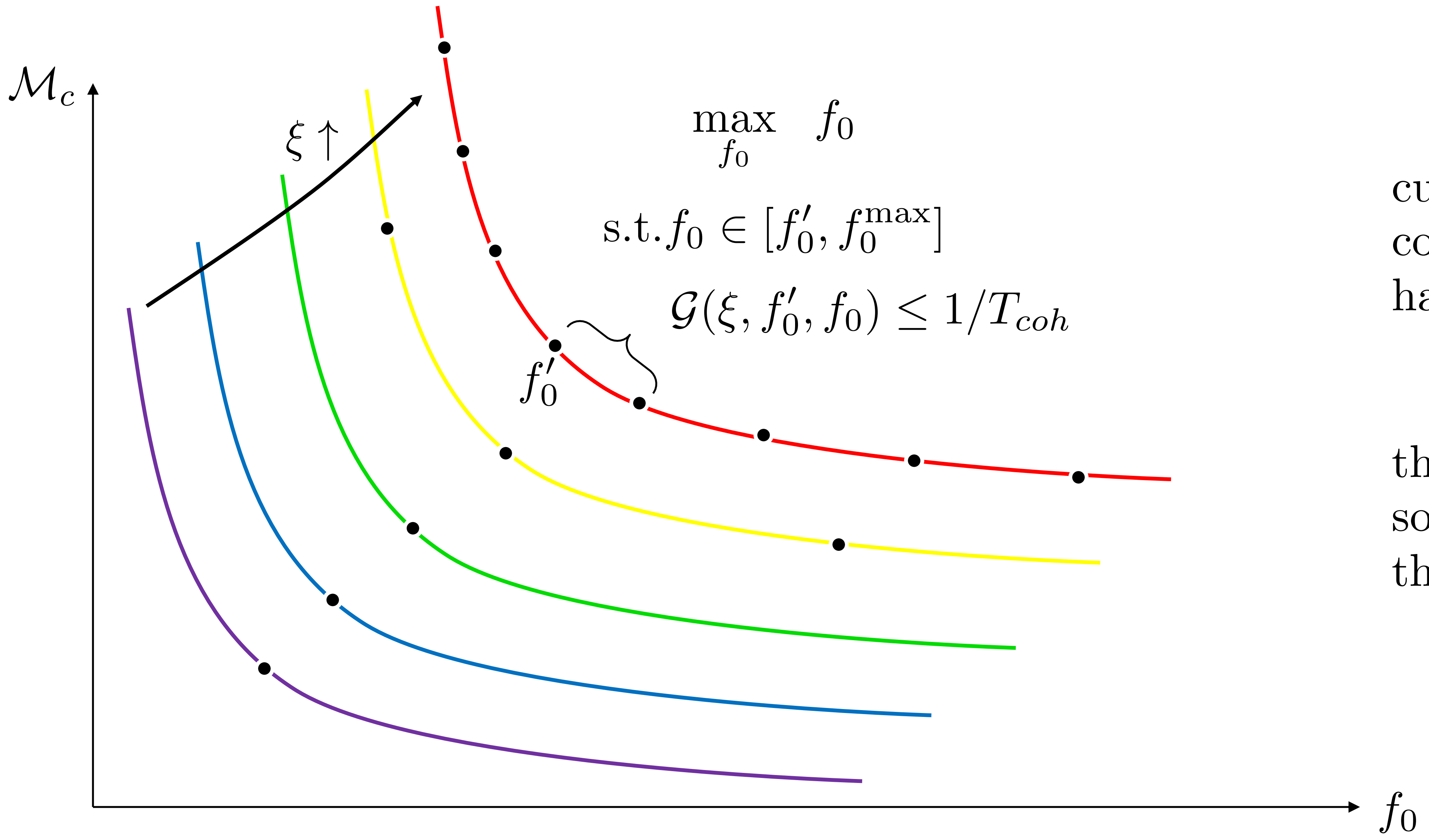
NOTE ON THE MAXIMUM Tcoh



There is a competing effect:

- Increasing the sensitivity requires to use longer coherence times.
- But for a fixed observing time, the number of points in the time axis is Tobs/Tcoh. Therefore, increasing the coherence time reduces the number of points, which messes with the statistics of the CR.
- But if Tcoh is too small, then the frequency bins get too wide to be reasonable

GRID 2: FIXED Tcoh

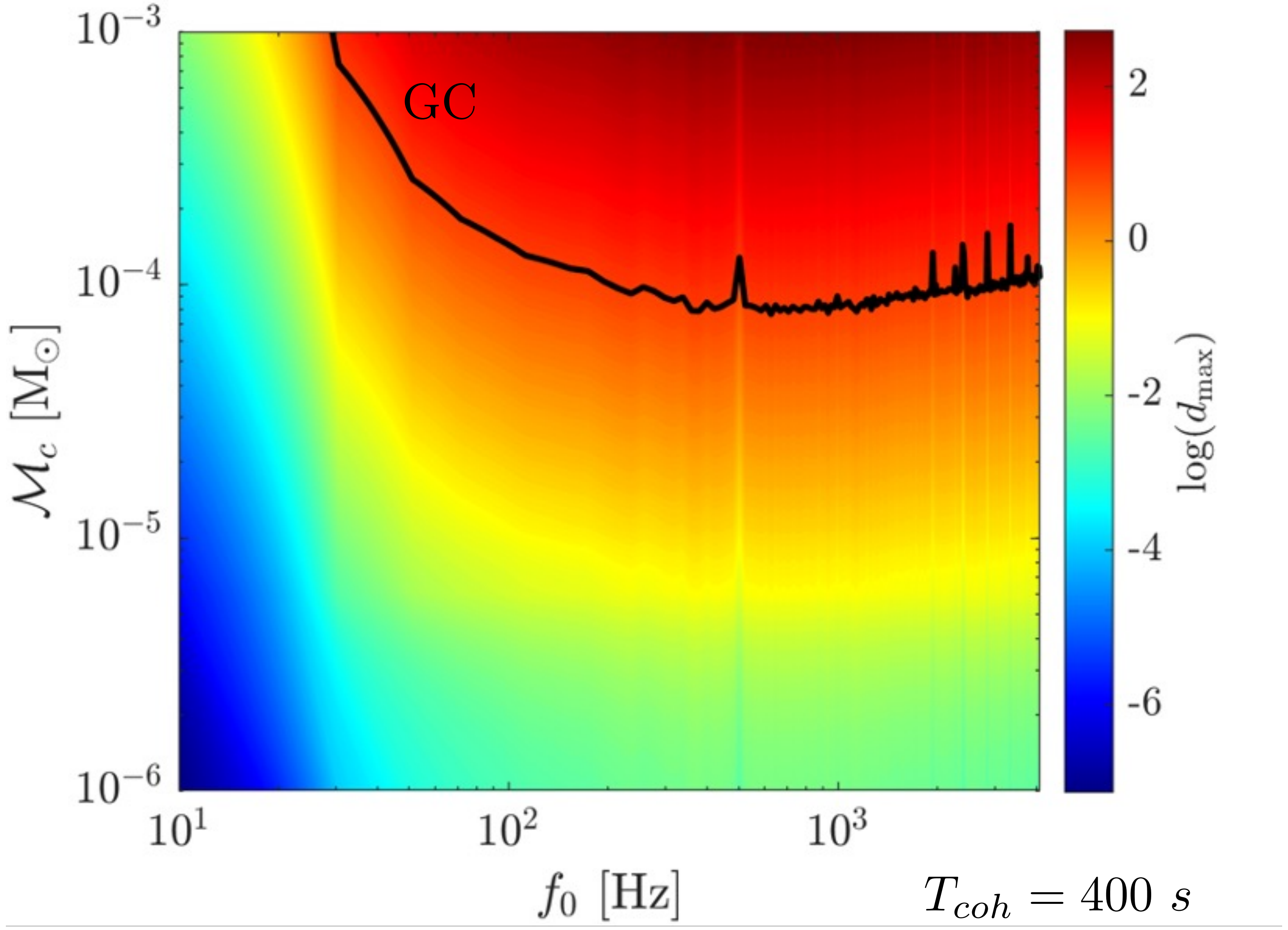


The idea is the same, but populating an iso-xi curve in order to make sure that the Tcoh can be kept constant. If the variation of the signals is higher (which happens at high xi) then more points are needed.

A way to accelerate this calculation is by noting that since the objective function is monotonic, the solution will be found at the feasible boundary. Therefore, the next frequency can be found at the point where

$$\mathcal{G}(\xi, f_0', f_0) = 1/T_{coh}$$

GRID 2: SENSITIVITY

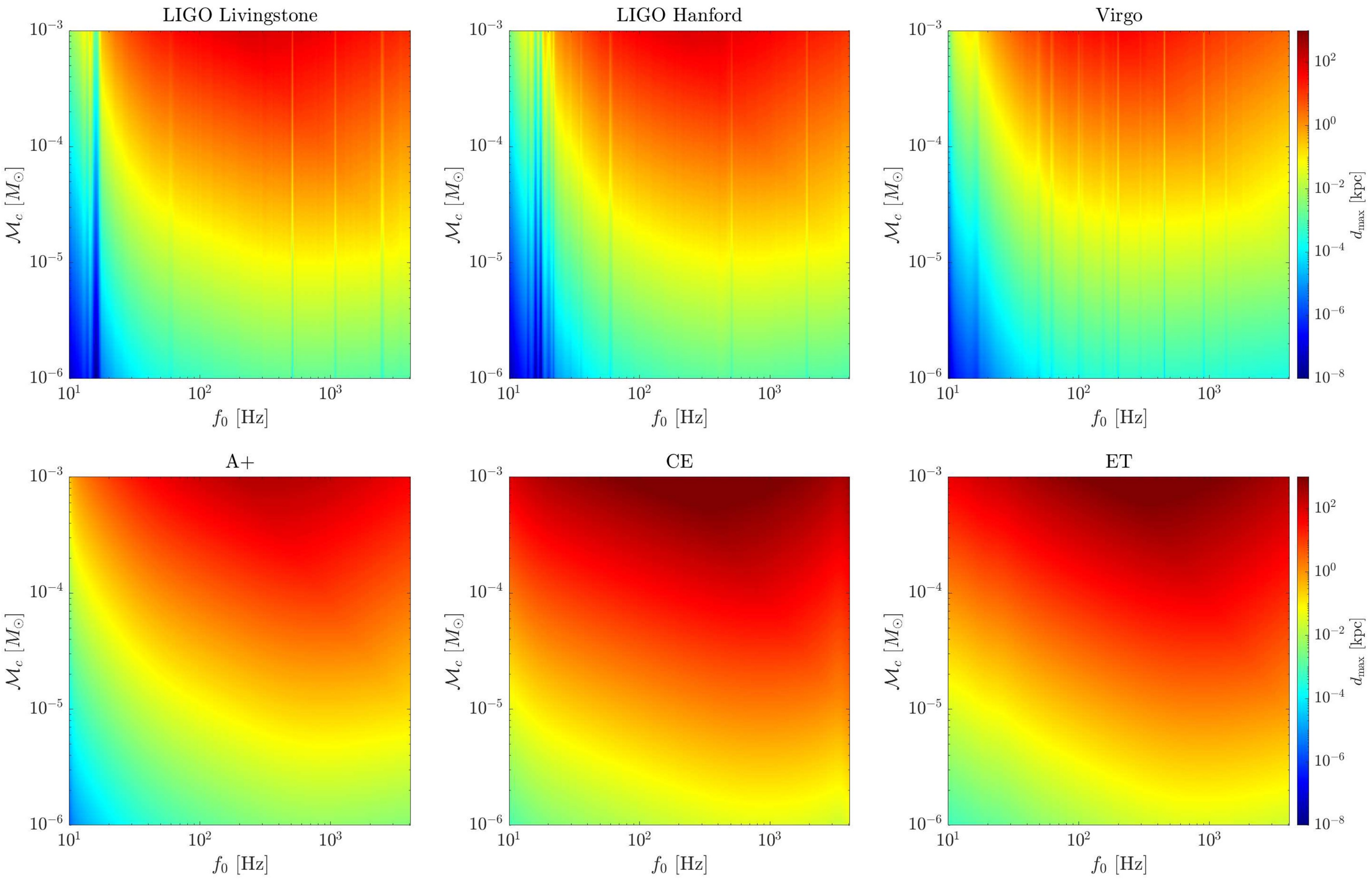


By fixing the coherence time we can reach the Galactic Center for a large portion of the parameter space, which justifies the use of this grid.

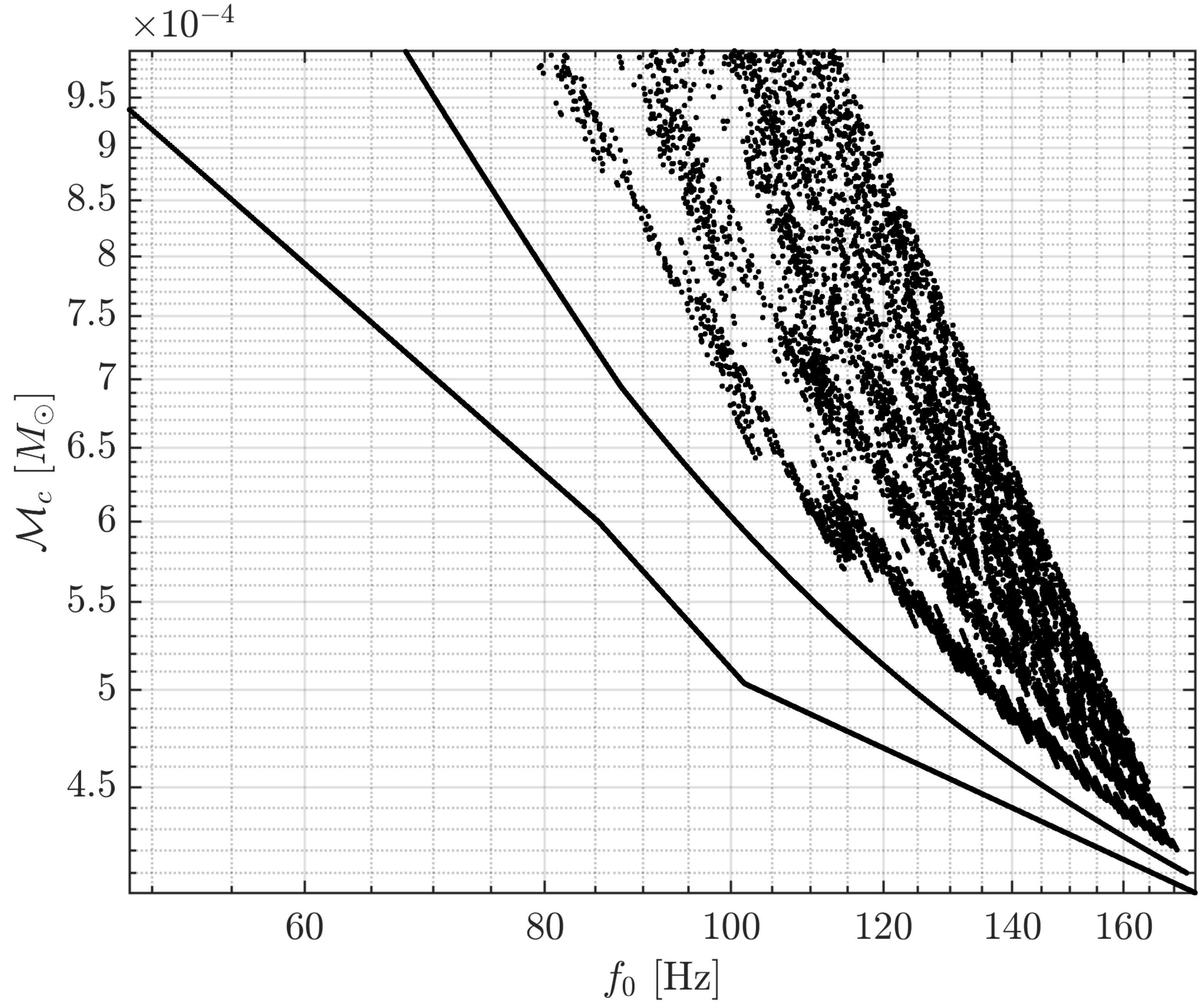
Reaching the GC or even the halo is interesting in the context of DM searches, so having this distance reach is a good point.

In any case, if we want to perform a GC search this grid is not optimal, as there is a part of the parameter space that cannot reach it, so placing points of the grid there "is a waste of computing time".

GRID 2: SENSITIVITY



GRID 3:
VARIABLE Tcoh, FIXED d



$\sim 18,000$ number of points

With this grid, the distance is maintained to the GC one and the Tcoh is allowed to change to not waste computing resources.

With "only" 18,000 points we can probe a considerable portion of the parameter space reaching the GC, which is interesting for DM searches.

NOTE: Since the maximum Tcoh is 400s, the Doppler shift is also contained in a single frequency bin, which implies that the search is technically omnidirectional

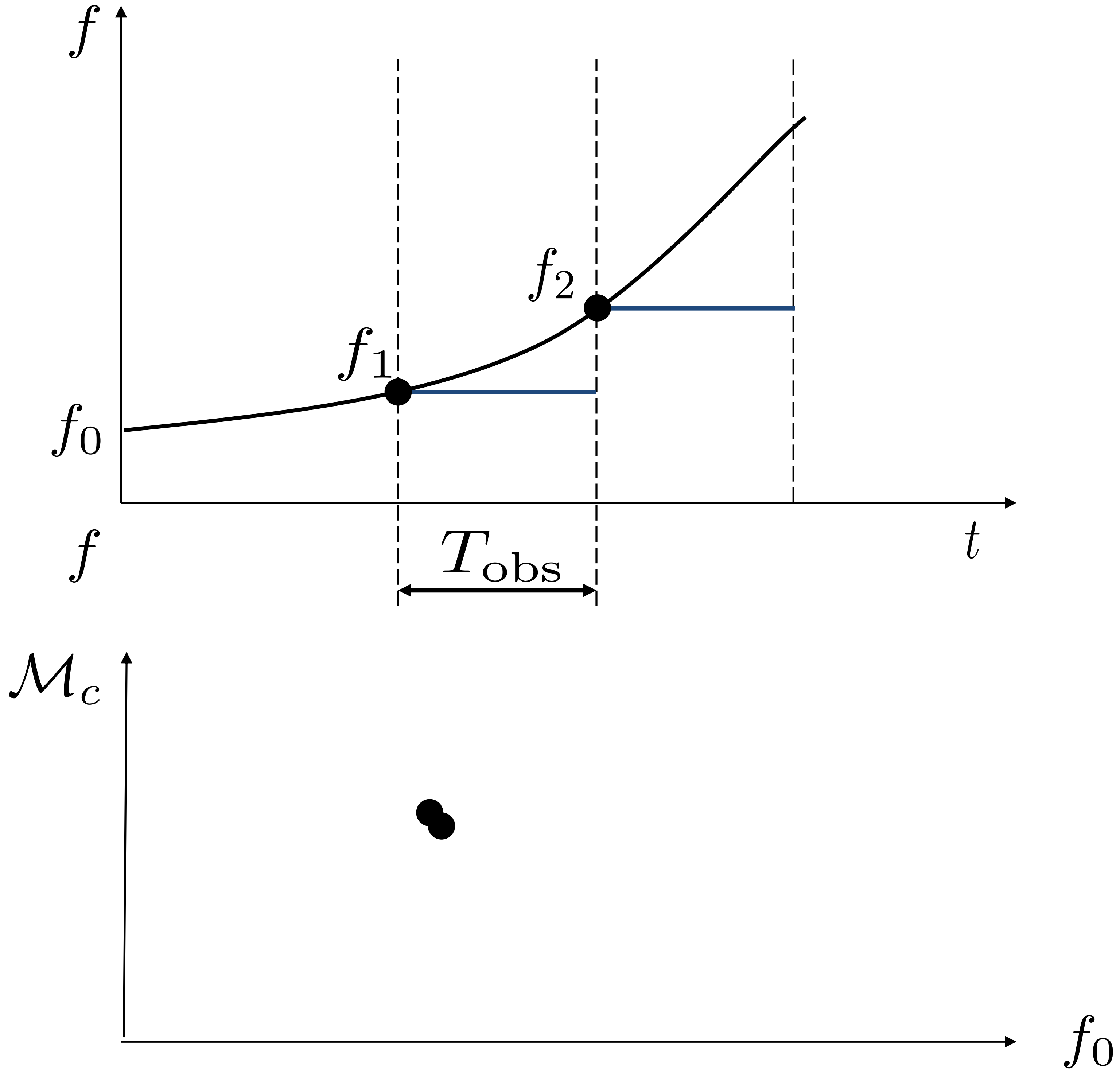
COHERENCE BETWEEN TRIGGERS

If candidates are selected, we need a way to know the parameters of the signal and combine results across time segments and interferometers

PARAMETER ESTIMATION

Each trigger will have an associated xi and frequency associated to it. This is referred to the starting time of the BSD file, yet we need it in terms of a reference time.

Thanks to the analytical formulas, each parameter can be expressed in the same reference time, which allows to combine easily the results among interferometers and among time segments.



INJECTION CAMPAIGN

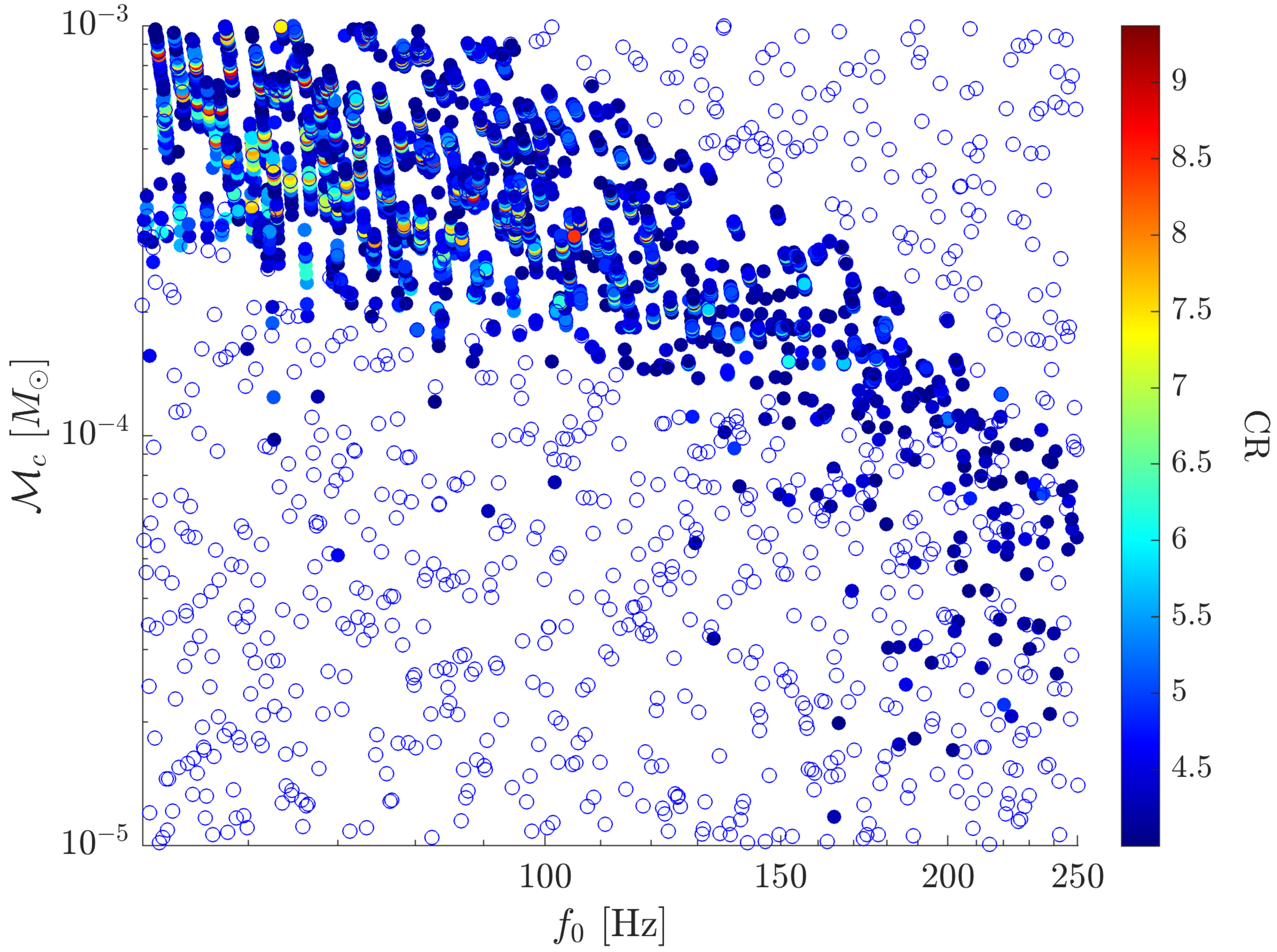
In order to test the method, a set of injections can be done and then the search pipeline run.

RESULTS

We inject a total of 1000 signals uniformly distributed (in log scale) in the parameter space of interest with random inclinations, but all placed at the GC and run the search in the GC grid.

We can consider that an injection is recovered if its distance with the parameters recovered is less than 4. This distance is defined as

$$d = \sqrt{\left(\frac{f_0 - f_0'}{\Delta f_0}\right)^2 + \left(\frac{\mathcal{M}_c - \mathcal{M}_c'}{\Delta \mathcal{M}_c}\right)^2}$$



RESULTS

Computing the strain for all the injections, we can obtain also an estimation of the efficiency. Here we are ignoring the potential dependency of the efficiency on the frequency, but it already gives an idea of the sensitivity of the method.

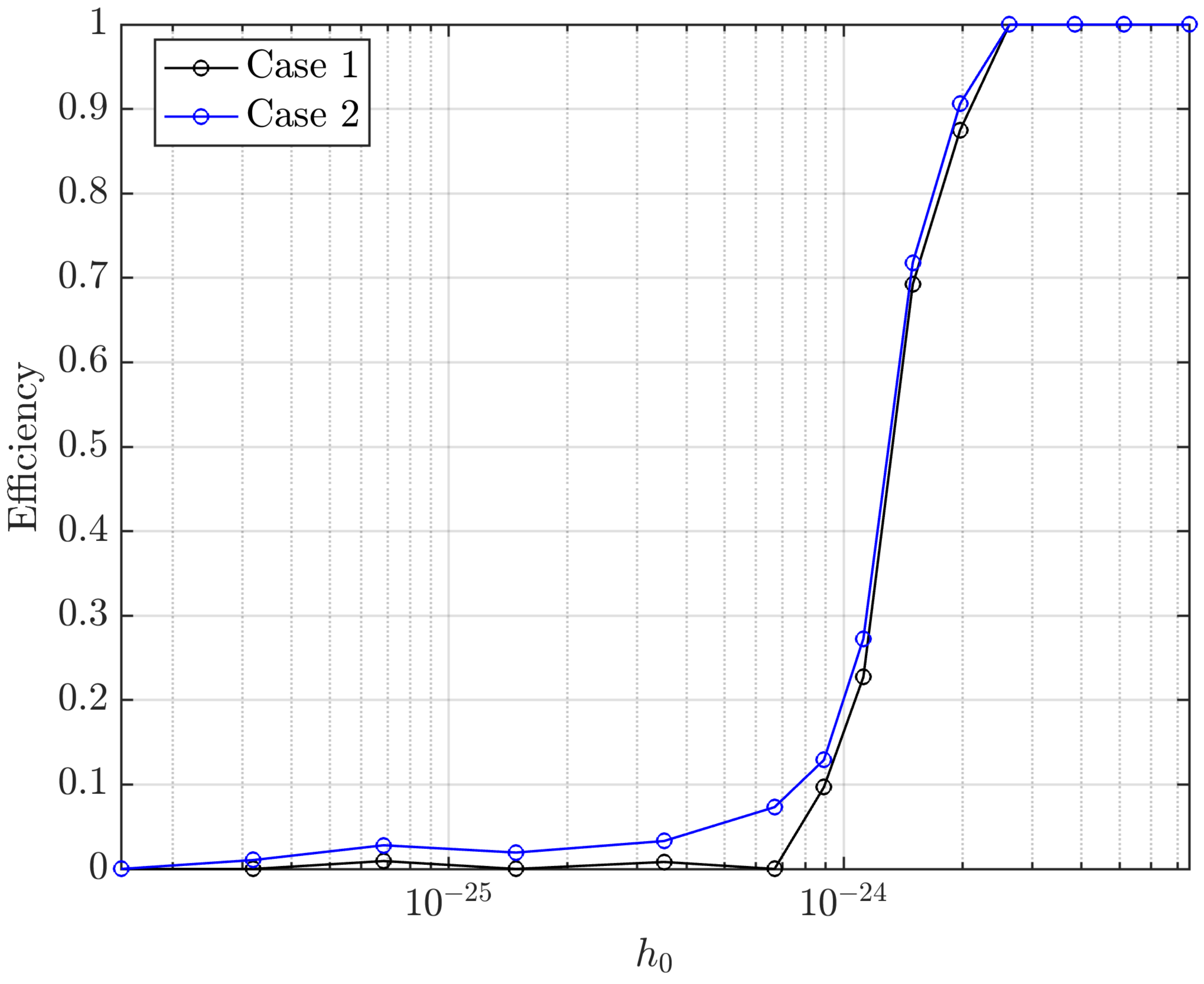
Two cases are here considered:

	LHO	LLO	T1	T2
CASE 1	⊘	✓	✓	⊘
CASE 2	✓	✓	✓	✓

The 95% of signals are recovered at

$$h^1_{95\%} = 2.34 \times 10^{-24}$$

$$h^2_{95\%} = 2.26 \times 10^{-24}$$



POTENTIAL UPPER LIMITS TO PBH

We can assume two formation mechanisms: a 2-body channel and a 3-body channel. Both of them are considered to happen during the early universe, as it is the dominant channel compared to late time formation.

$$\frac{\mathrm{d}R_{\mathrm{PBH},2}}{\mathrm{d}m_1\mathrm{d}m_2} = \frac{1.6\times10^6}{\mathrm{Gpc}^3\mathrm{yr}^1} f_{\mathrm{PBH}}^{\frac{53}{37}} \left[\frac{t}{t_0}\right]^{-\frac{34}{37}} \left[\frac{M}{M_\odot}\right]^{-\frac{32}{37}} \times \eta^{-\frac{34}{37}} S\left[\psi, f_{\mathrm{PBH}}, M\right] \frac{\psi(m_1)\psi(m_2)}{m_1 m_2}$$

$$\frac{\mathrm{d}R_{\mathrm{PBH},3}}{\mathrm{d}m_1\mathrm{d}m_2} \approx \frac{7.9\times10^4}{\mathrm{Gpc}^3\,\mathrm{yr}} \left[\frac{t}{t_0}\right]^{\frac{\gamma}{7}-1} f_{\mathrm{PBH}}^{\frac{144\gamma}{259}+\frac{47}{37}} \times \left[\frac{\langle m\rangle}{M_\odot}\right]^{\frac{5\gamma-32}{37}} \left(\frac{M}{2\langle m\rangle}\right)^{\frac{179\gamma}{259}-\frac{2122}{333}} (4\eta)^{-\frac{3\gamma}{7}-1} \times \mathcal{K} \frac{e^{-3.2(\gamma-1)}\gamma}{28/9-\gamma} \bar{\mathcal{F}}(m_1, m_2) \frac{\psi(m_1)\psi(m_2)}{m_1 m_2},$$

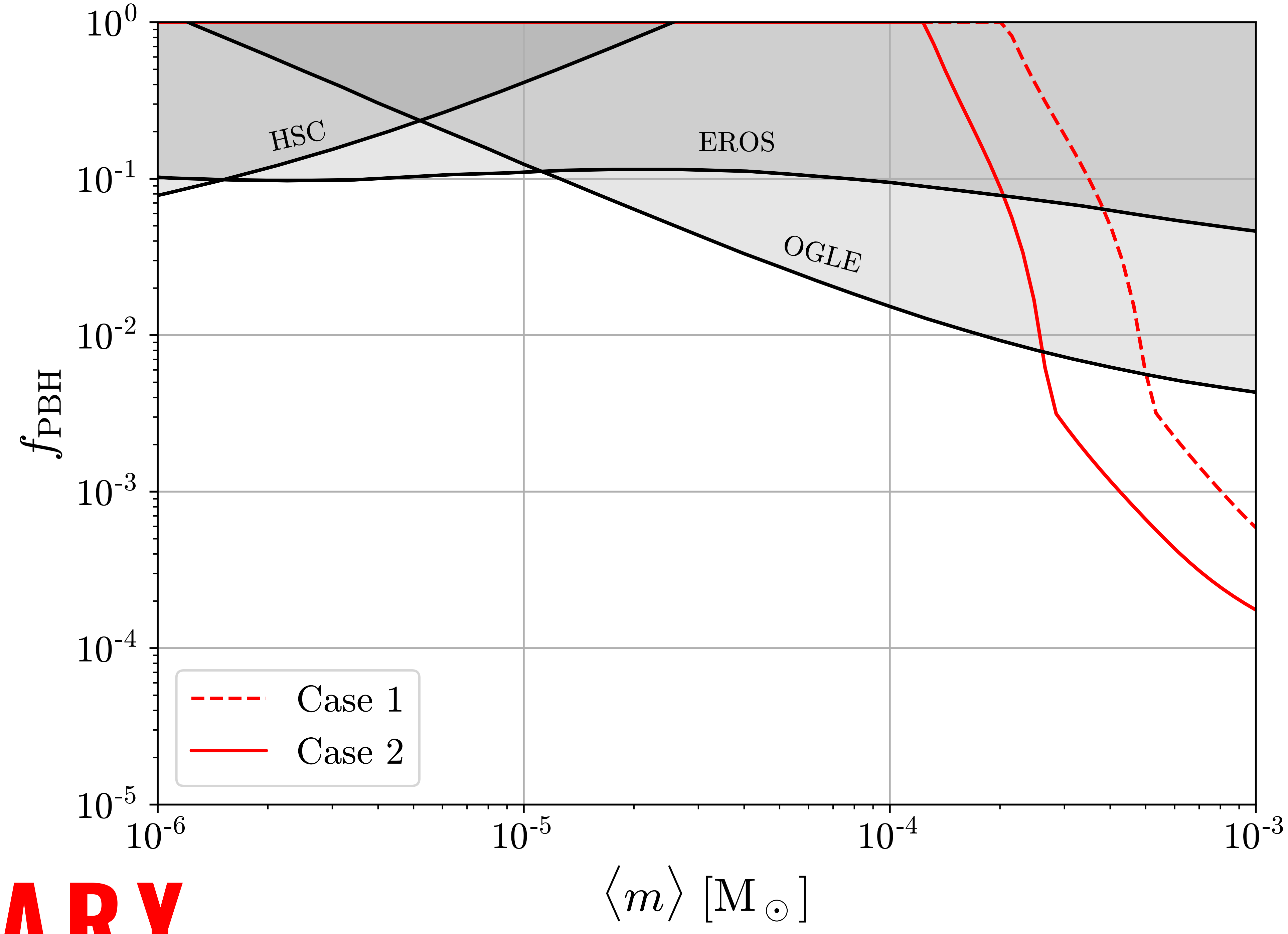
See M. Andrés-Carcasona et al. (2024) and references therein for the various terms

Then, an upper limit can be set by excluding the region where the number of expected events, N, is above 3. N is computed, accounting for the DM distribution surrounding the MW as

$$N = T\int \frac{\mathrm{d}z}{1+z}\frac{\mathrm{d}V_c}{\mathrm{d}z}\bar{\delta}(d(z))R(t(z)) \times p_{\mathrm{det}}(\mathrm{SNR}_c/\mathrm{SNR}(z))$$

O. Pujolas et al. (2021)

POTENTIAL UPPER LIMITS TO PBH



PRELIMINARY

NEXT STEPS

The next steps that we plan to do in order to continue this work are important to ensure the good outcome of the project.

TEST

We need to run more tests, with injections, changing the setup, optimizing parameters,...

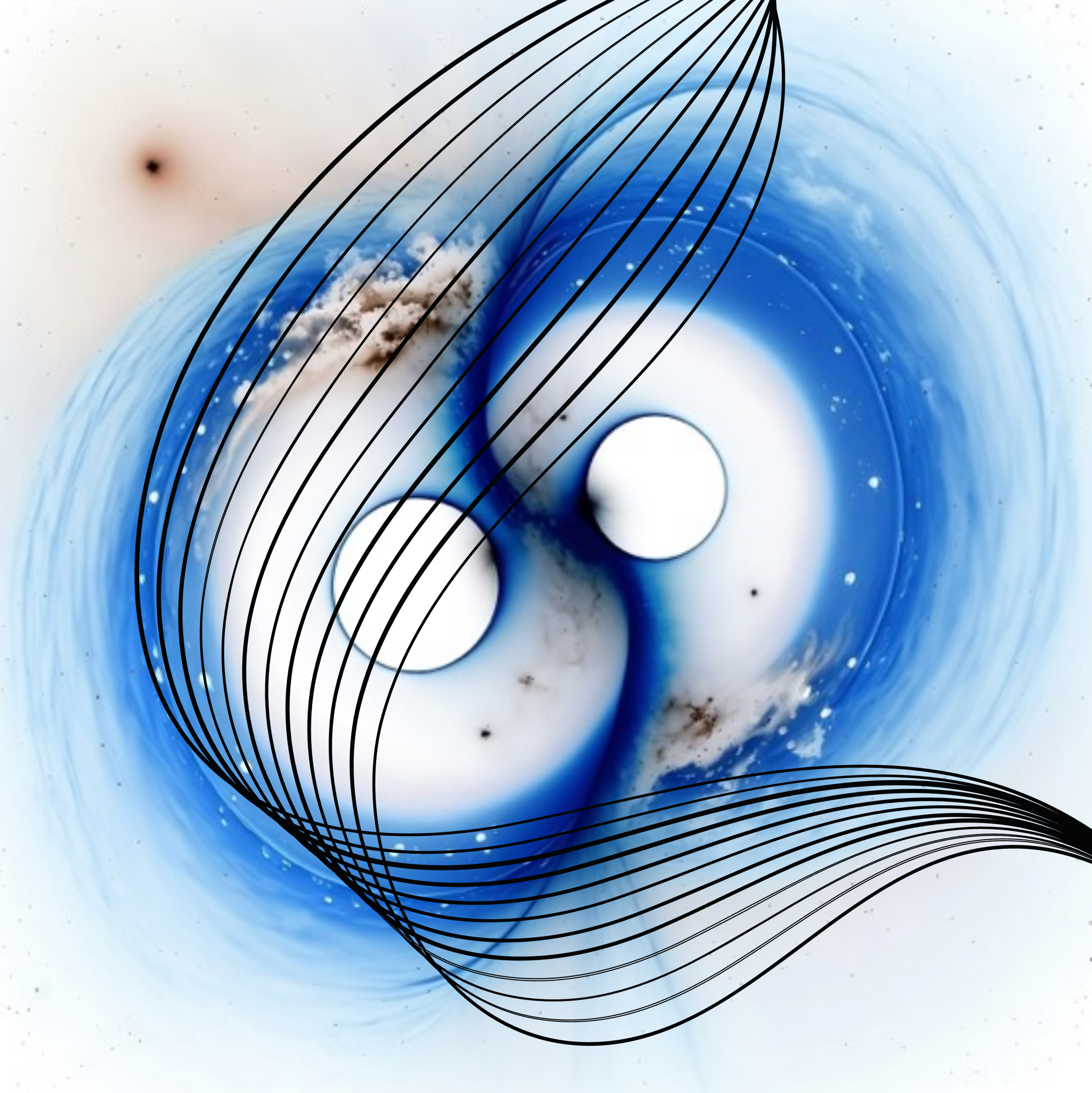
REVIEW

We plan to start the internal LVK review ASAP.

SEARCH

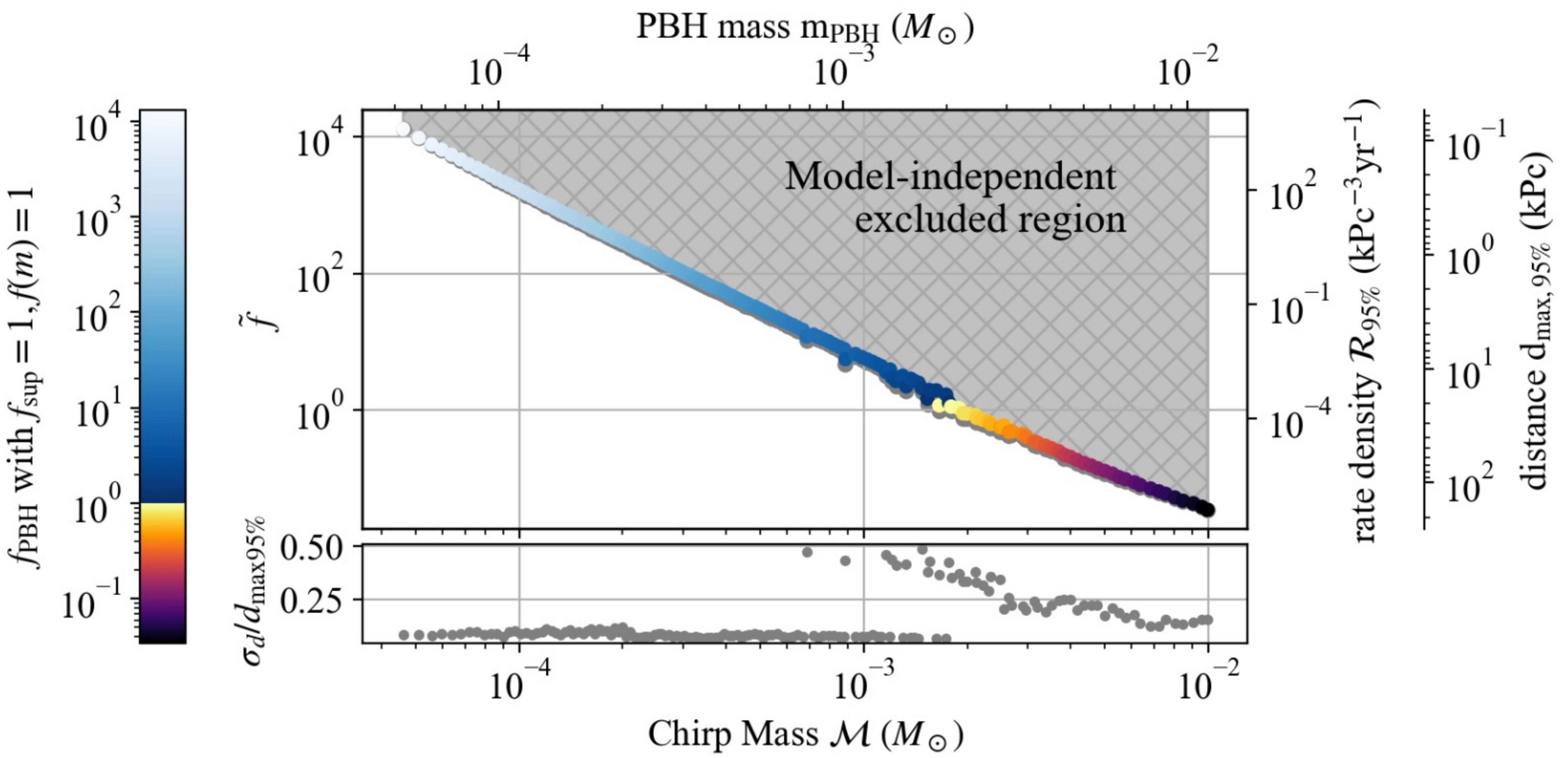
Perform the actual search and place the upper limits of PBH populations with actual results

THANK'S FOR WATCHING



EXTRA SLIDES

COMPARISON TO RECENT SEARCHES



Andrew L. Miller et al. (2024)

Main differences:

- We include the effect of 3-body channel.
- We assume a NFW profile of DM in the milky way.
- We include the suppression factor.
- We are able to probe $f_{PBH} < 1$ below 10^{-3} solar masses.