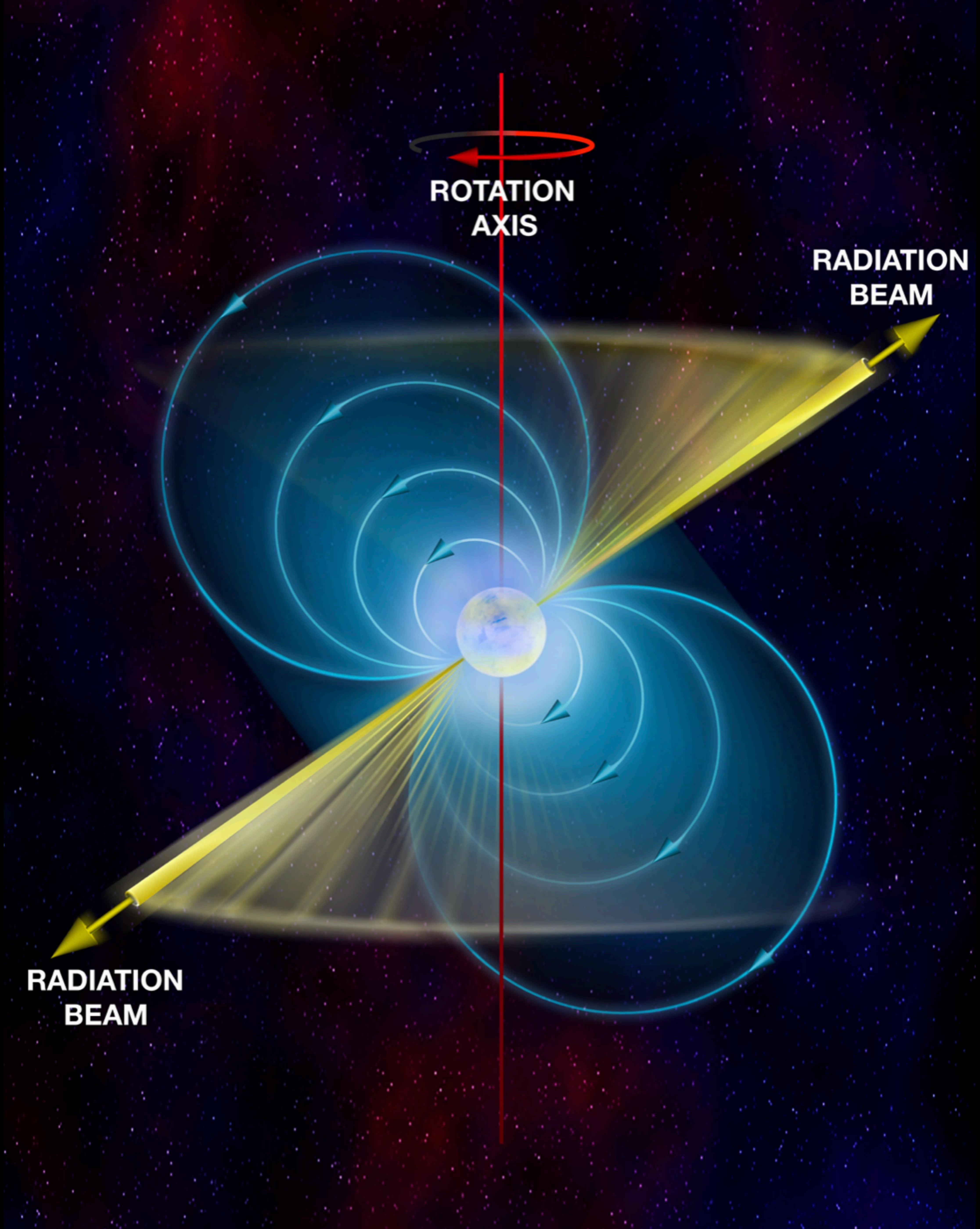
Parameter estimation for a two-component model of neutron stars with a Kalman filter

By Nicholas O'Neill, Pat Meyers and Andrew Melatos.

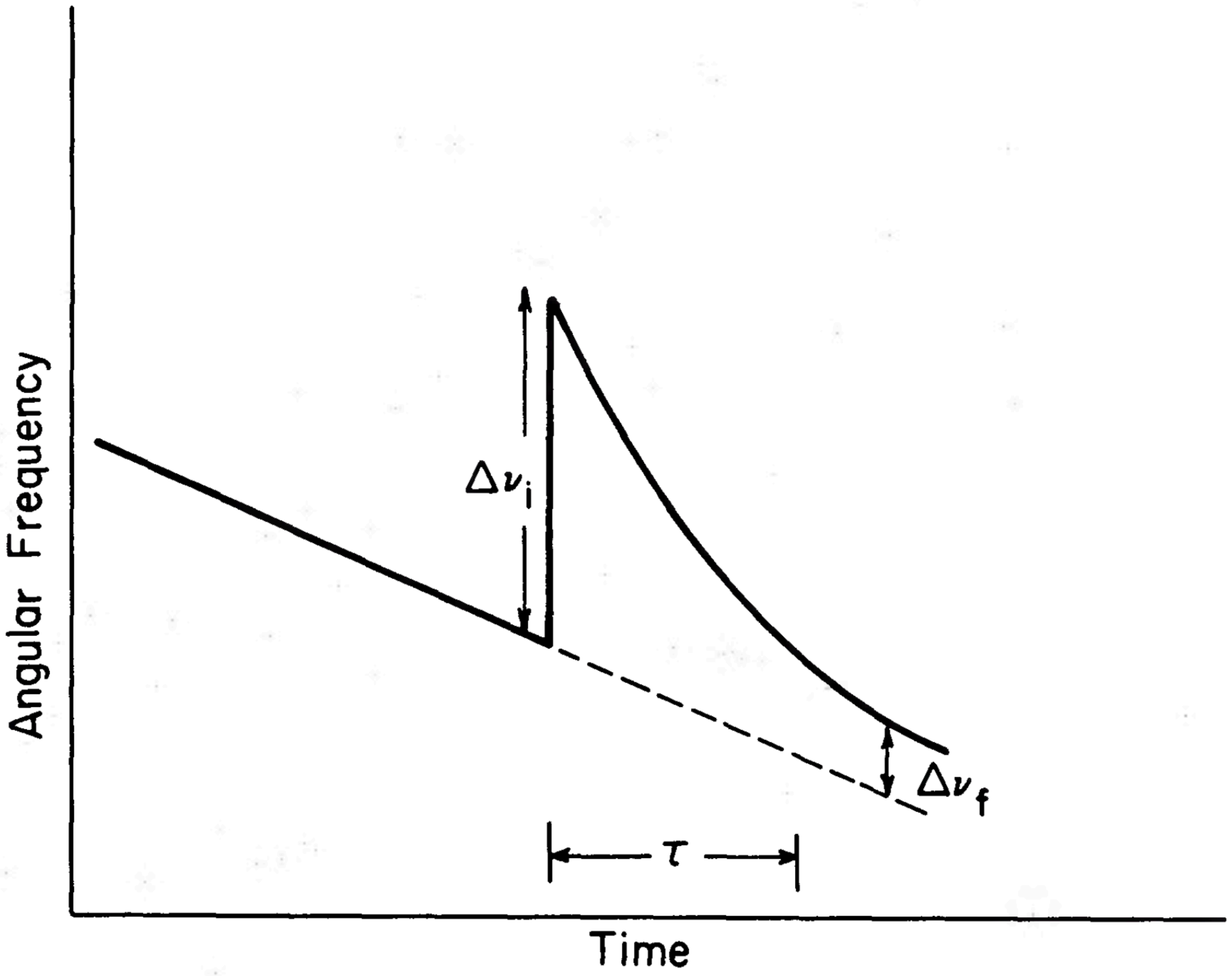
THE UNIVERSITY OF MELBOURNE

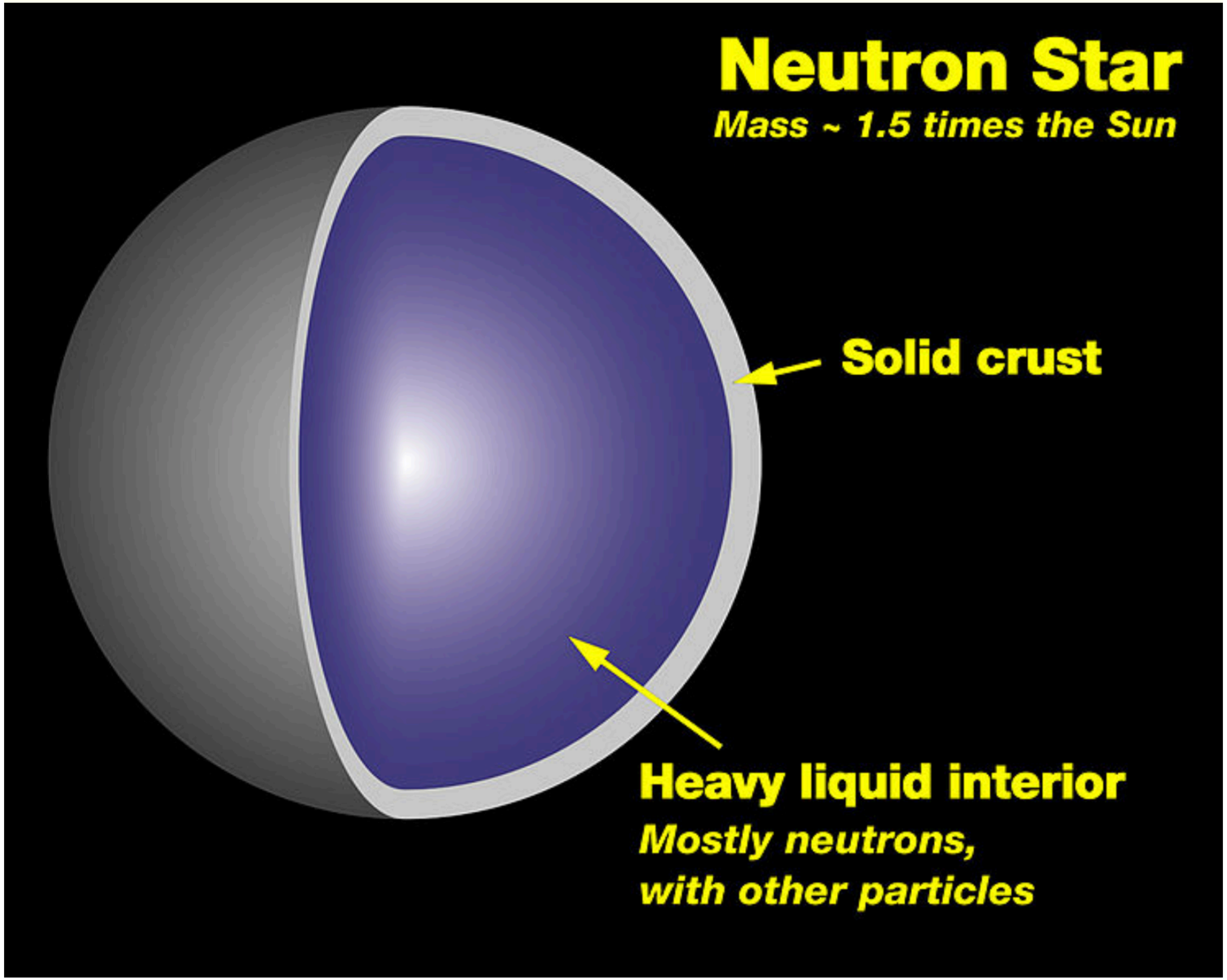
OzGrav
ARC Centre of Excellence for Gravitational Wave Discovery



Picture Credit: B. Saxton, NRAO/AUI/NSF

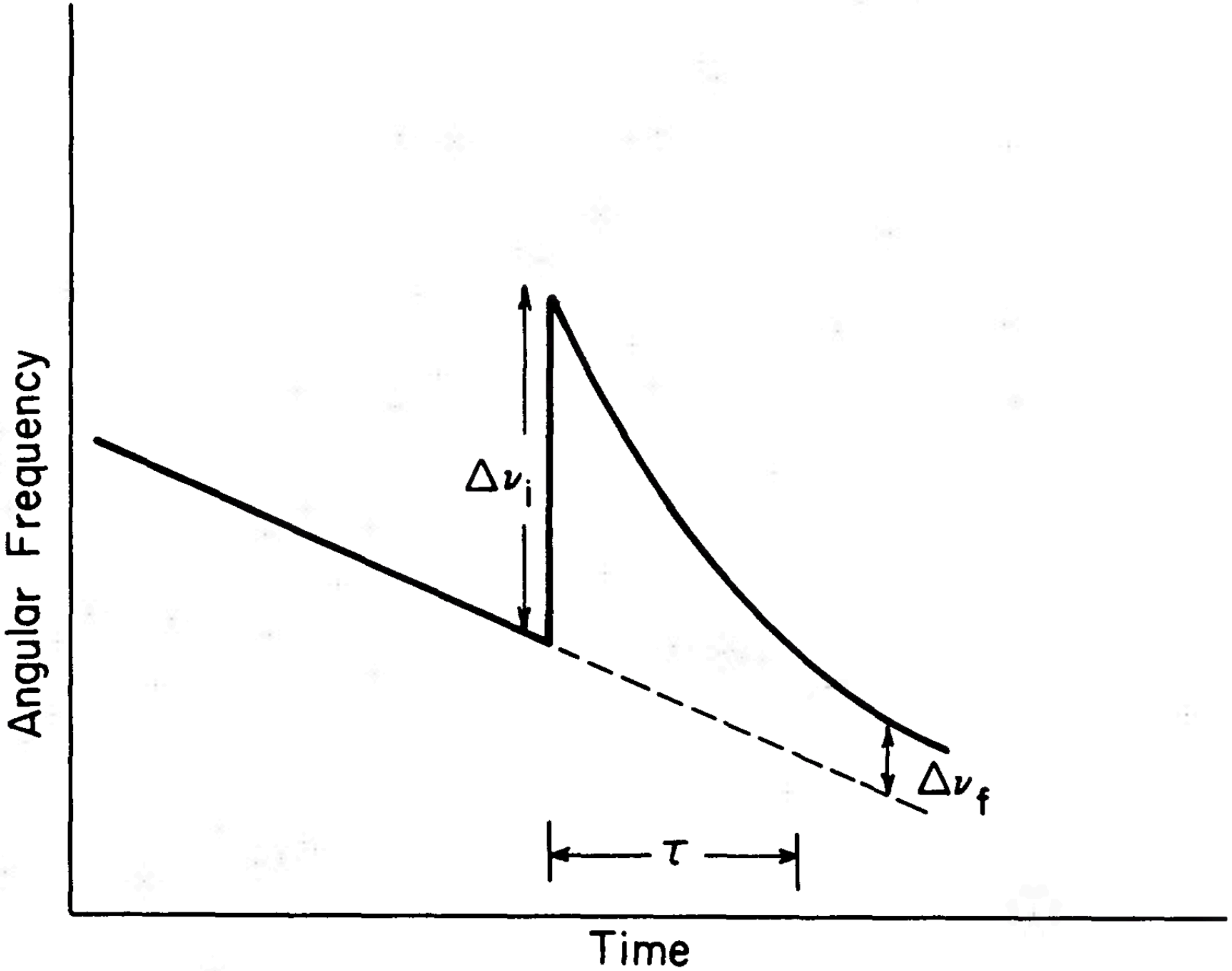
Pulsar glitch



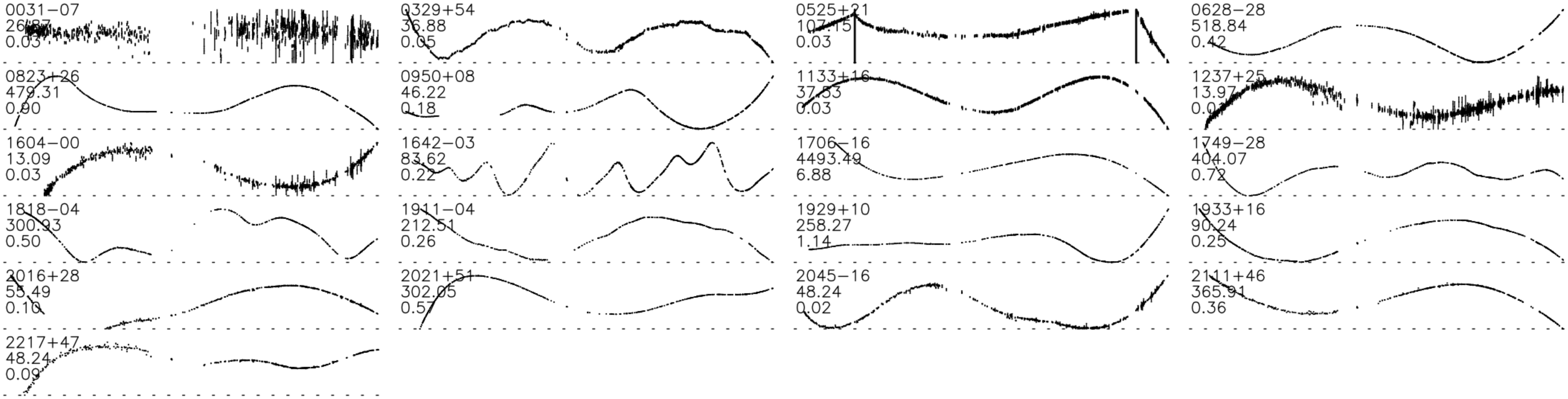


Picture from: NASA/Marshall Space Flight Center

Pulsar glitch



Pulsar timing noise examples



G. Hobbs *et al* 2006 *Chin. J. Astron. Astrophys.* **6** 169

We fit this model to data to find the parameters.

$$I_c \frac{d\Omega_c}{dt} = -\frac{I_c}{\tau_c}(\Omega_c - \Omega_s) + N_c + \xi_c$$

$$I_s \frac{d\Omega_s}{dt} = -\frac{I_s}{\tau_s}(\Omega_s - \Omega_c) + N_s + \xi_s$$

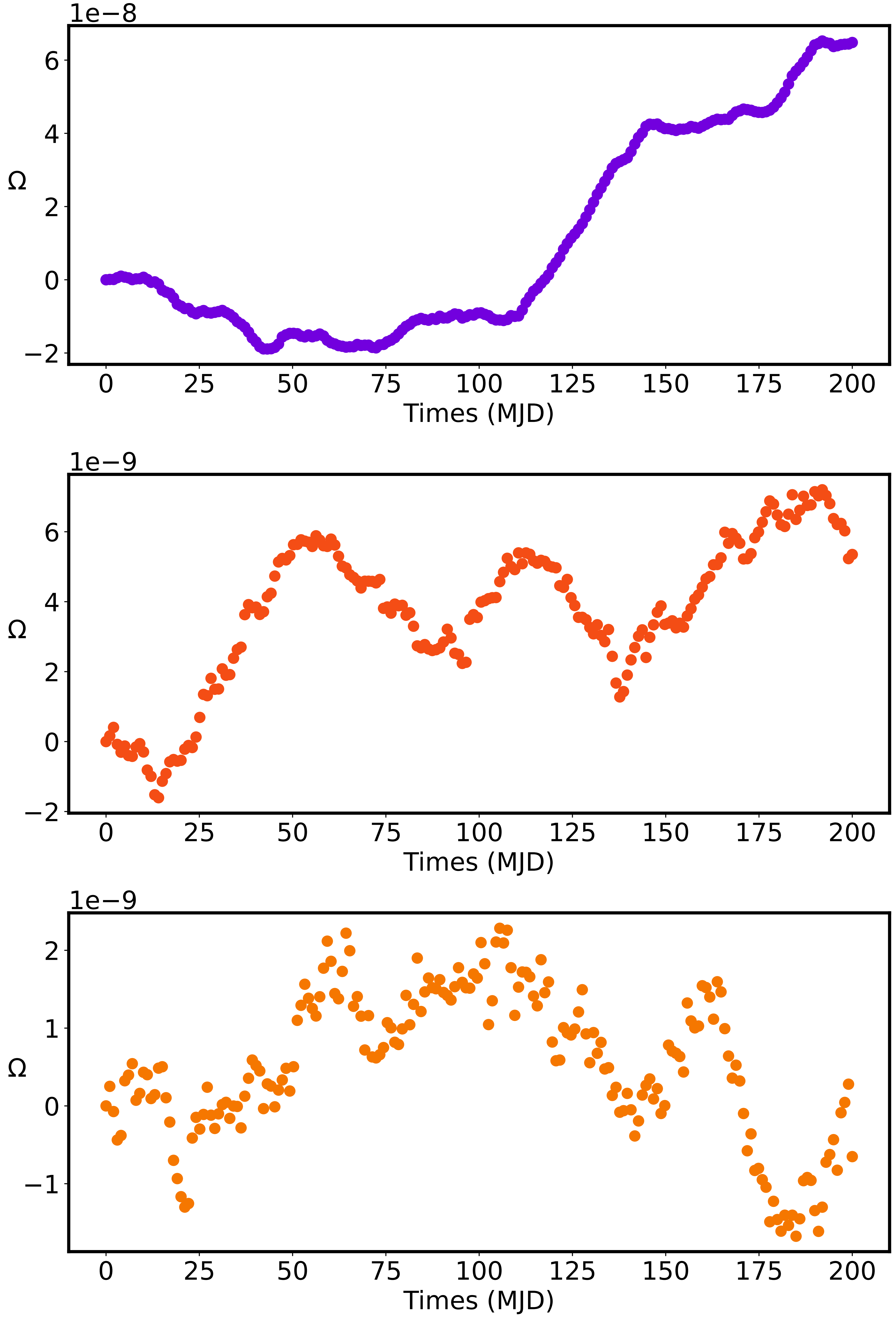
$$\langle \xi_c(t) \xi_c(t') \rangle = \sigma_c^2 \delta(t - t')$$

$$\langle \xi_s(t) \xi_s(t') \rangle = \sigma_s^2 \delta(t - t')$$

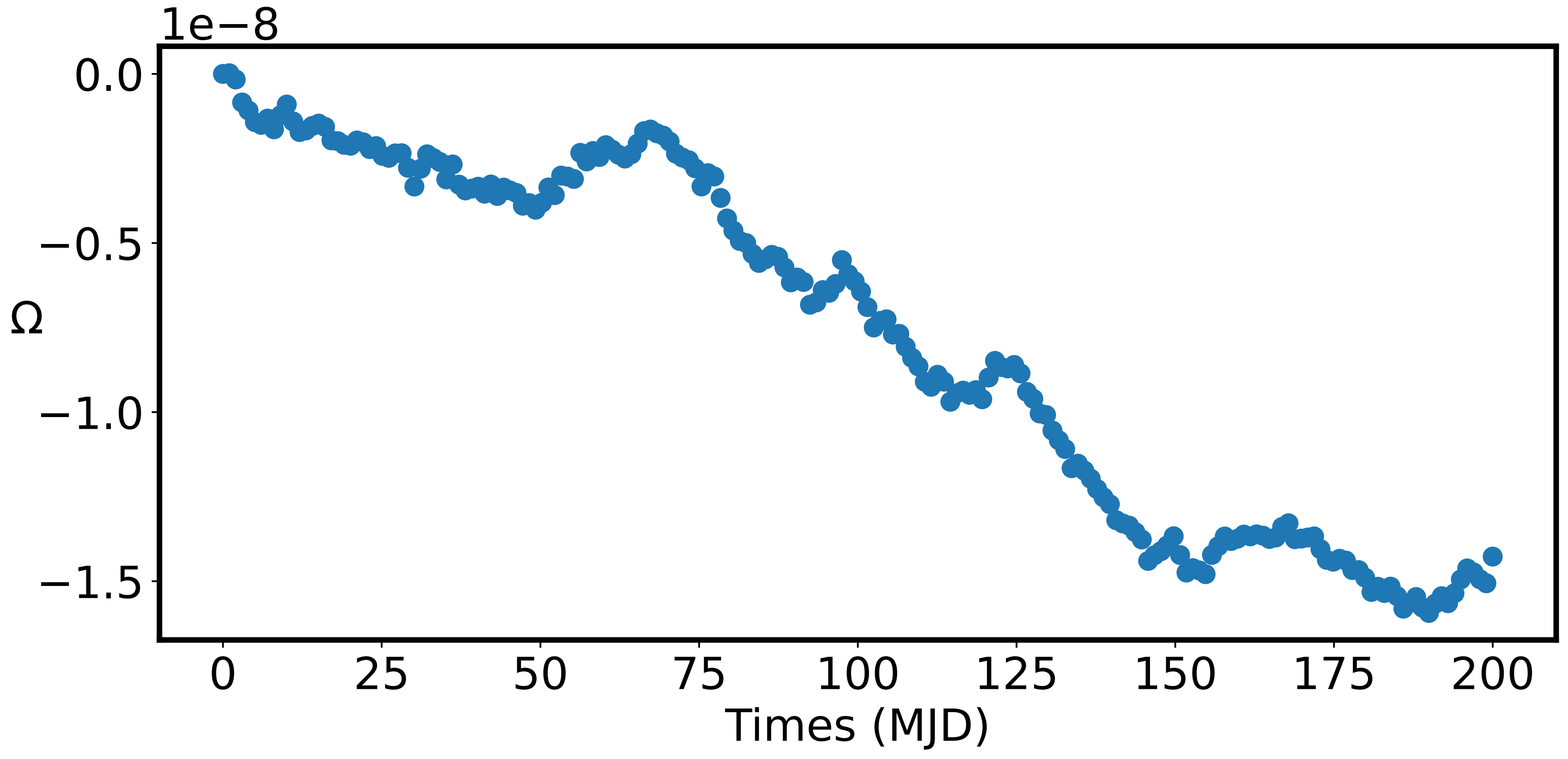
$$\langle \xi_c(t) \xi_s(t') \rangle = 0$$

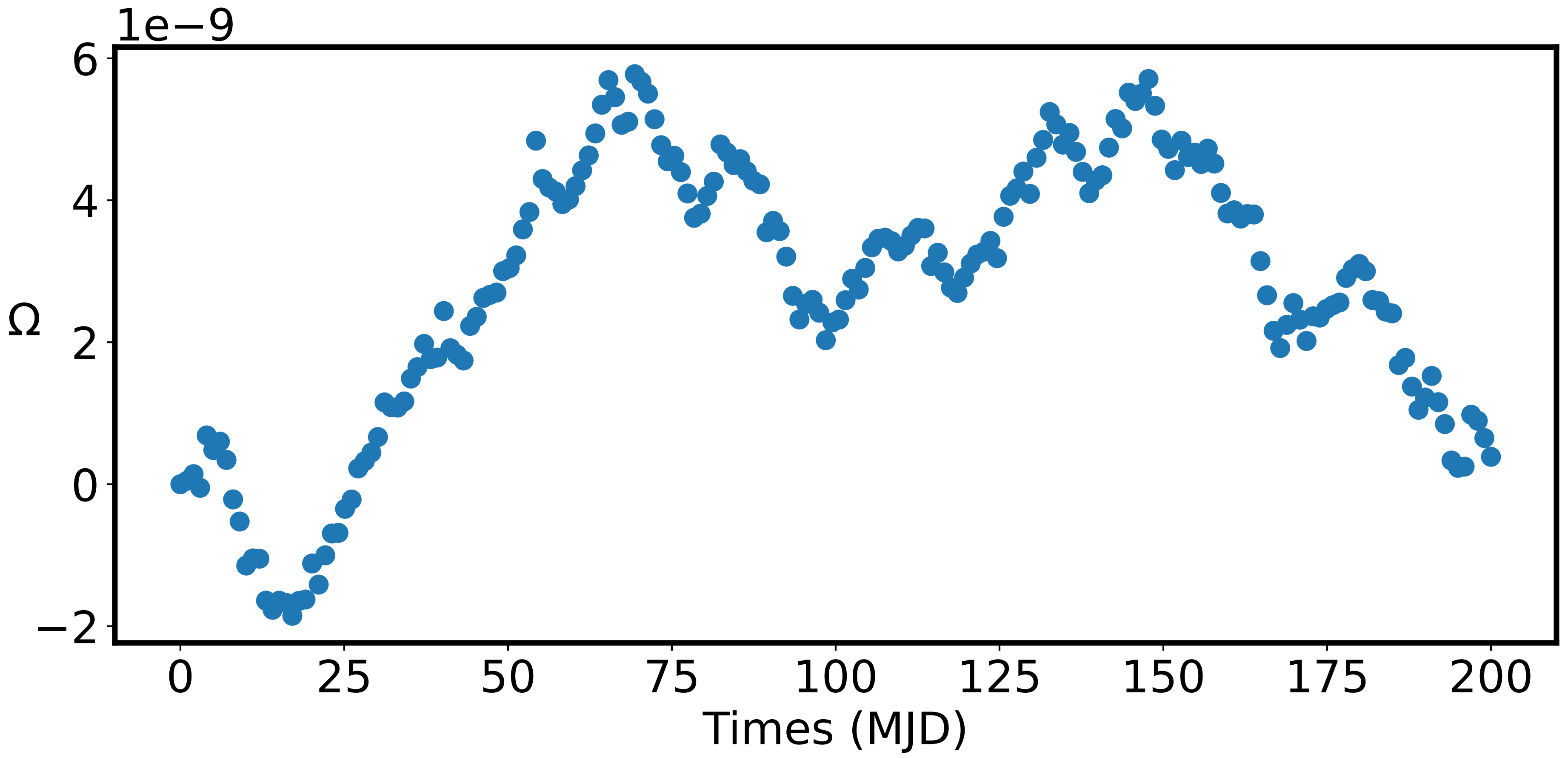
Simulating pulsar data with different parameters gives very different behaviour.

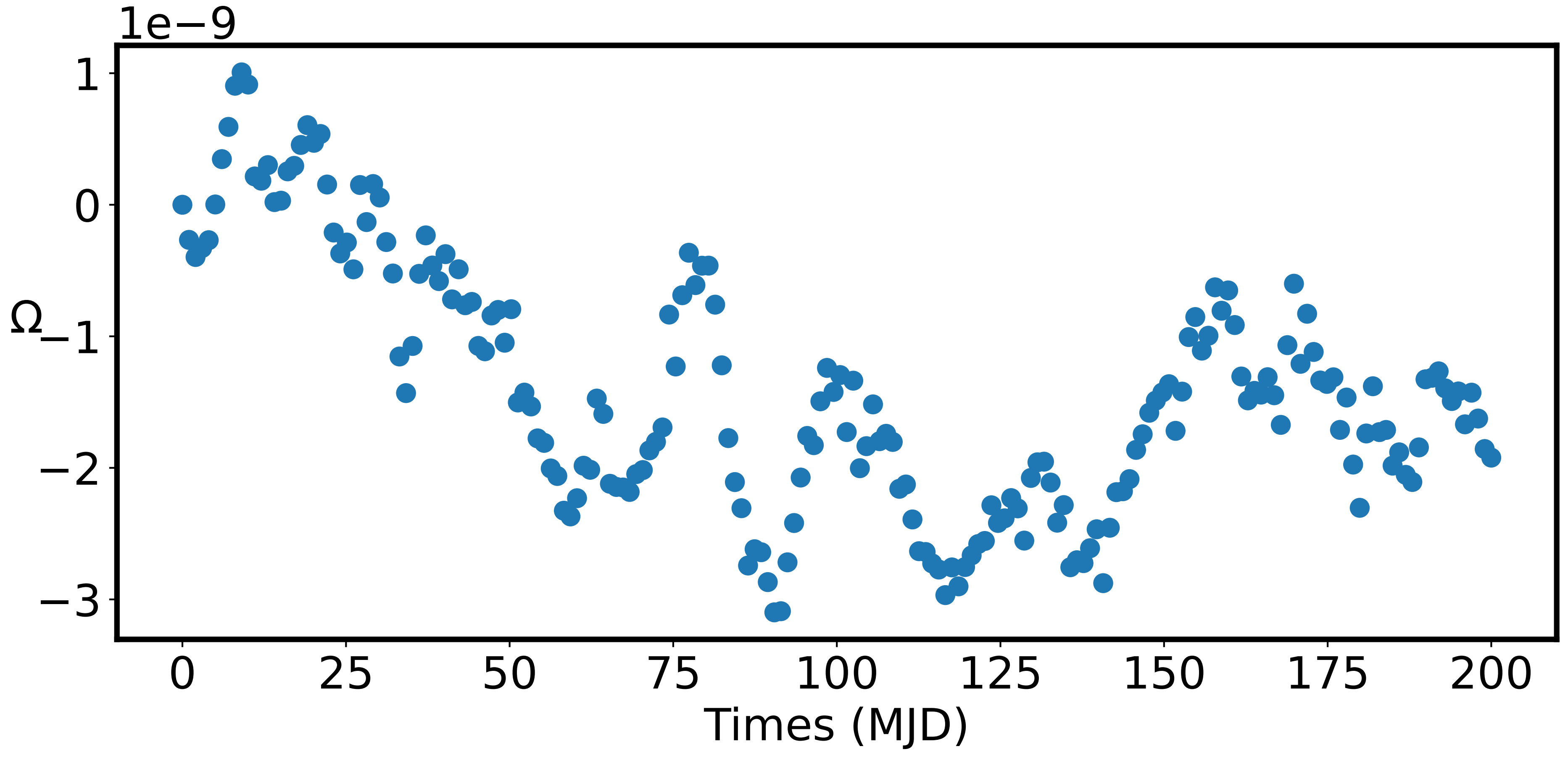
The core noise strength decreases from top to bottom.



However, the same parameters can give very different looking behaviours just by chance.







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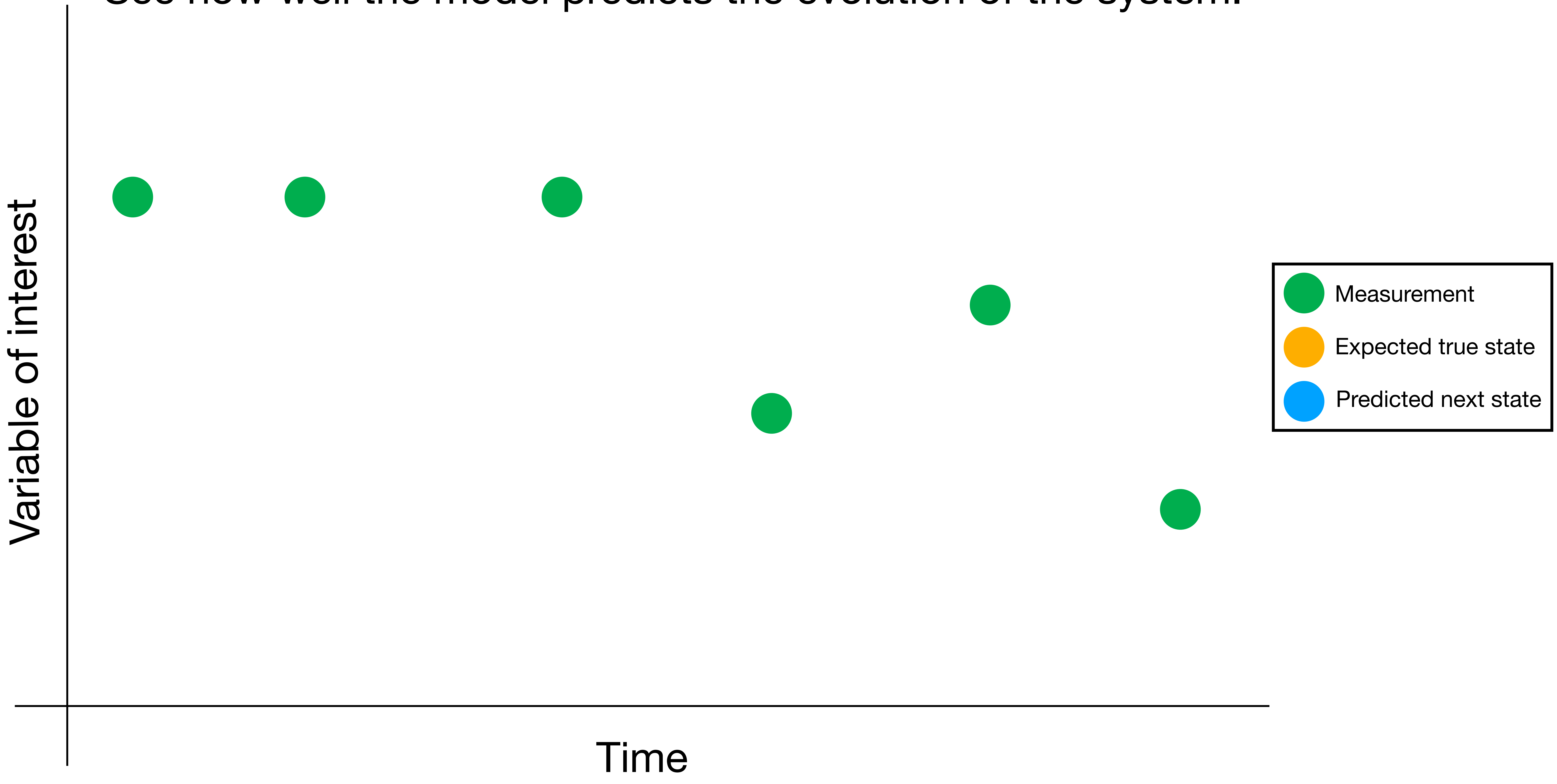
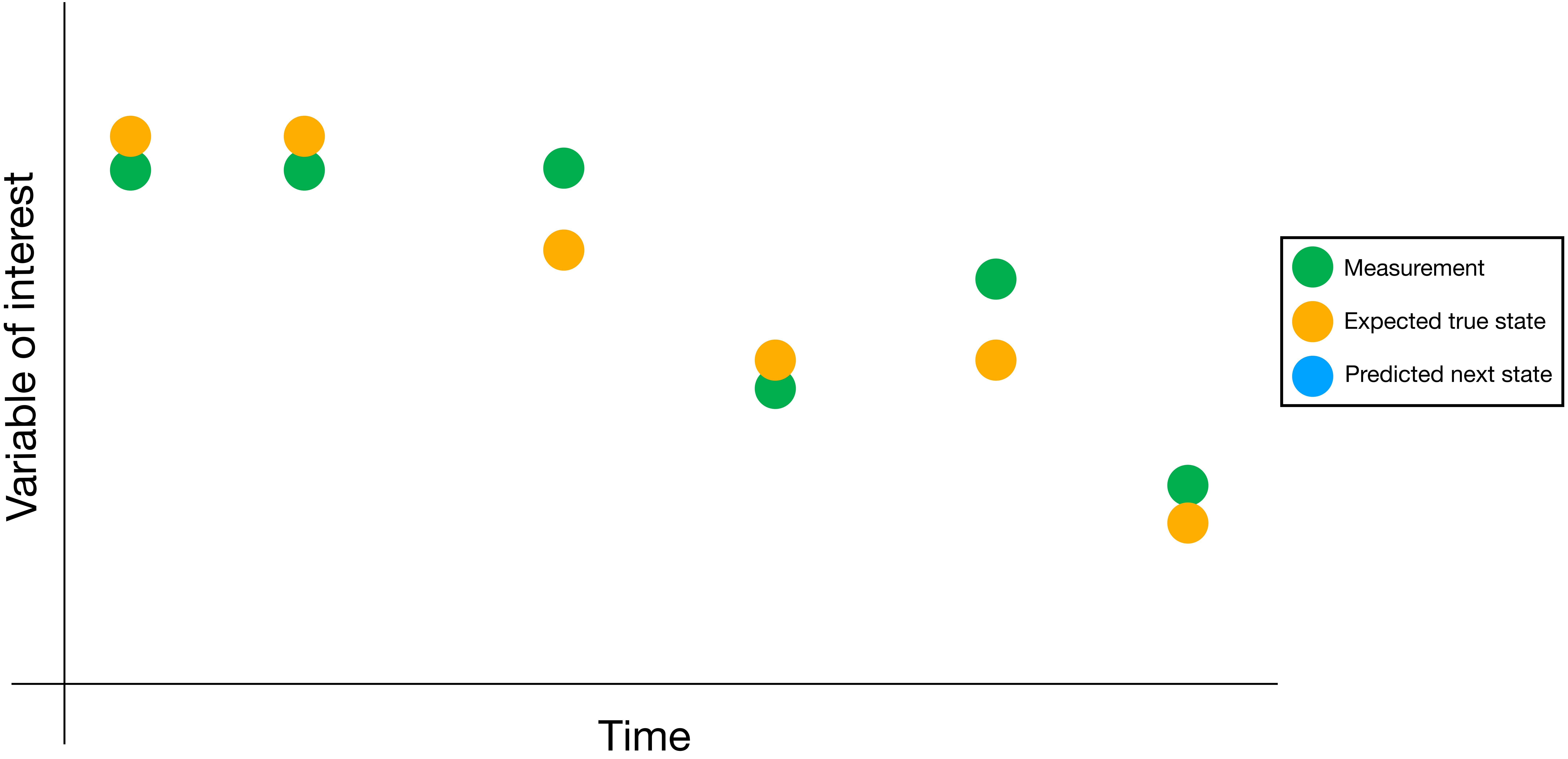
$$\langle \xi_c(t)\xi_s(t') \rangle = 0$$

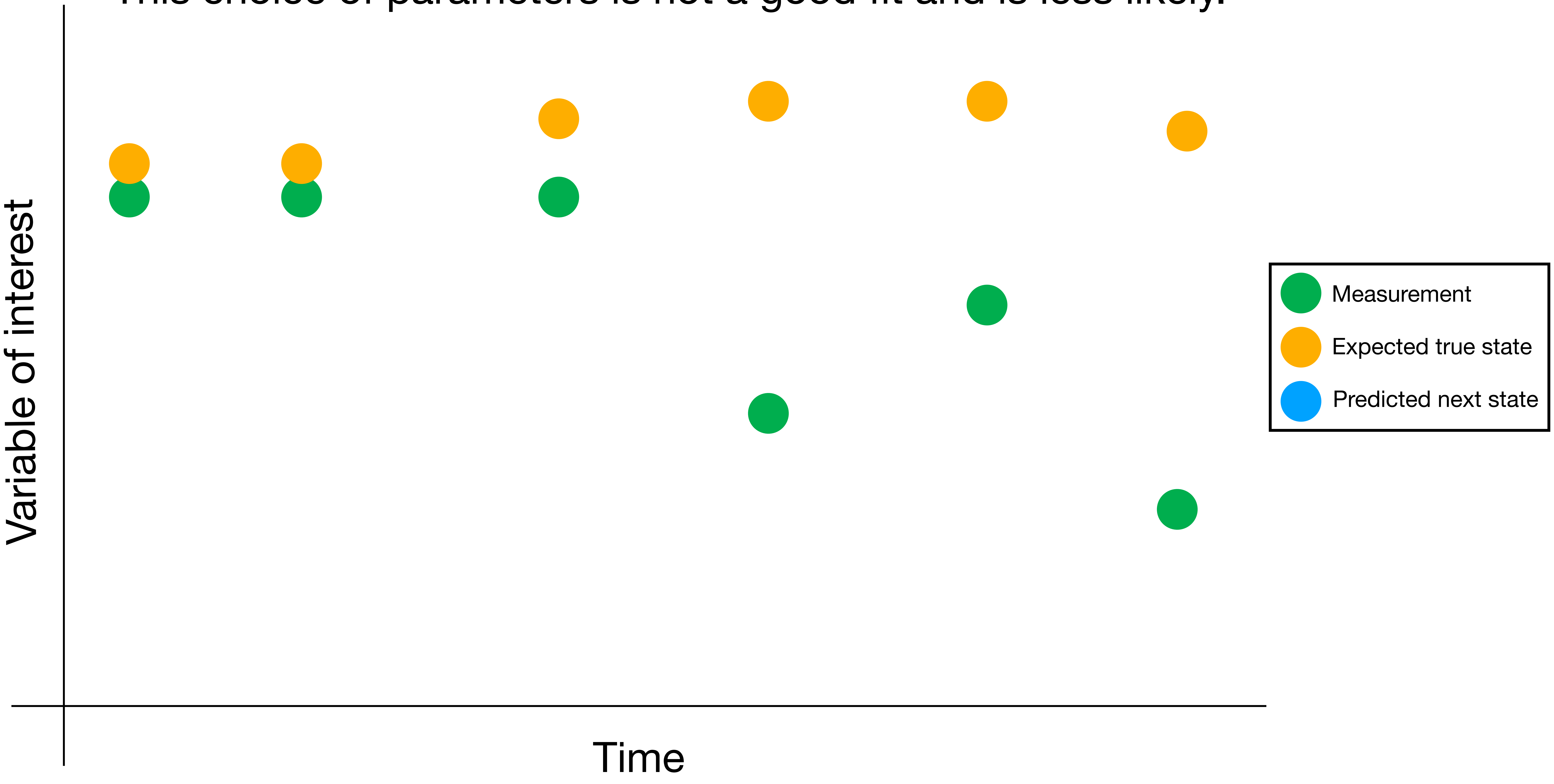
See how well the model predicts the evolution of the system.



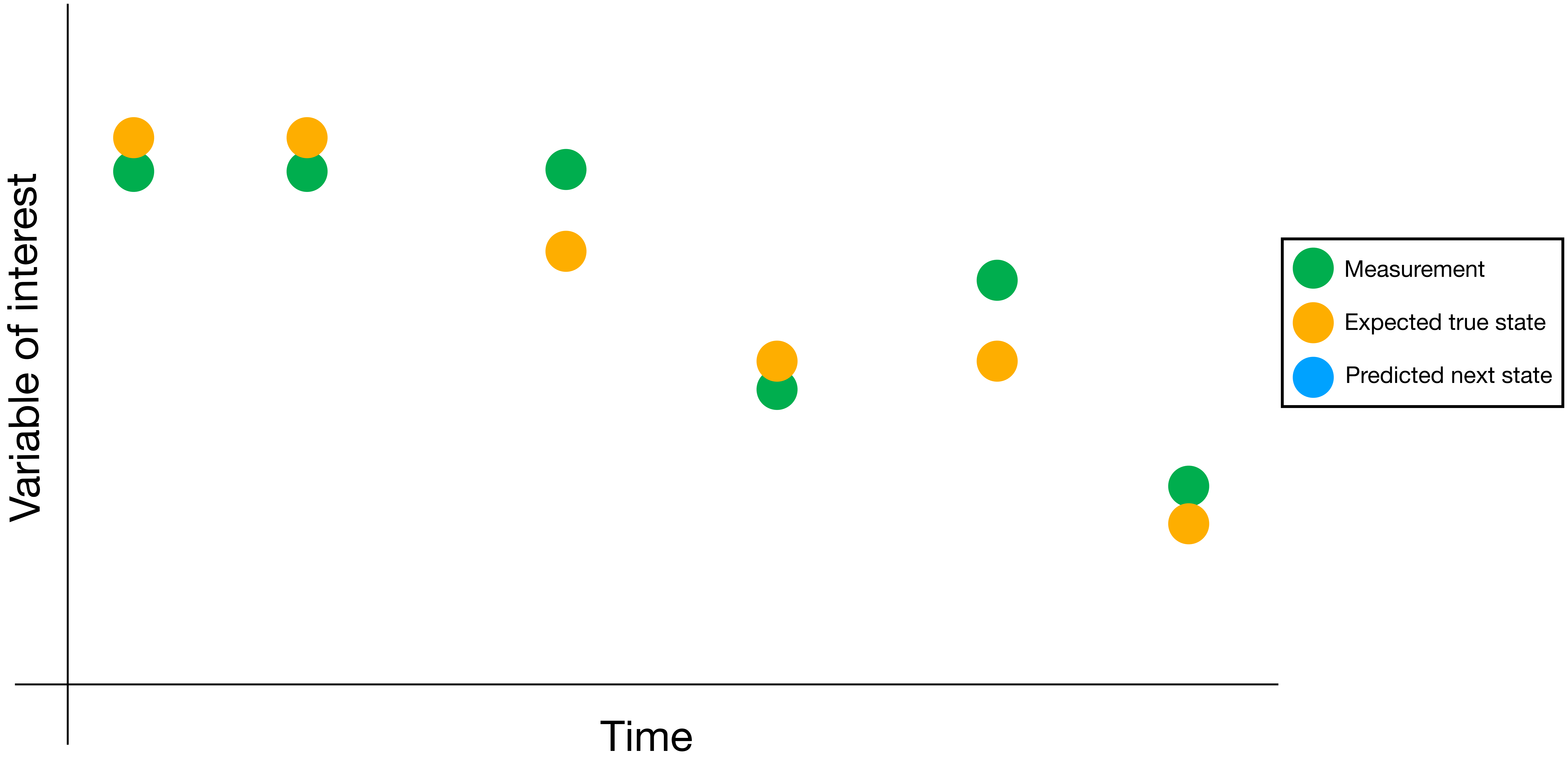
A good choice of parameters will model the data well.



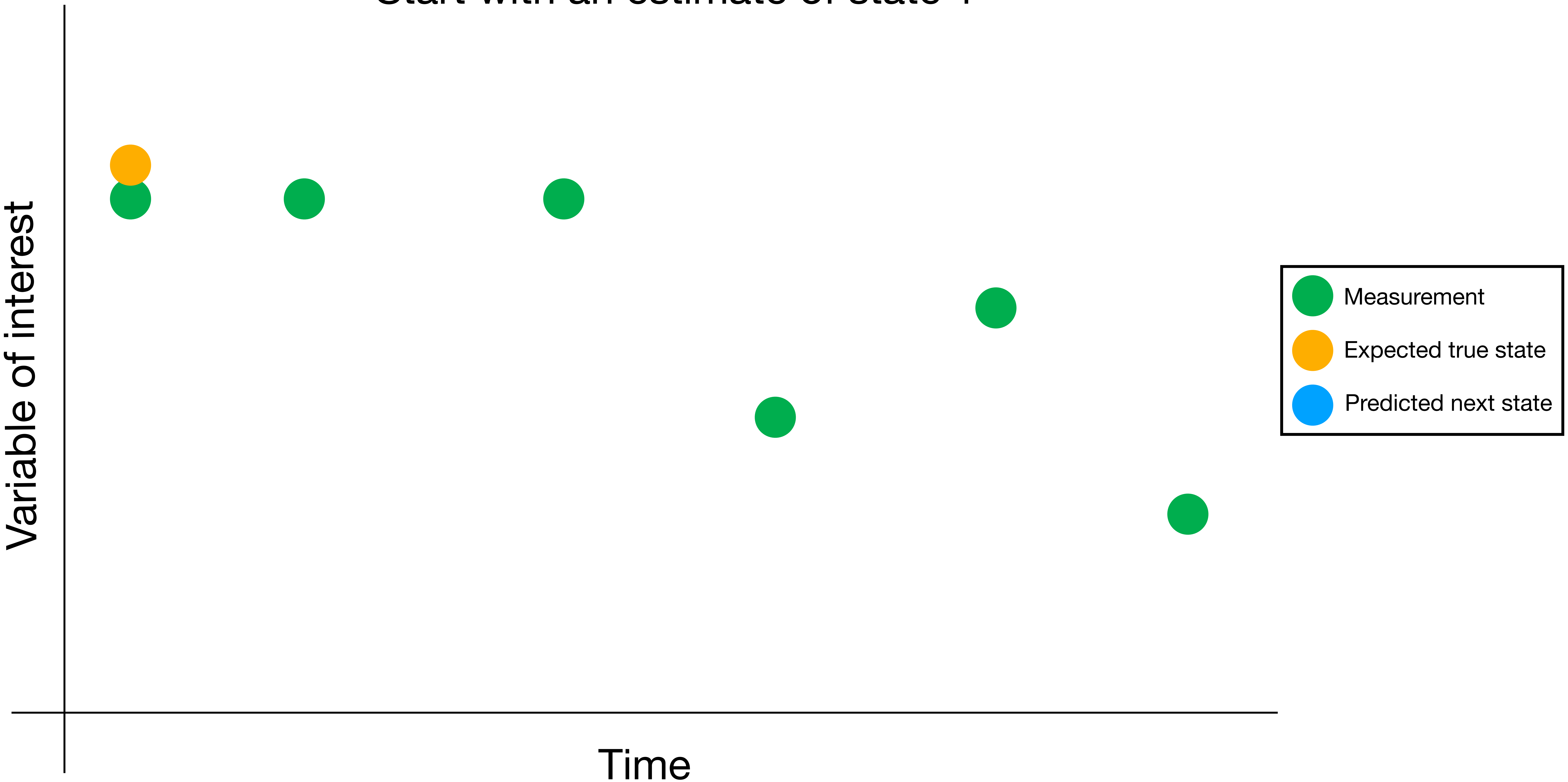
This choice of parameters is not a good fit and is less likely.



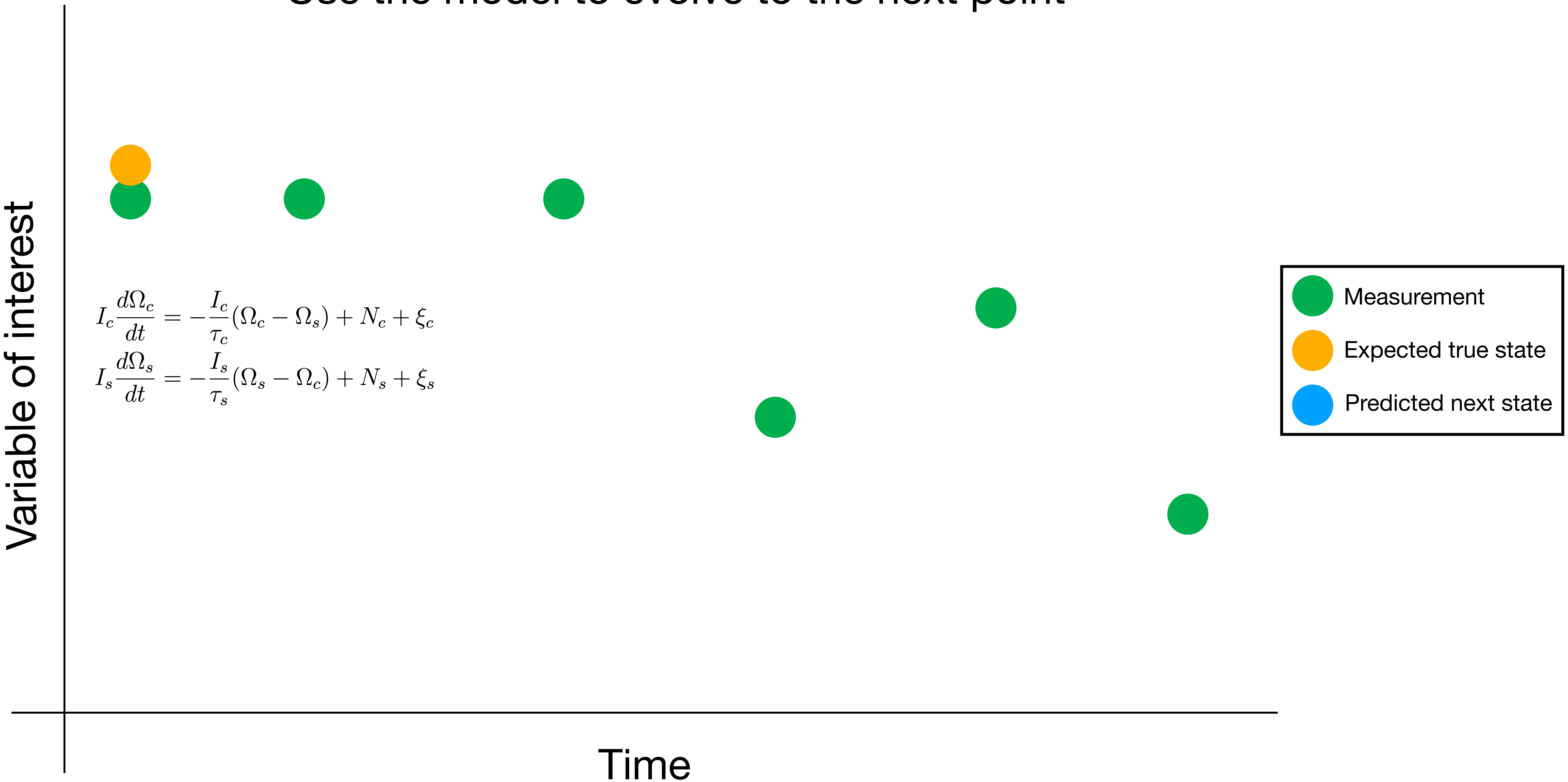
The measurements are the true state plus noise.



Start with an estimate of state 1



Use the model to evolve to the next point



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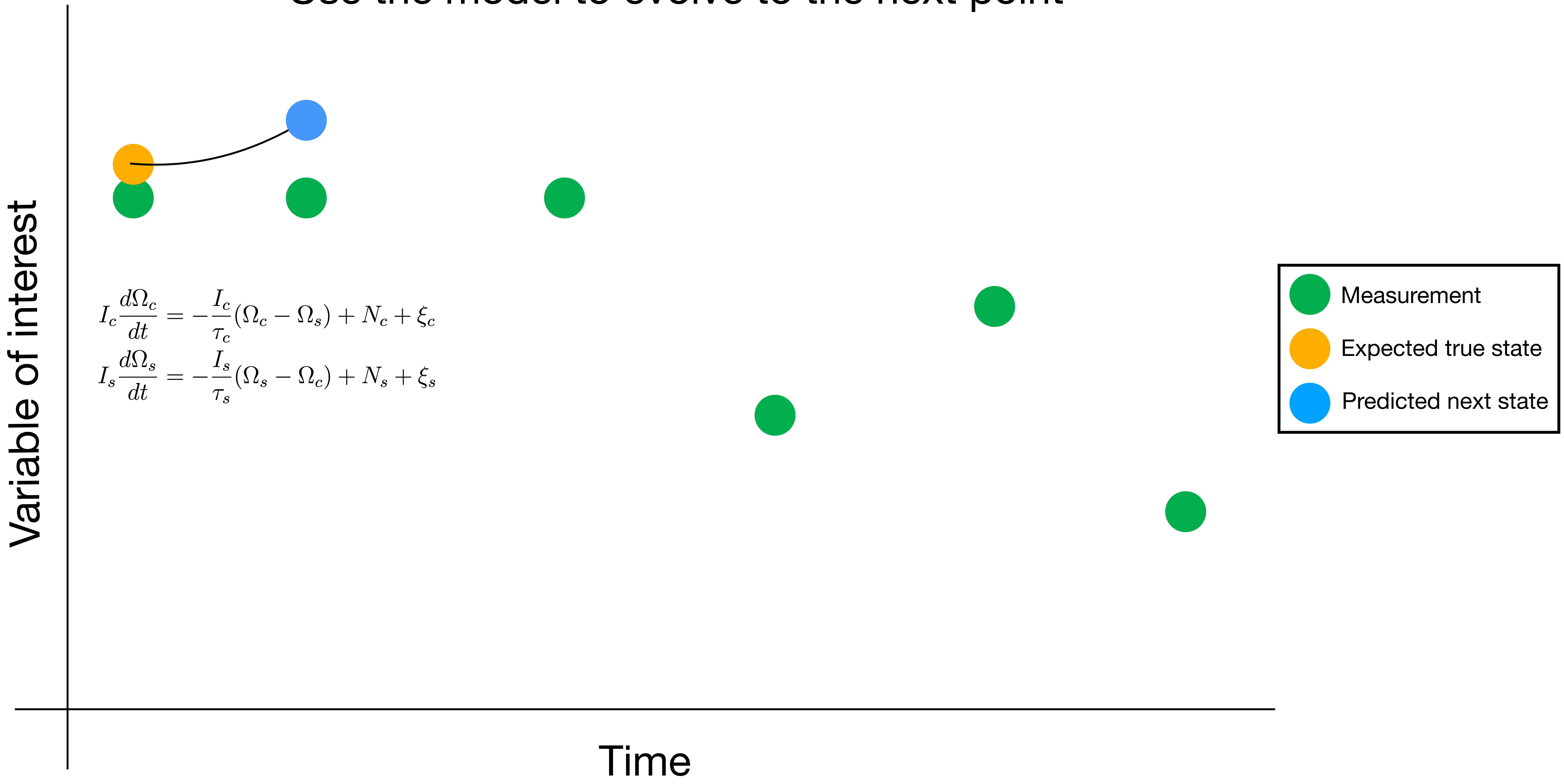
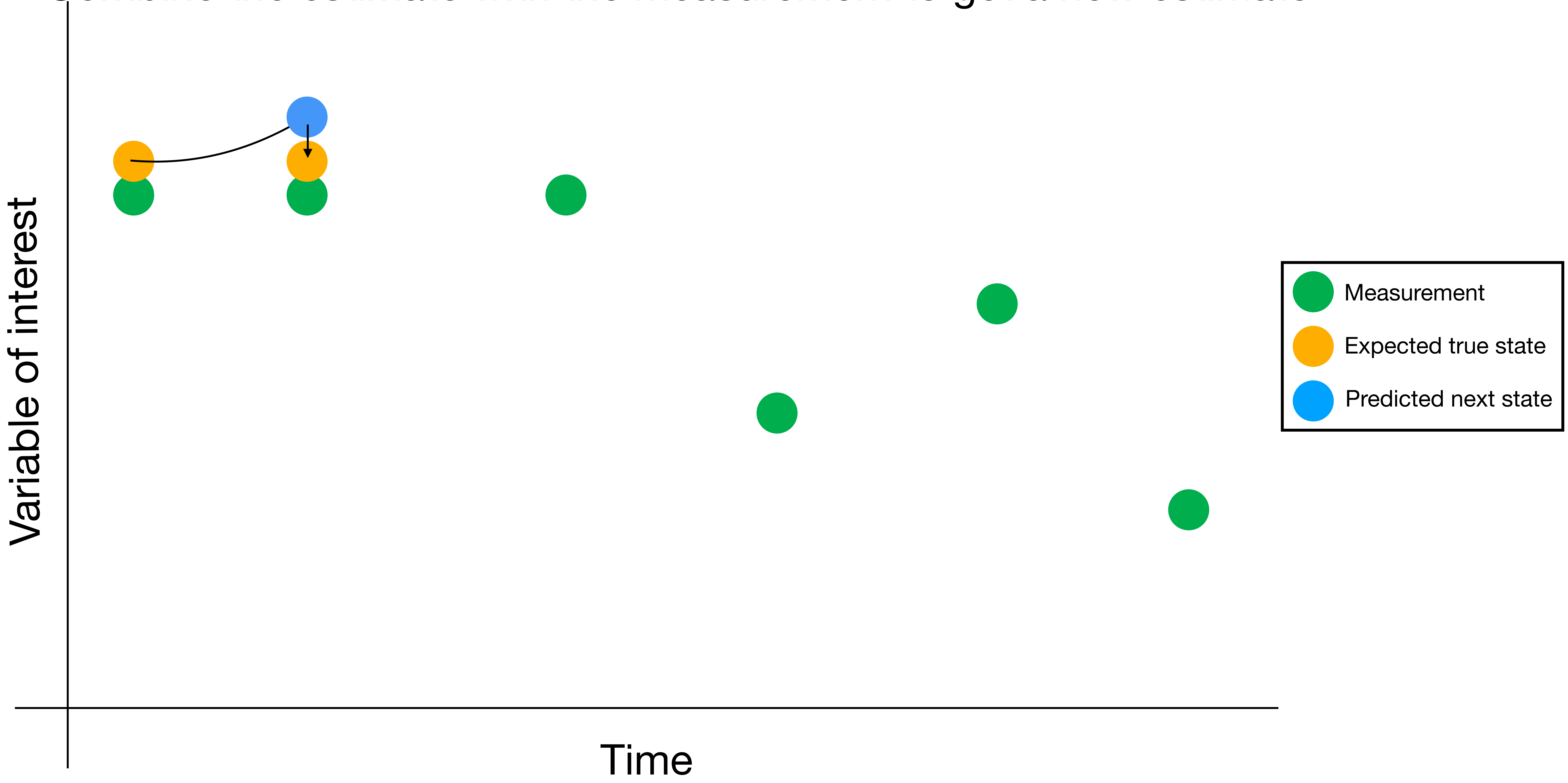
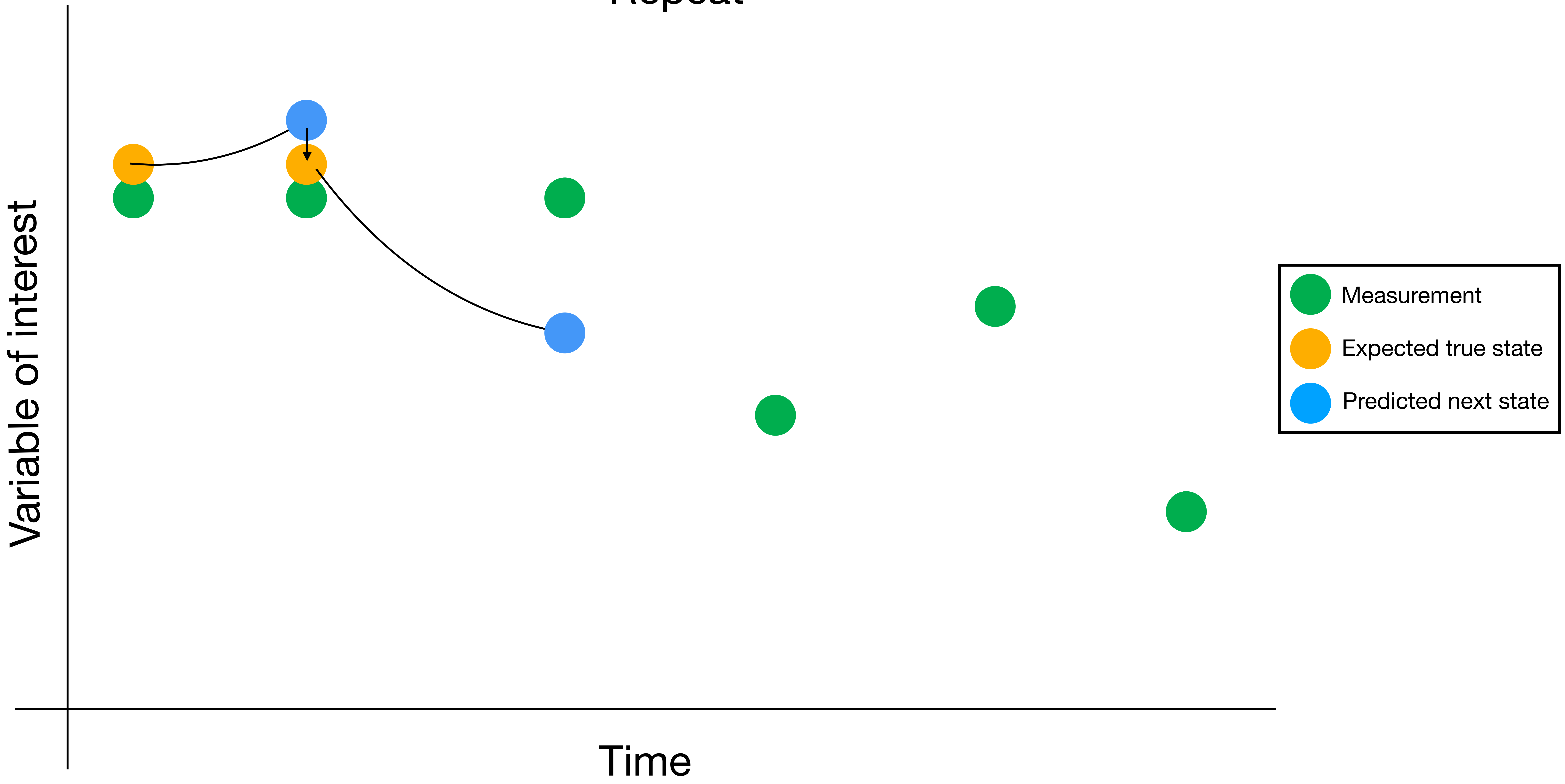


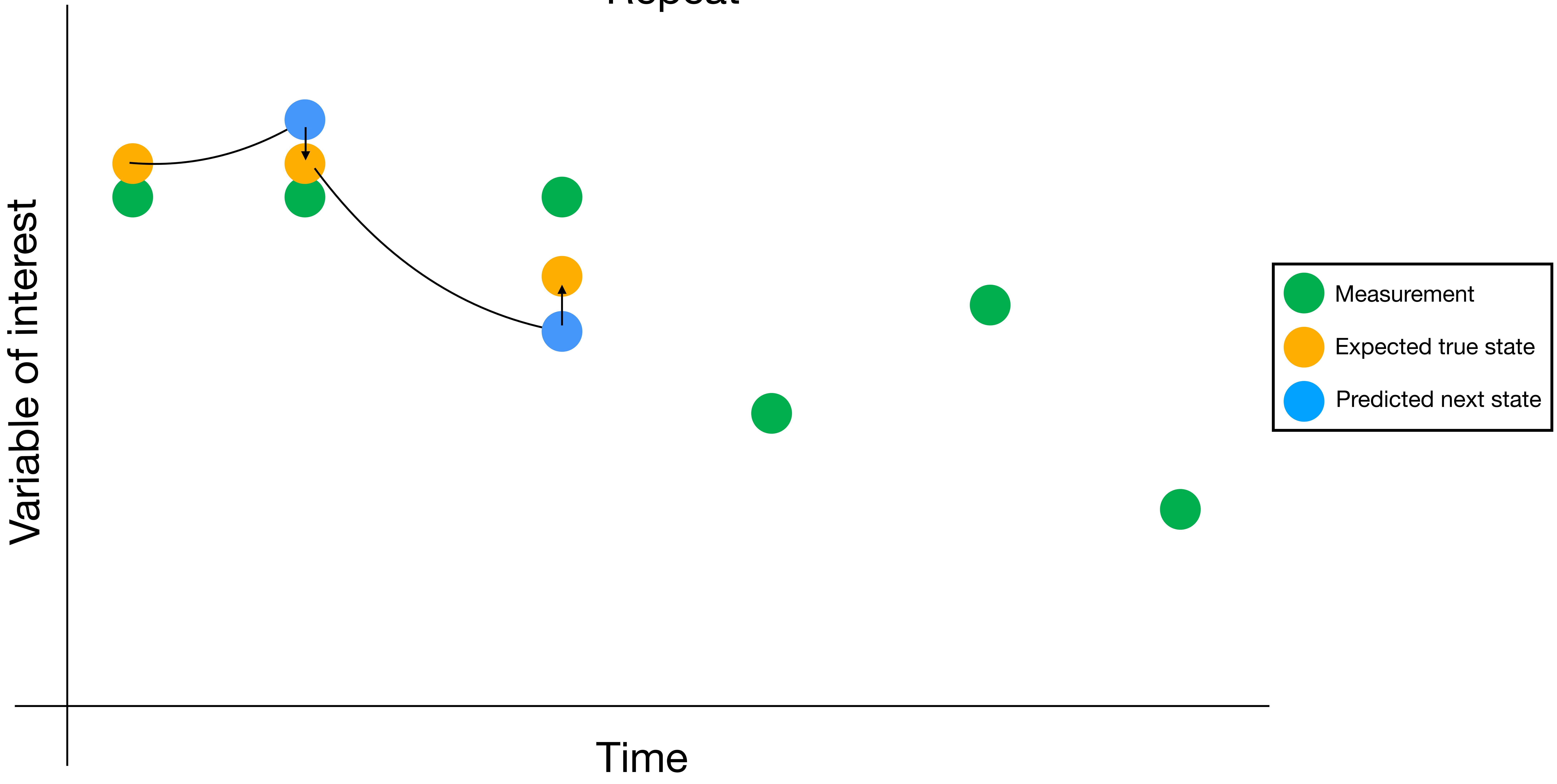
Combine the estimate with the measurement to get a new estimate



Repeat


Variable of interest
Time
Measurement
Expected true state
Predicted next state

Repeat


Variable of interest
Time
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Expected true state
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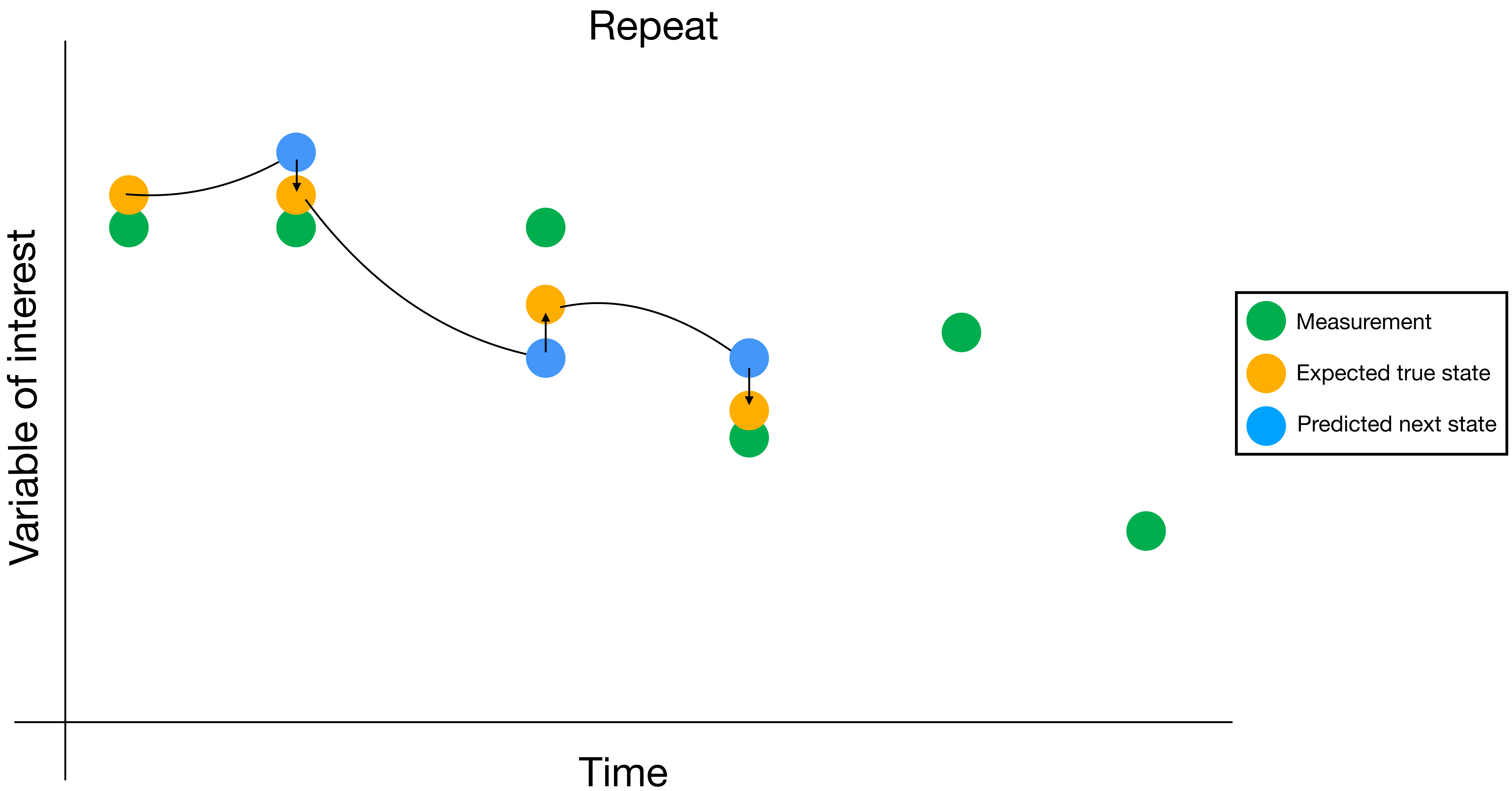
Repeat
Variable of interest
Time
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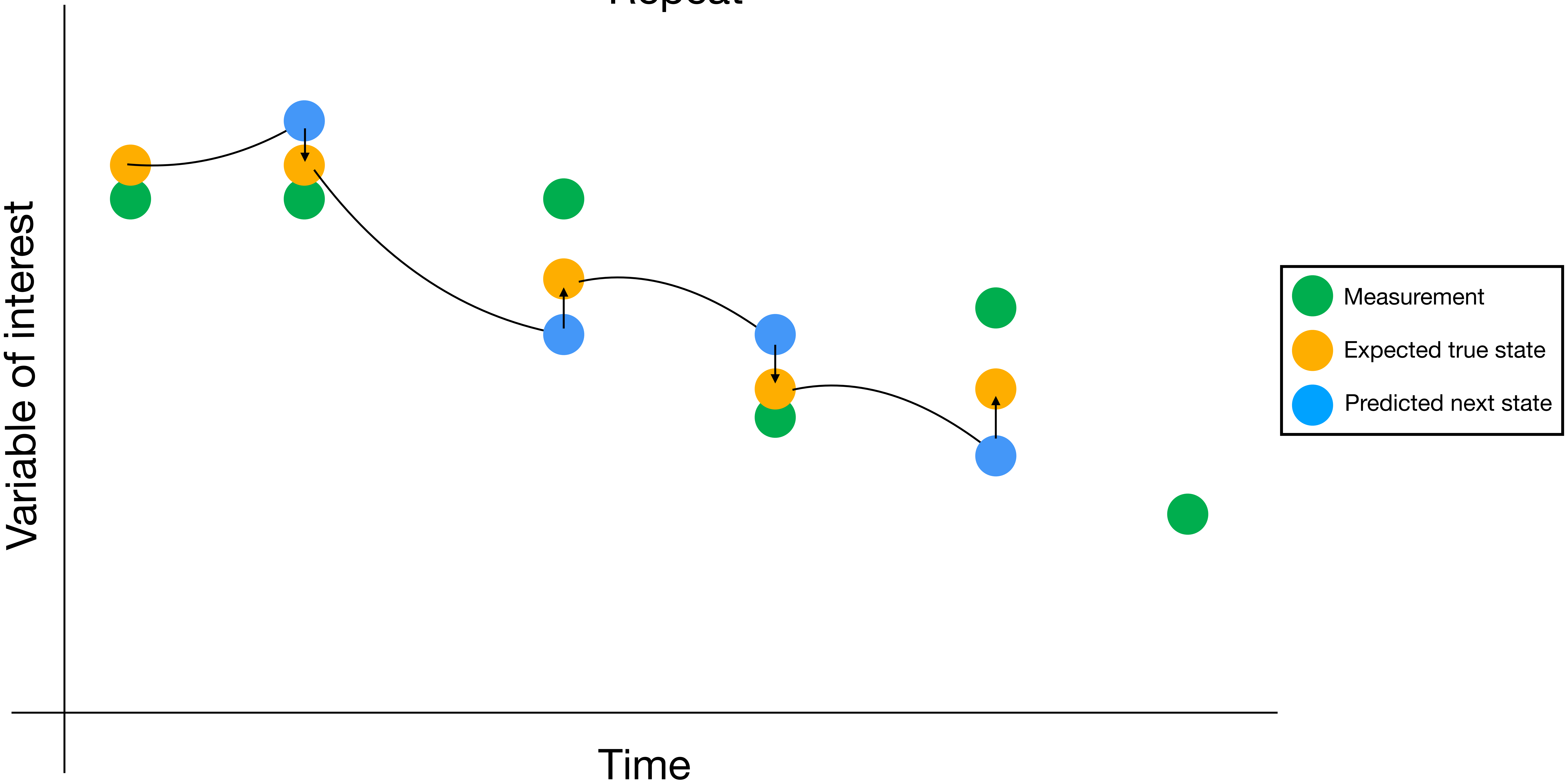
Repeat


Variable of interest
Time
Measurement
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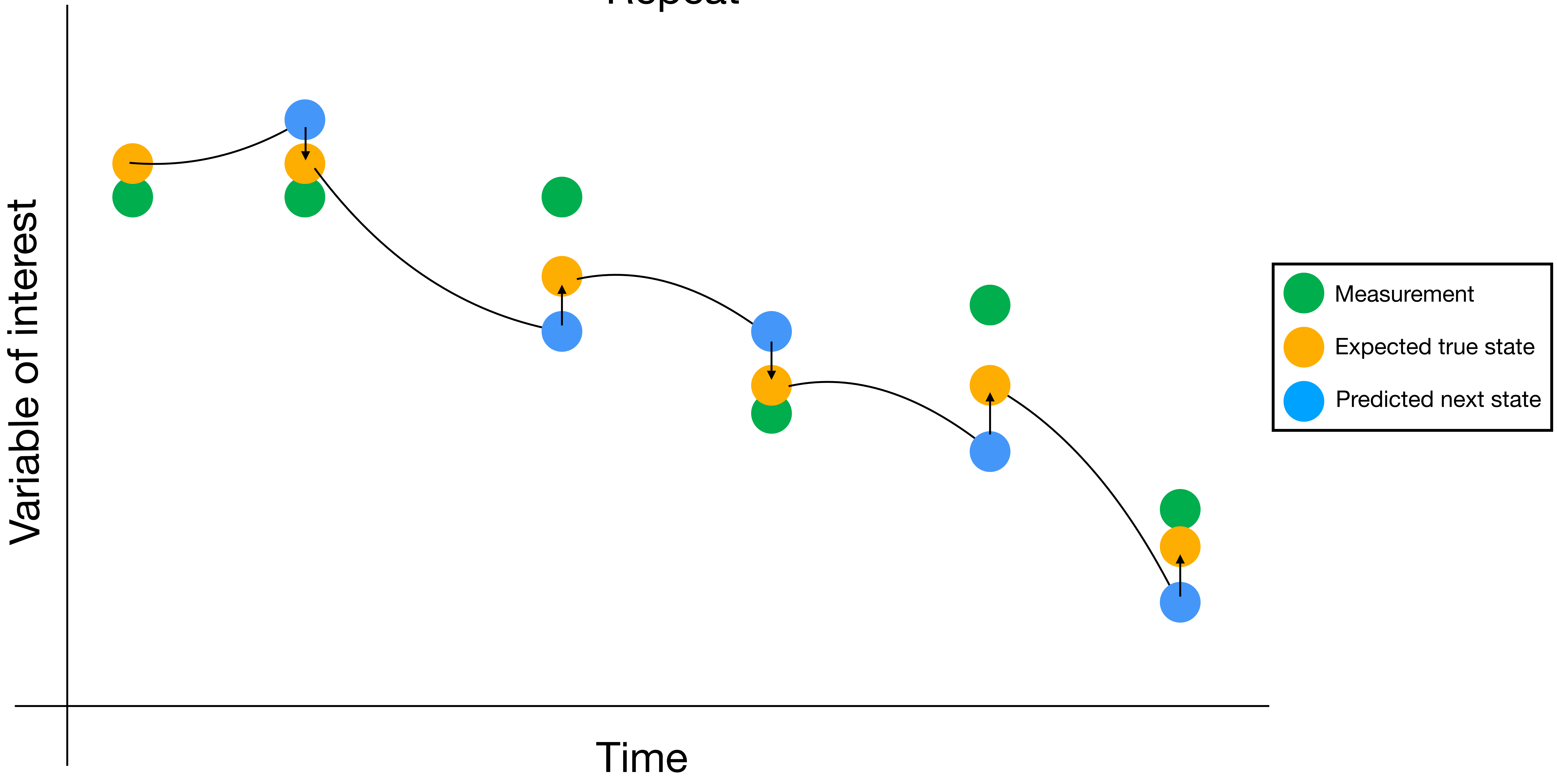
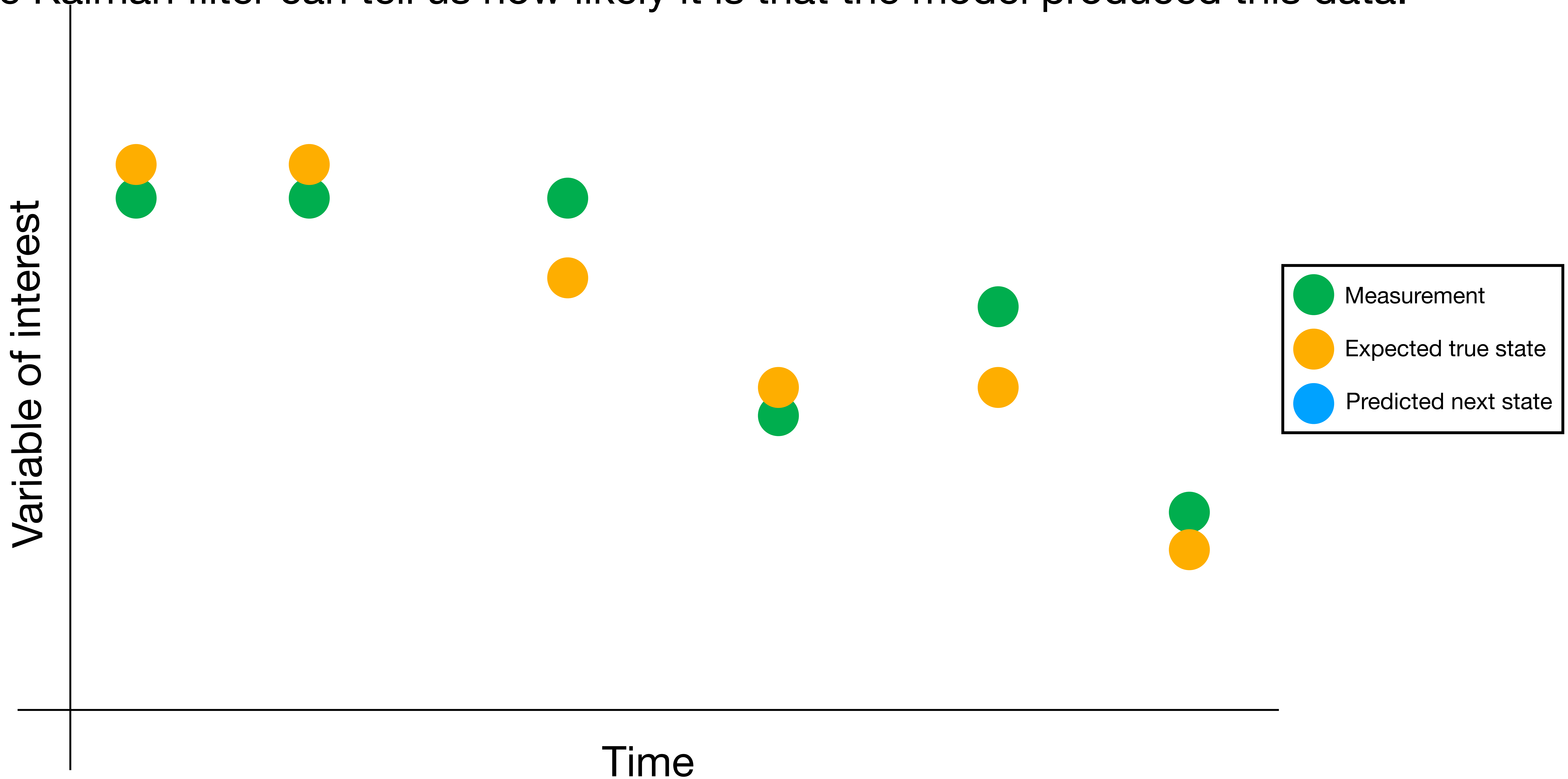
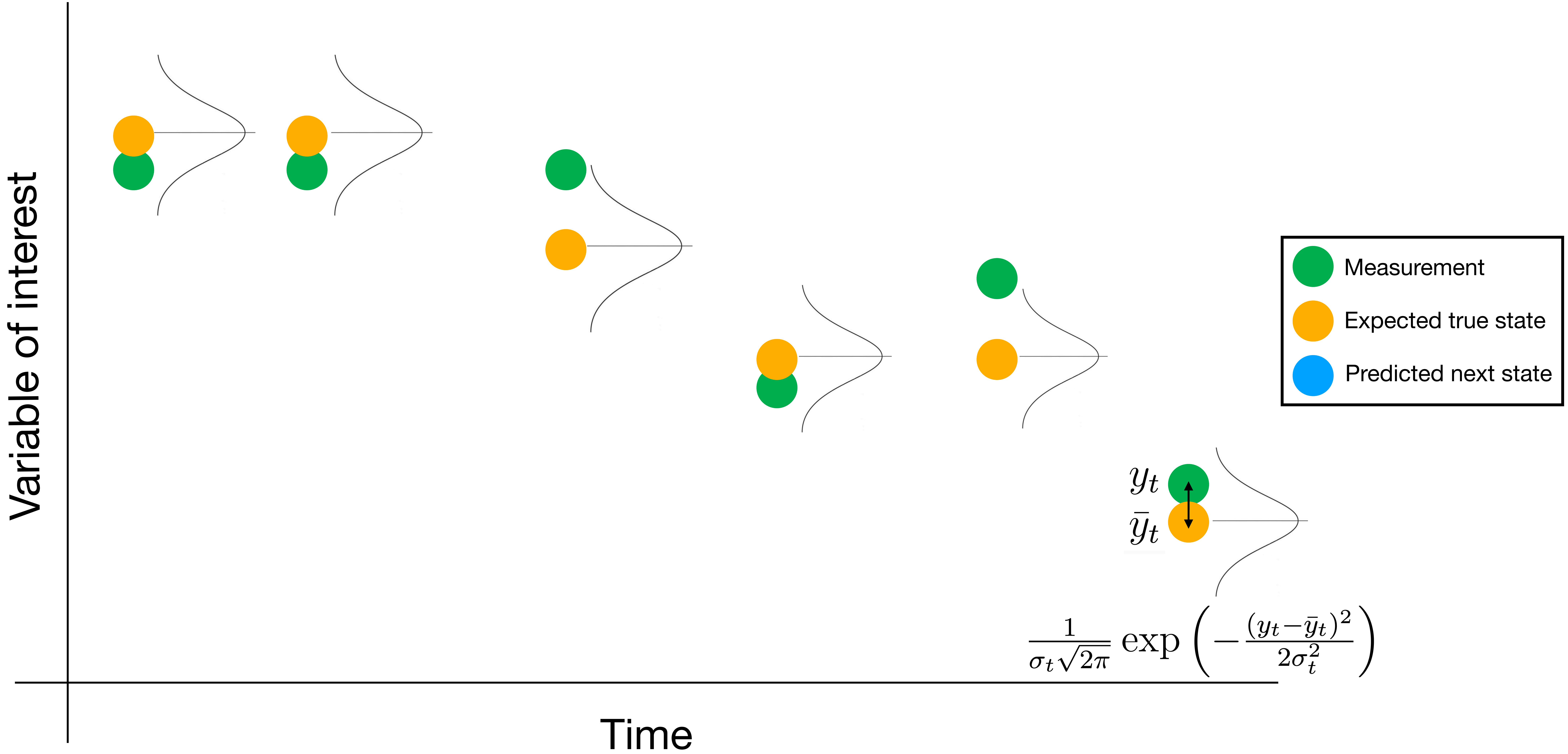
Repeat


Variable of interest
Time
Measurement
Expected true state
Predicted next state

The Kalman filter can tell us how likely it is that the model produced this data.



Work out the probability that the measurements were produced by this model.



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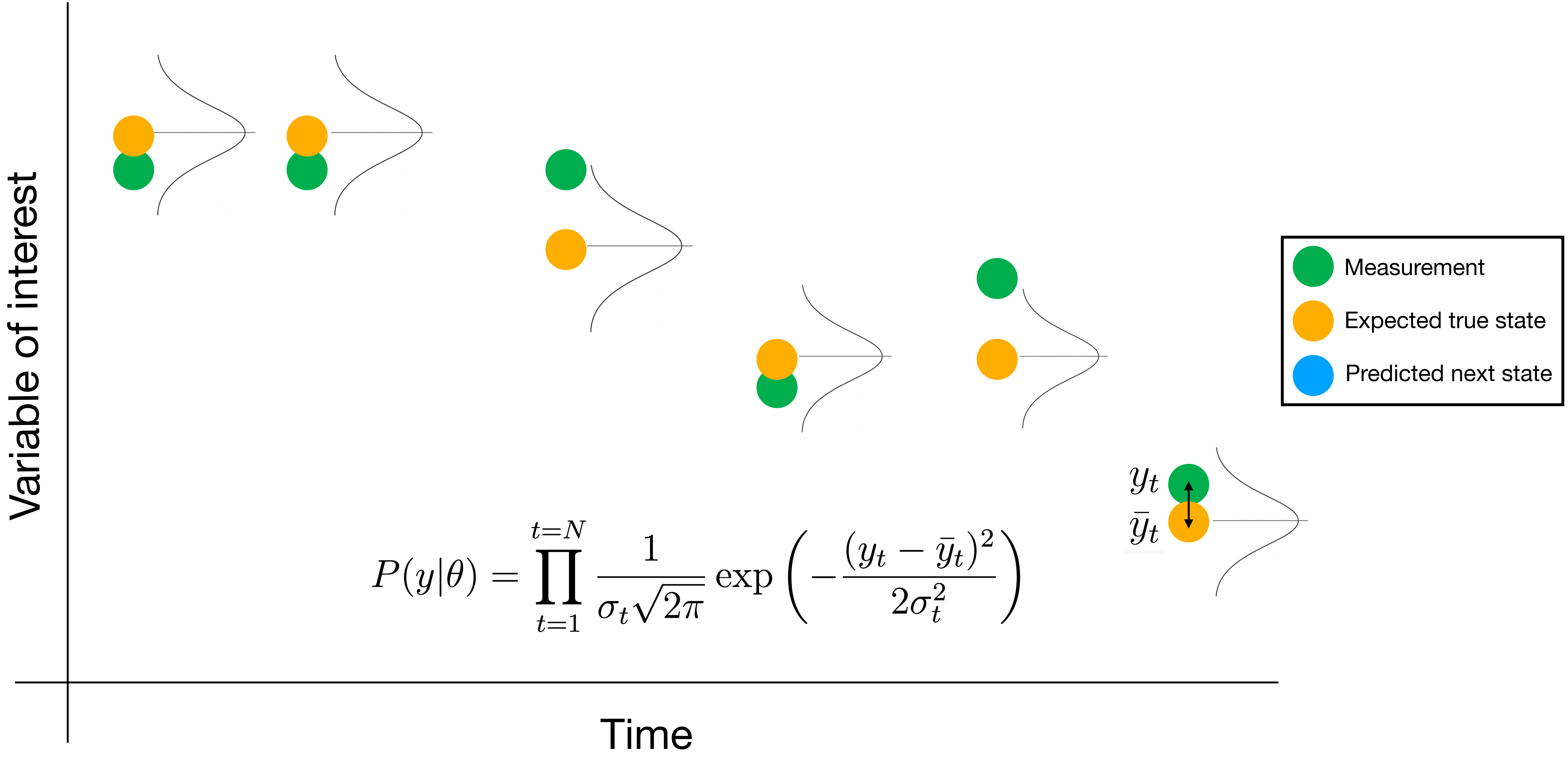

$$P(y|\theta) = \prod_{t=1}^{t=N} \frac{1}{\sigma_t\sqrt{2\pi}} \exp\left(-\frac{(y_t - \bar{y}_t)^2}{2\sigma_t^2}\right)$$

We fit this model to data to find the parameters.

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$$\langle \xi_c(t)\xi_c(t')\rangle = \sigma_c^2 \delta(t-t')$$

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$$\langle \xi_c(t)\xi_s(t')\rangle = 0$$

The parameters are changed from $\tau_c, \tau_s, \sigma_c, \sigma_s, N_c$, and N_s to the more physically meaningful or useful parameter choices $r, \tau, \sigma_c^2/I_c^2, \sigma_s^2/I_s^2, \langle\dot{\Omega}_c\rangle$ and $\langle\Omega_c - \Omega_s\rangle$.

$$r = \frac{\tau_s}{\tau_c} = \frac{I_s}{I_c}$$

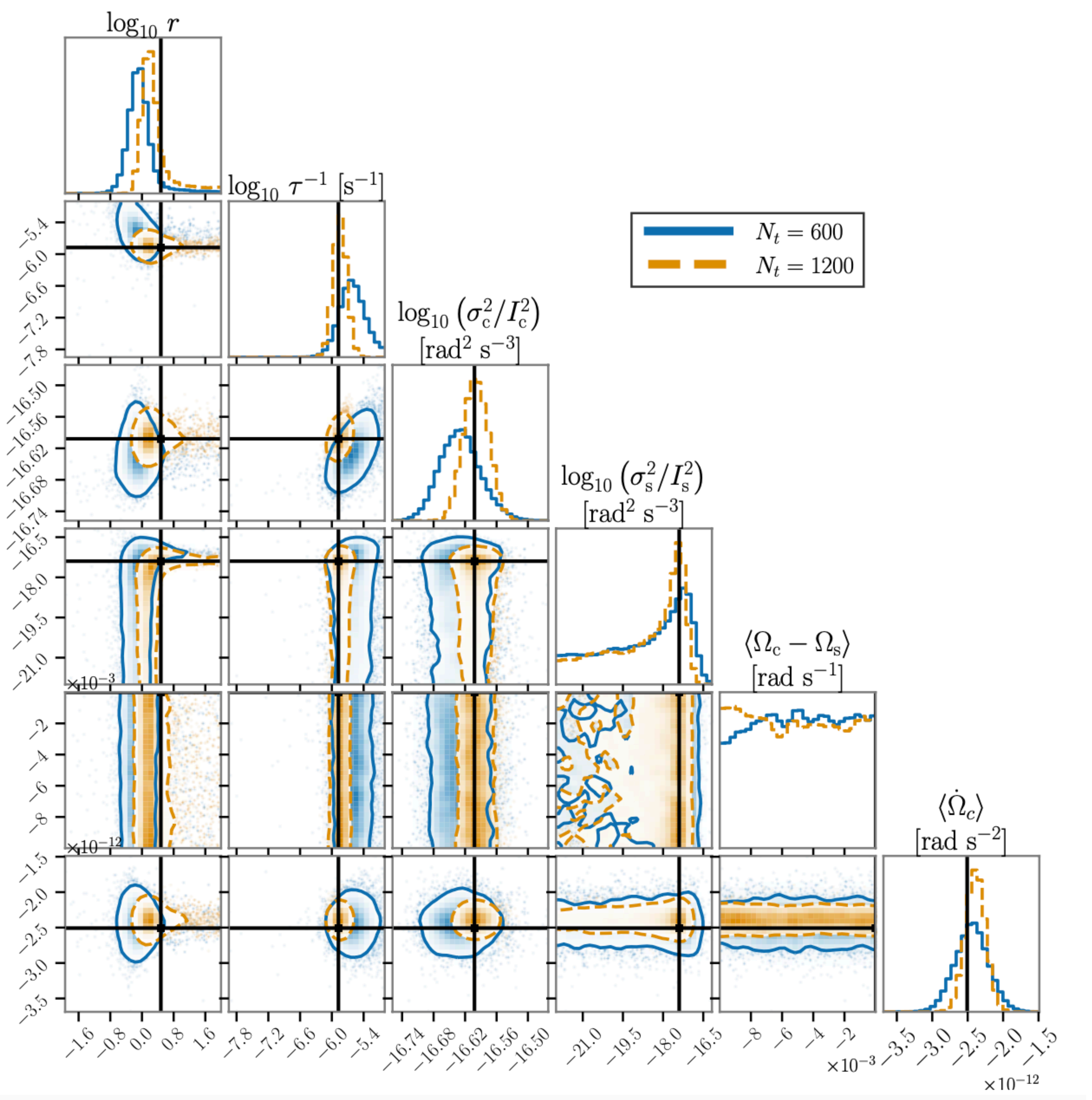
$$\frac{1}{\tau} = \frac{1}{\tau_c} + \frac{1}{\tau_s}$$

$$\langle\dot{\Omega}_c\rangle = \frac{\tau_c N_c/I_c + \tau_s N_s/I_s}{\tau_c + \tau_s}$$

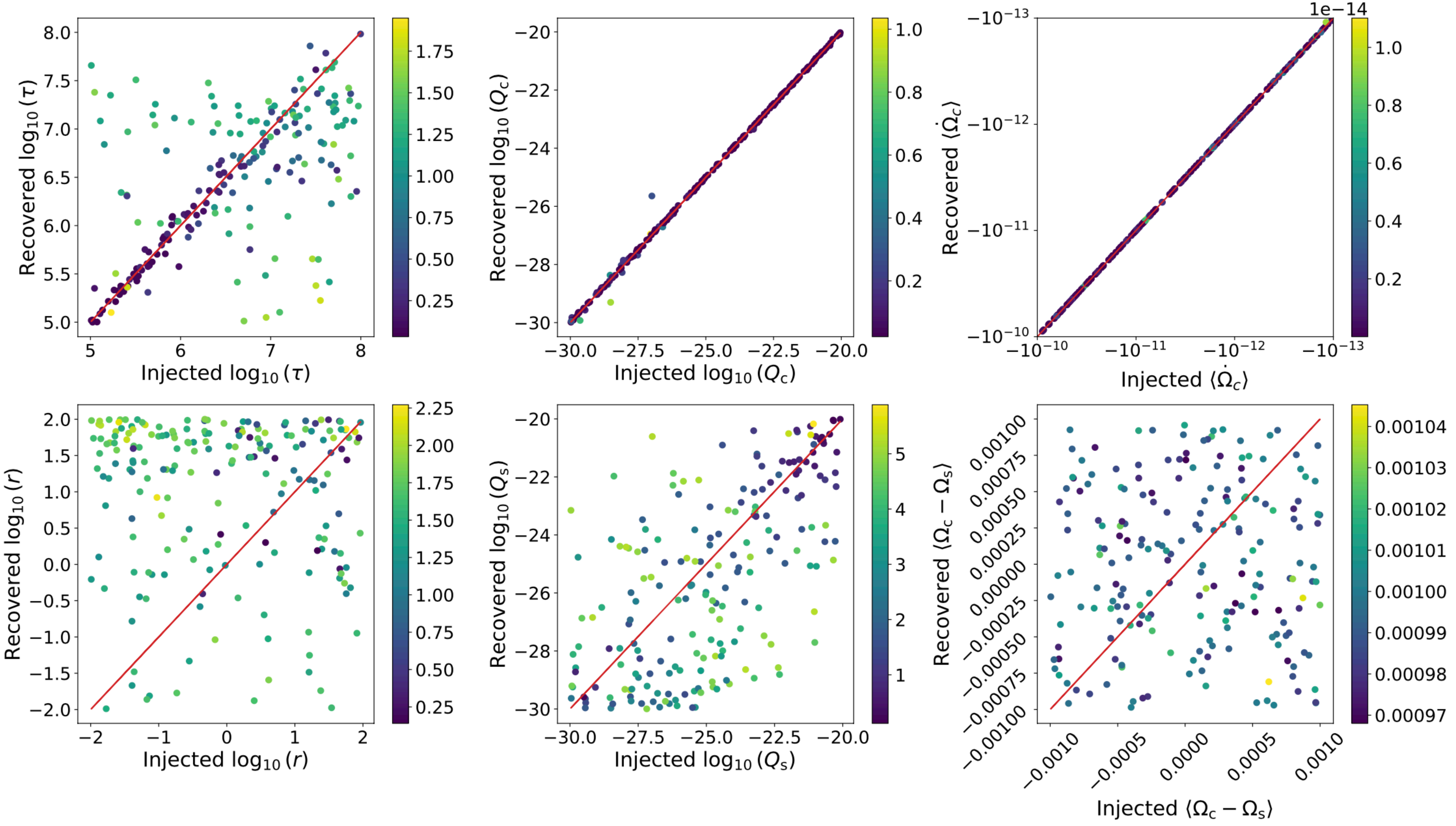
$$\langle\Omega_c - \Omega_s\rangle = \tau\left(\frac{N_c}{I_c} - \frac{N_s}{I_s}\right)$$

Probability distribution for parameters calculated using simulated neutron star frequency data.

Only crust data is available here. Not all parameters can be recovered.

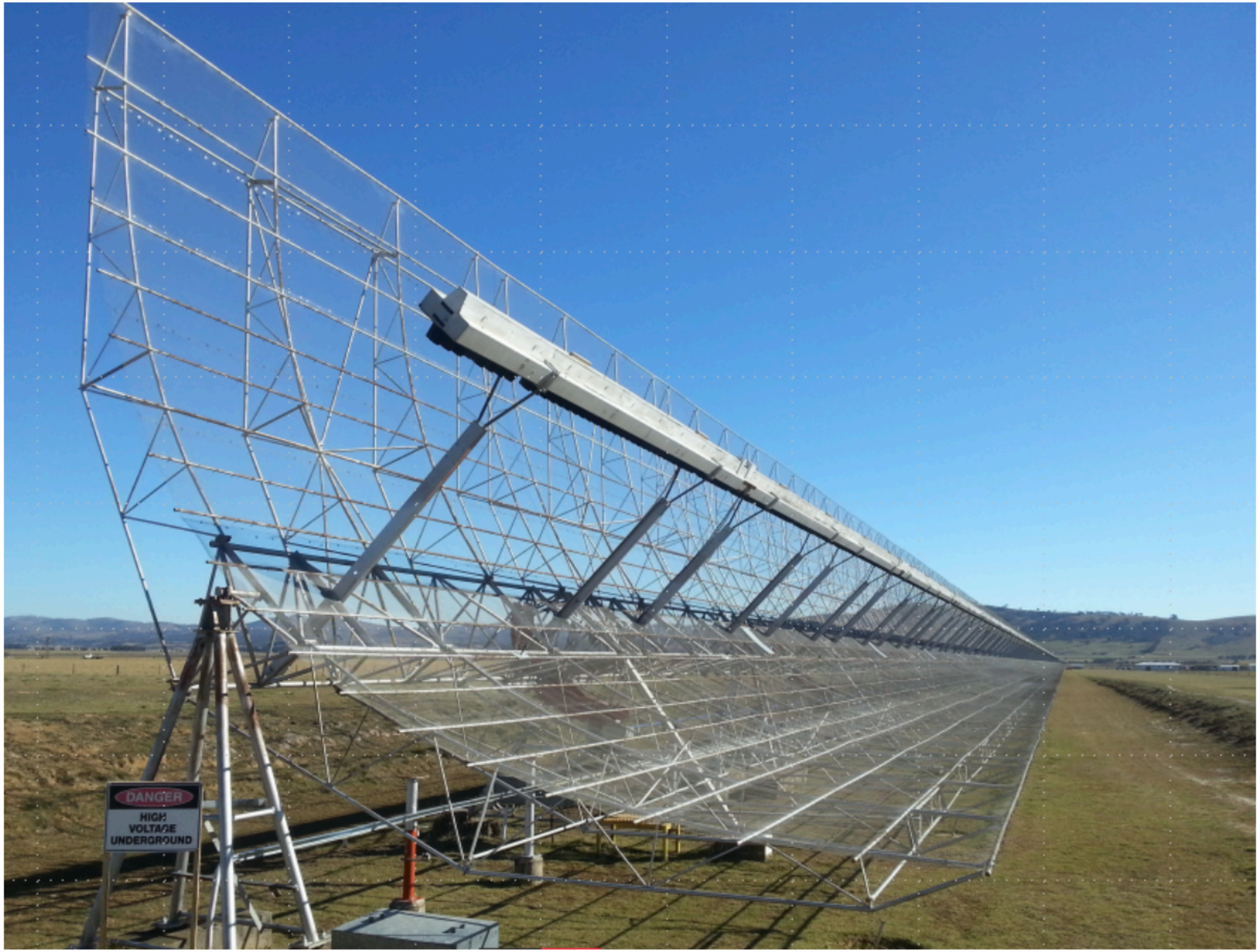


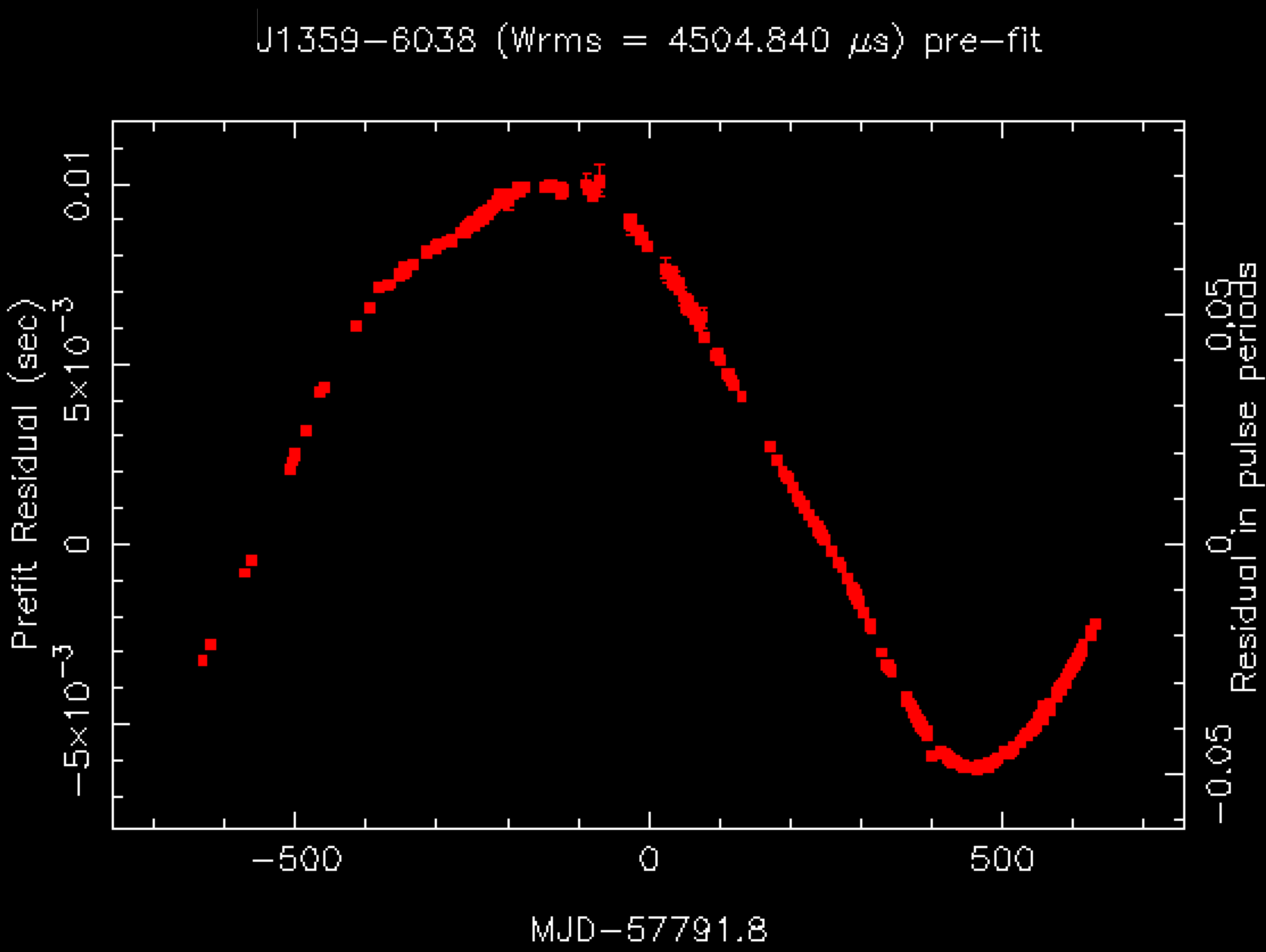
Recovered two-component model parameters for 200 simulations.



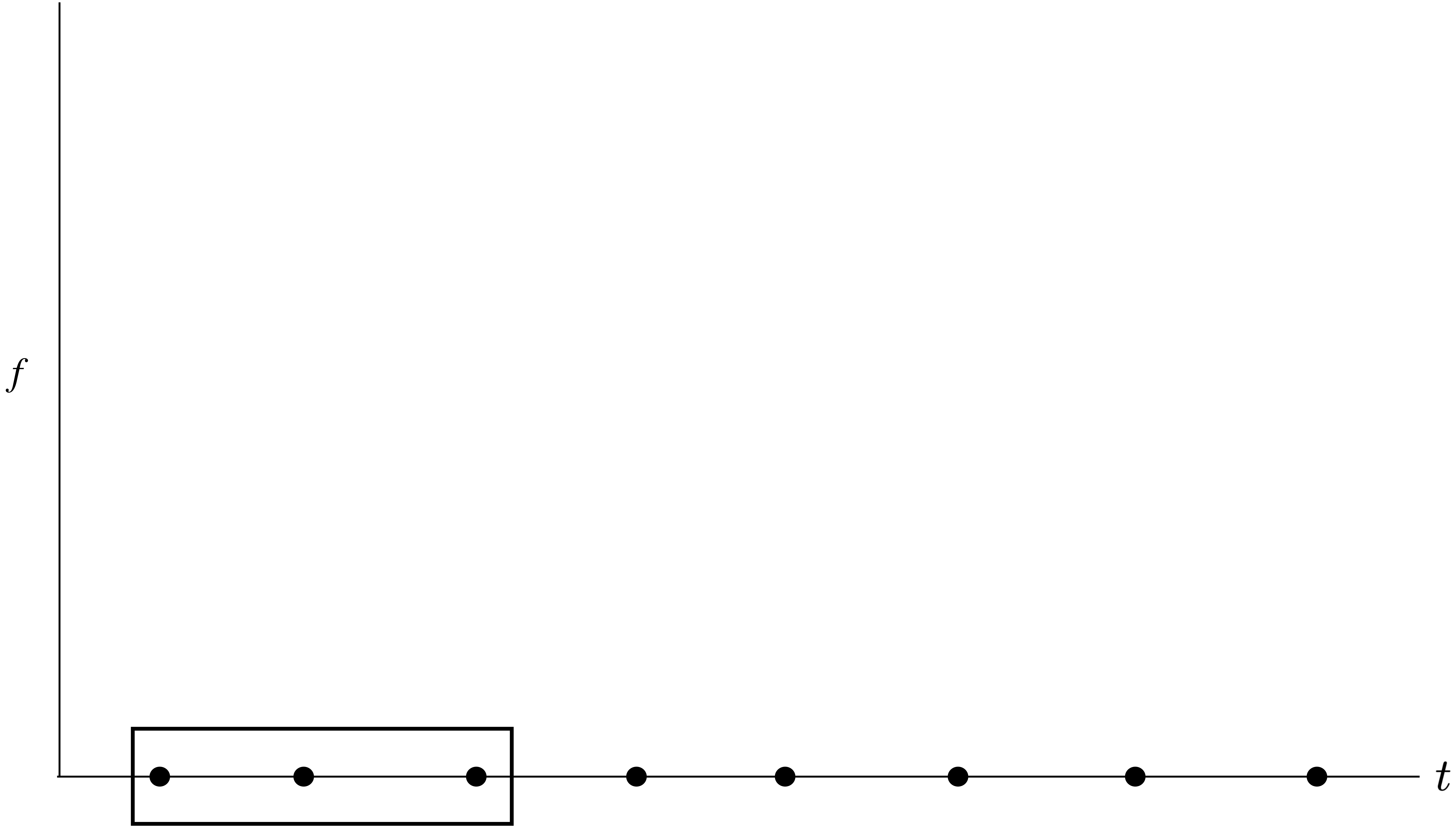
Real data

Data comes from the UTMOST pulsar timing programme carried out by Molonglo radio telescope.




J1359−6038 (Wrms = 4504.840 μs) pre−fit
Prefit Residual (sec)
Residual in pulse periods
MJD−57791.8
0.01
5×10−3
0
−5×10−3
0.05
0
−0.05
−500
0
500

We fit frequencies to short subsets of the TOAs



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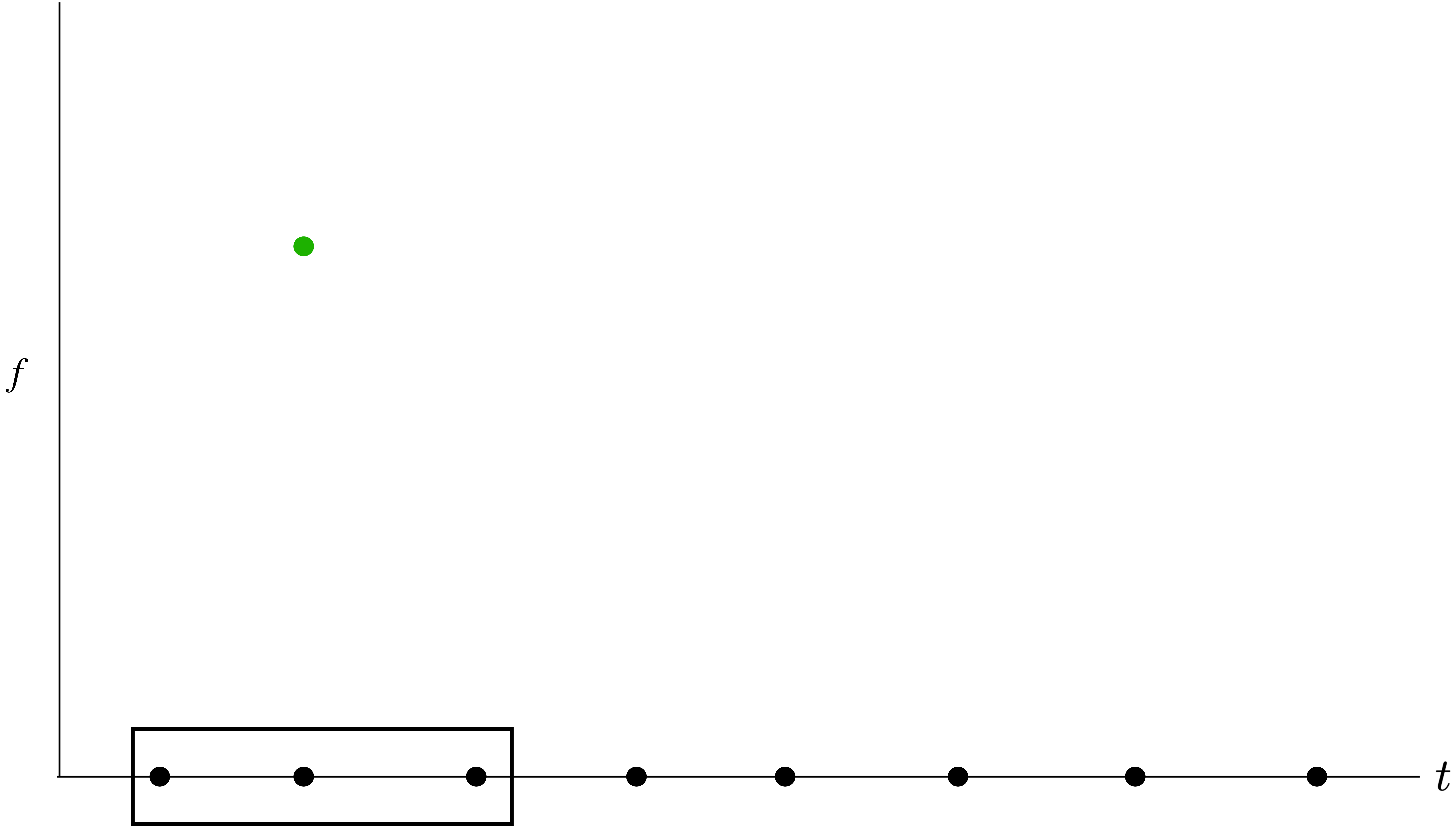

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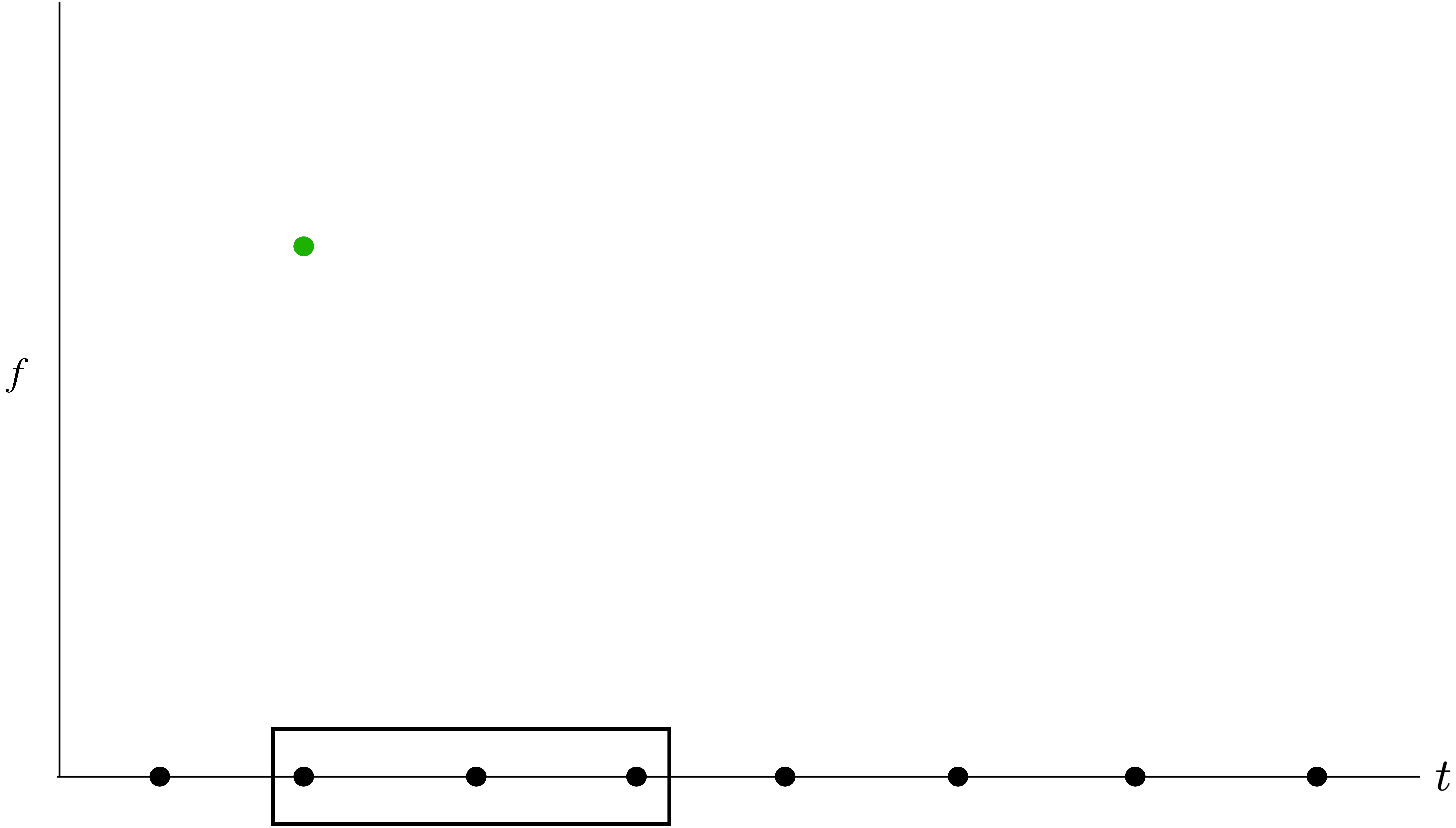

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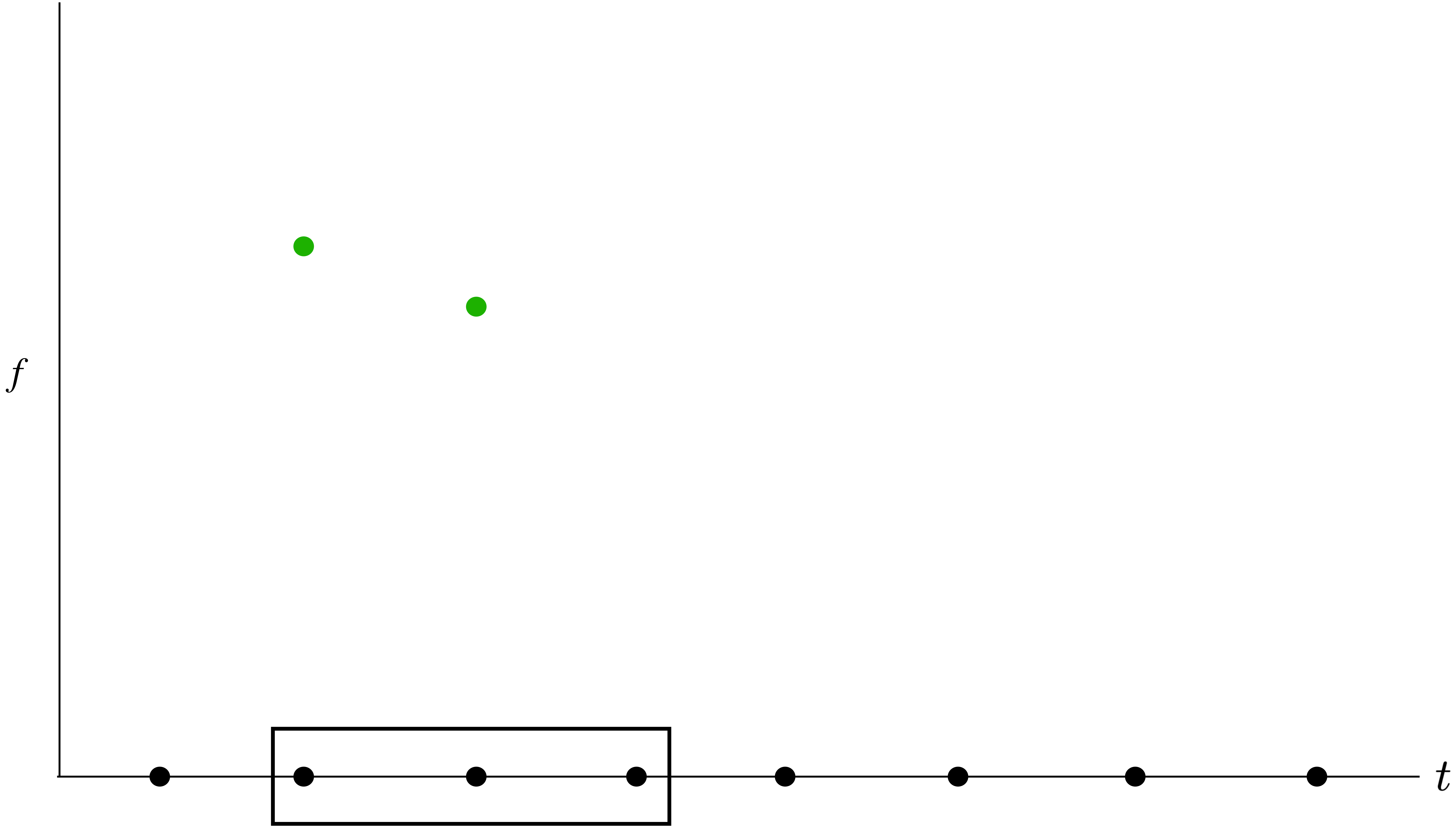

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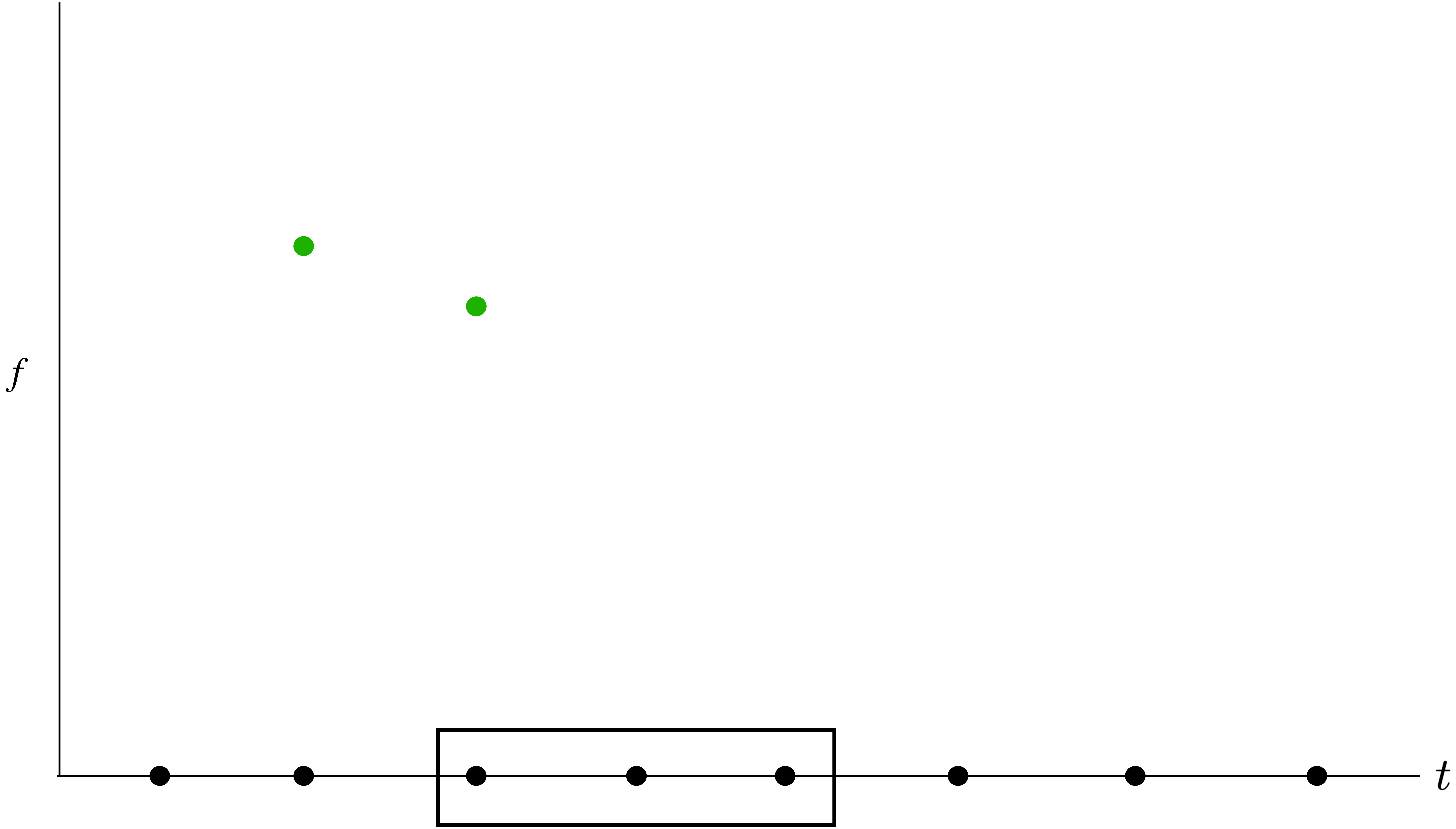

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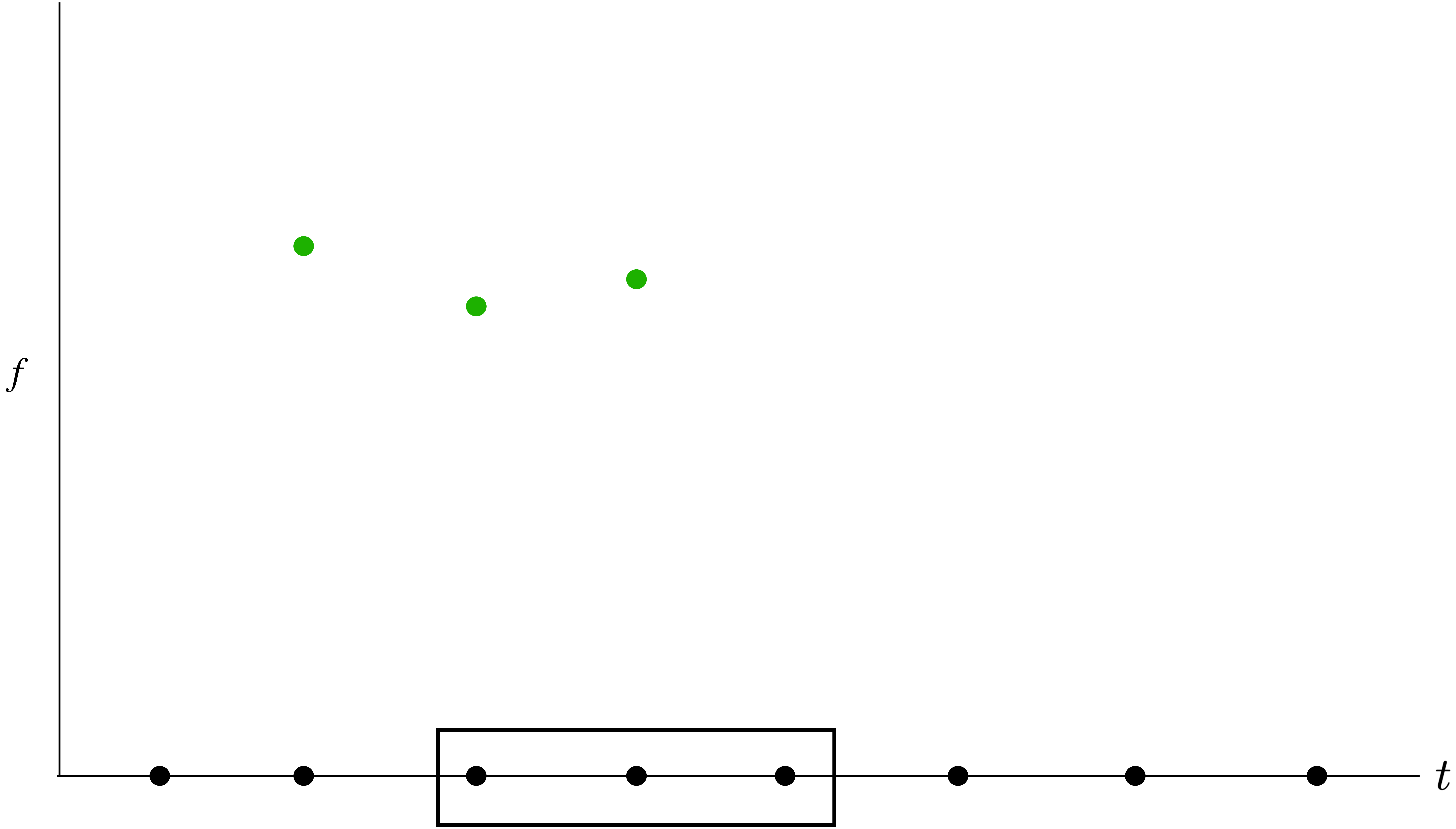

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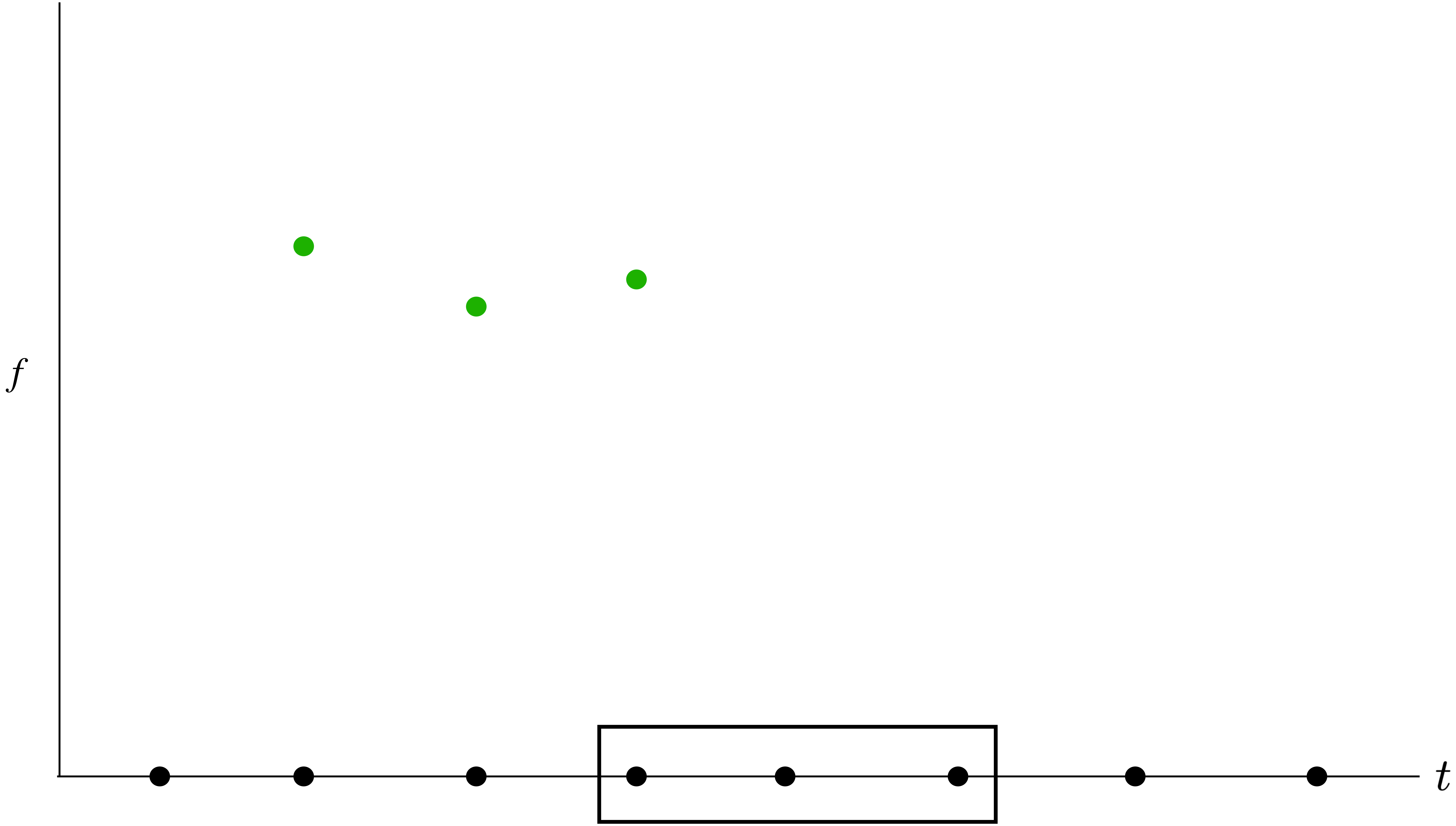

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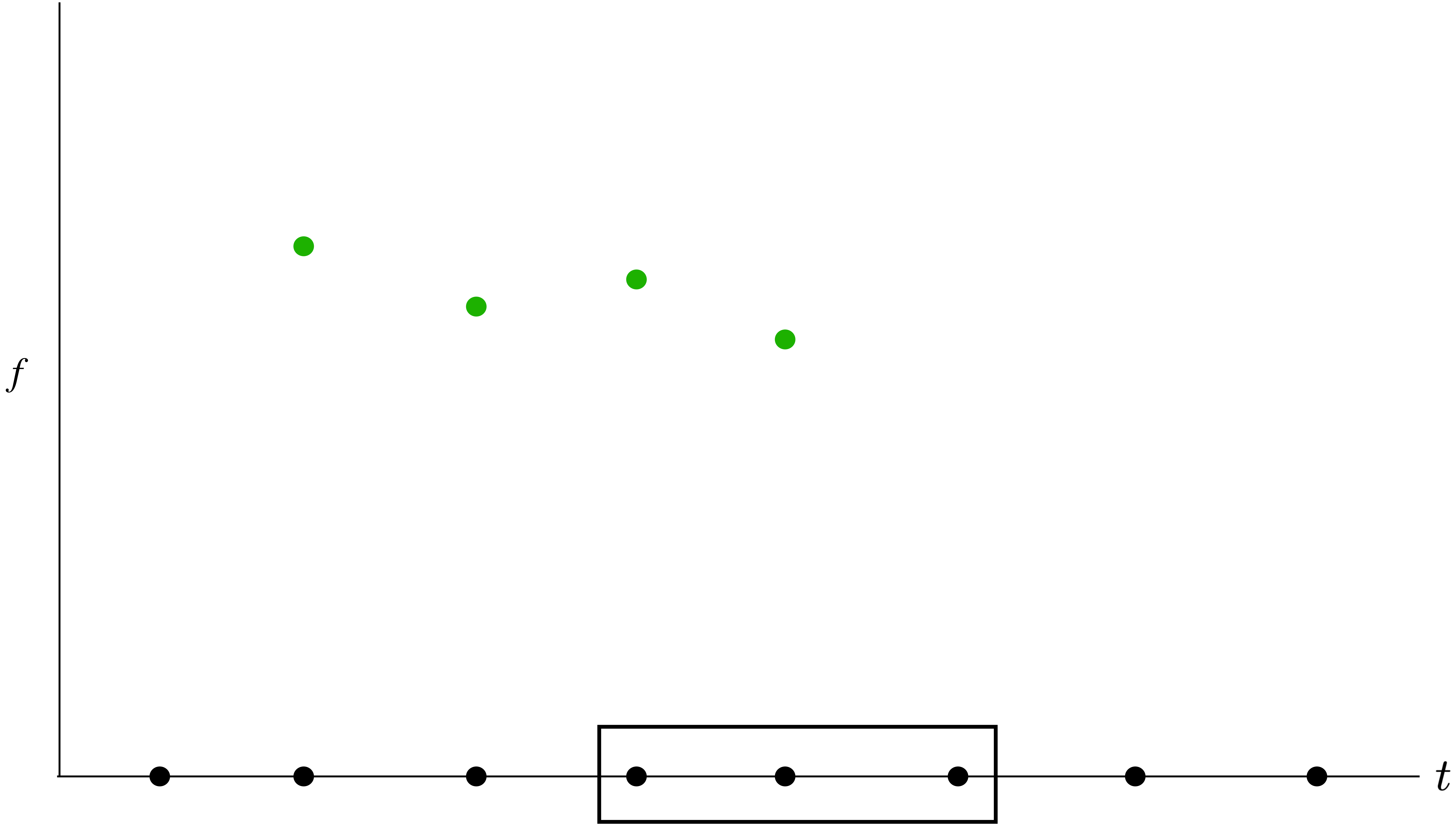

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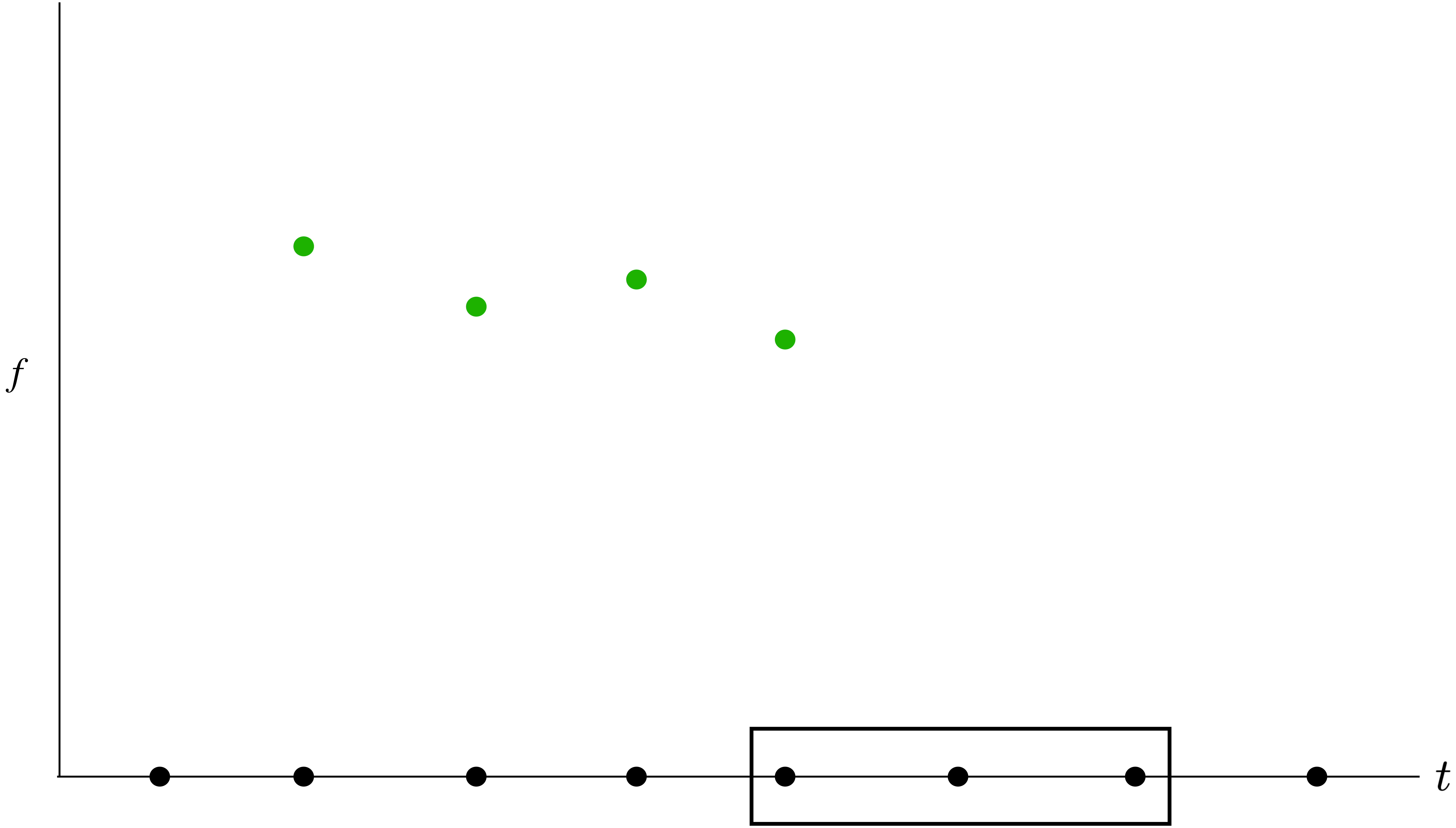

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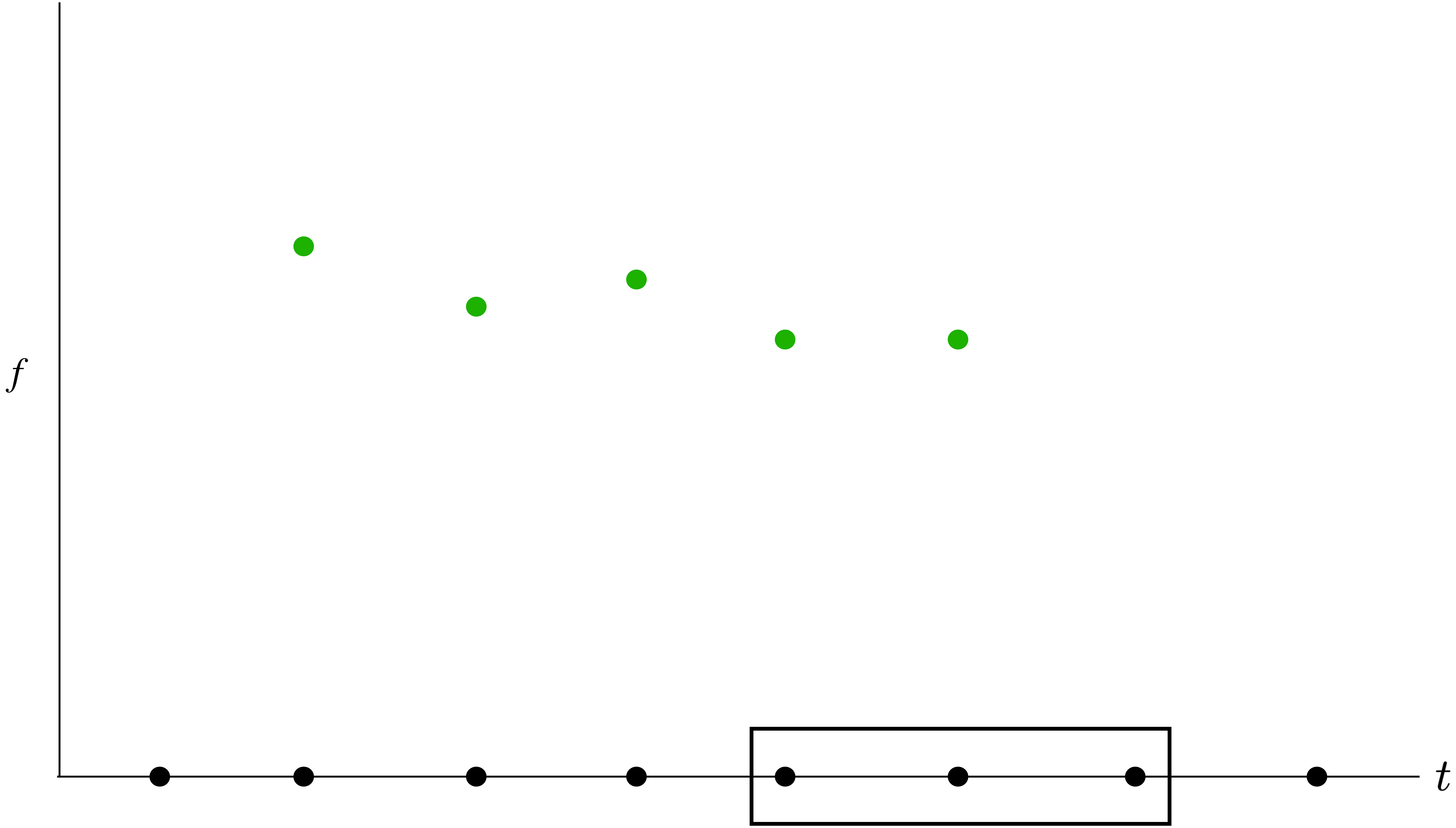

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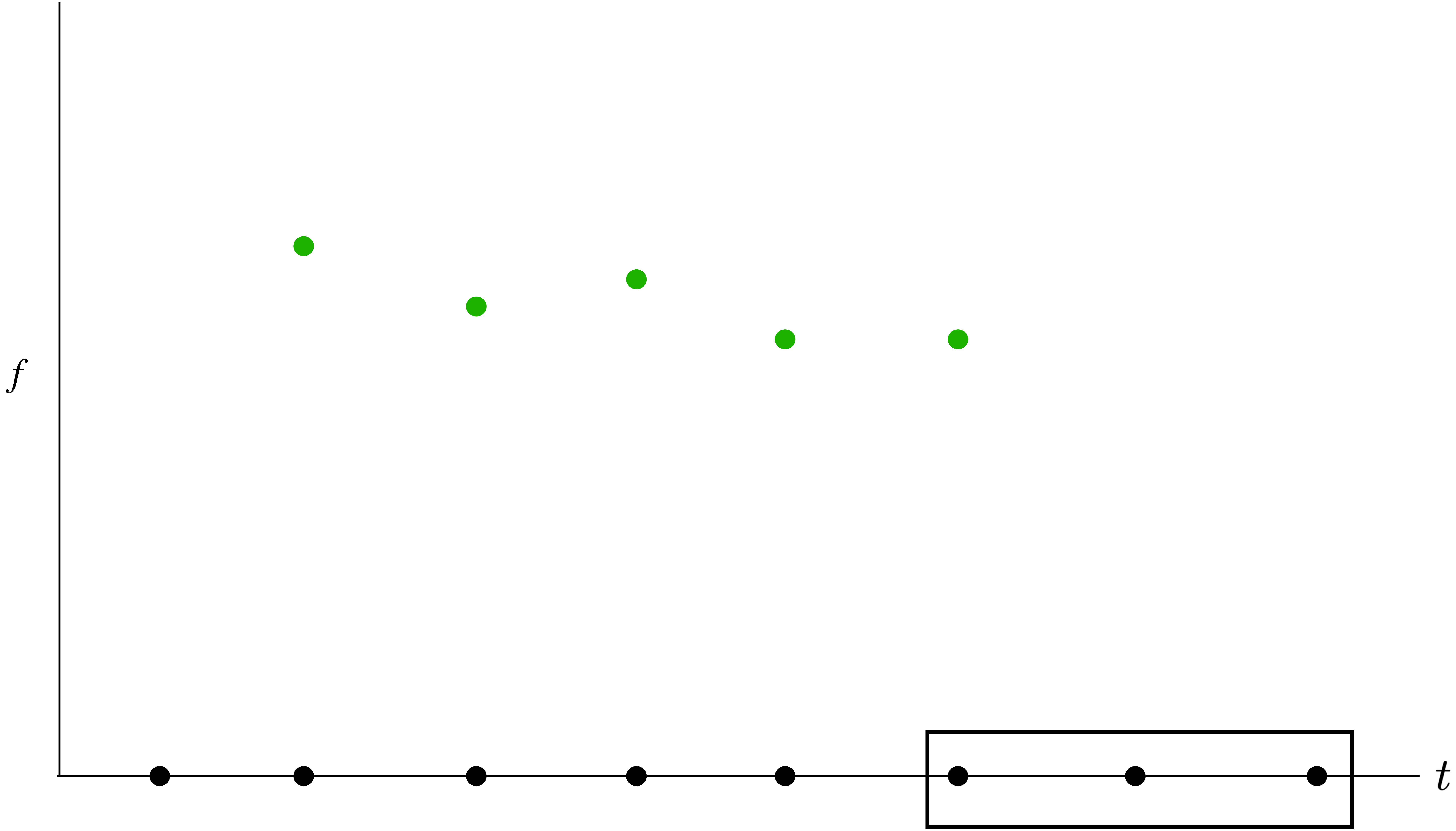

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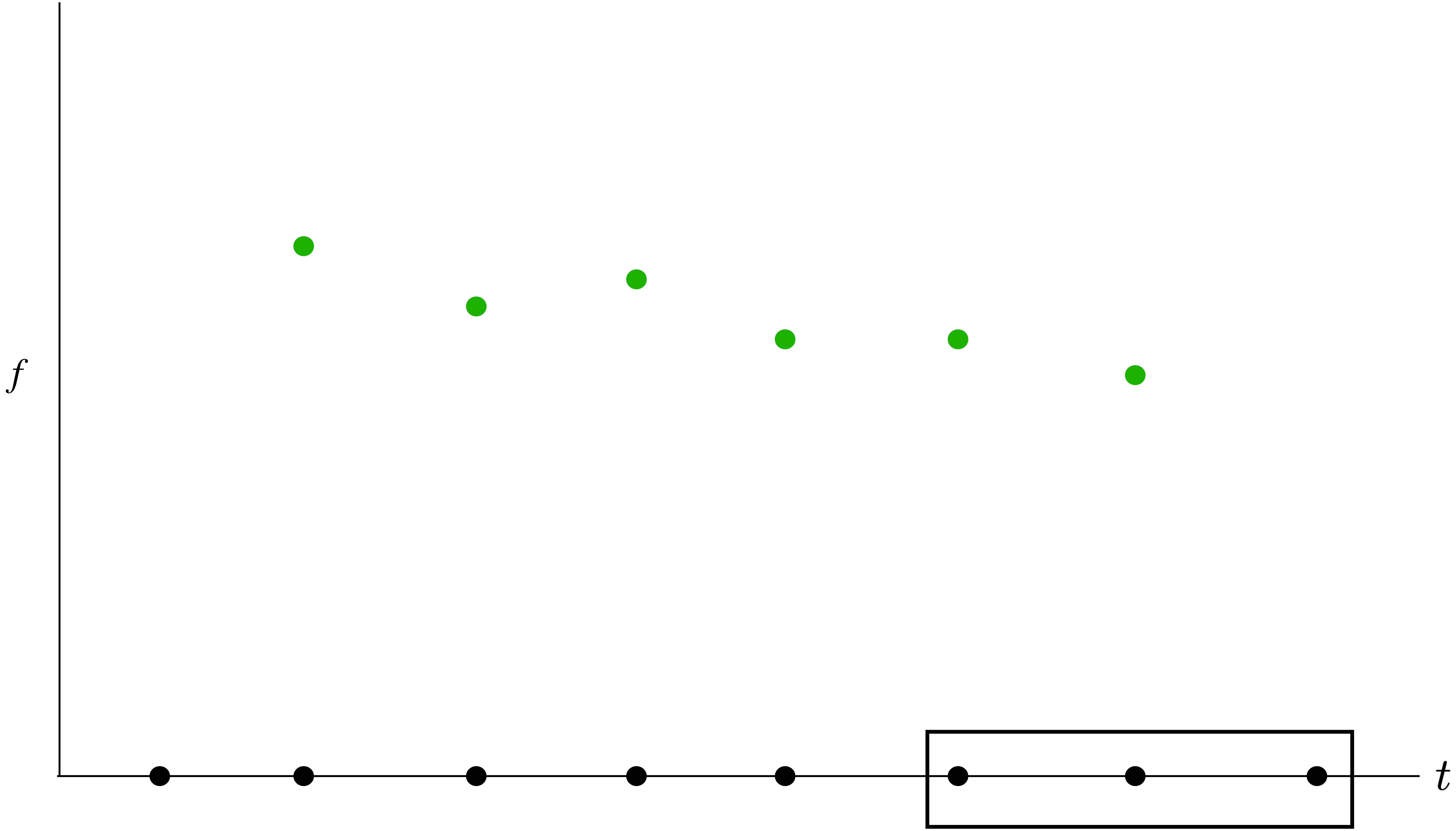

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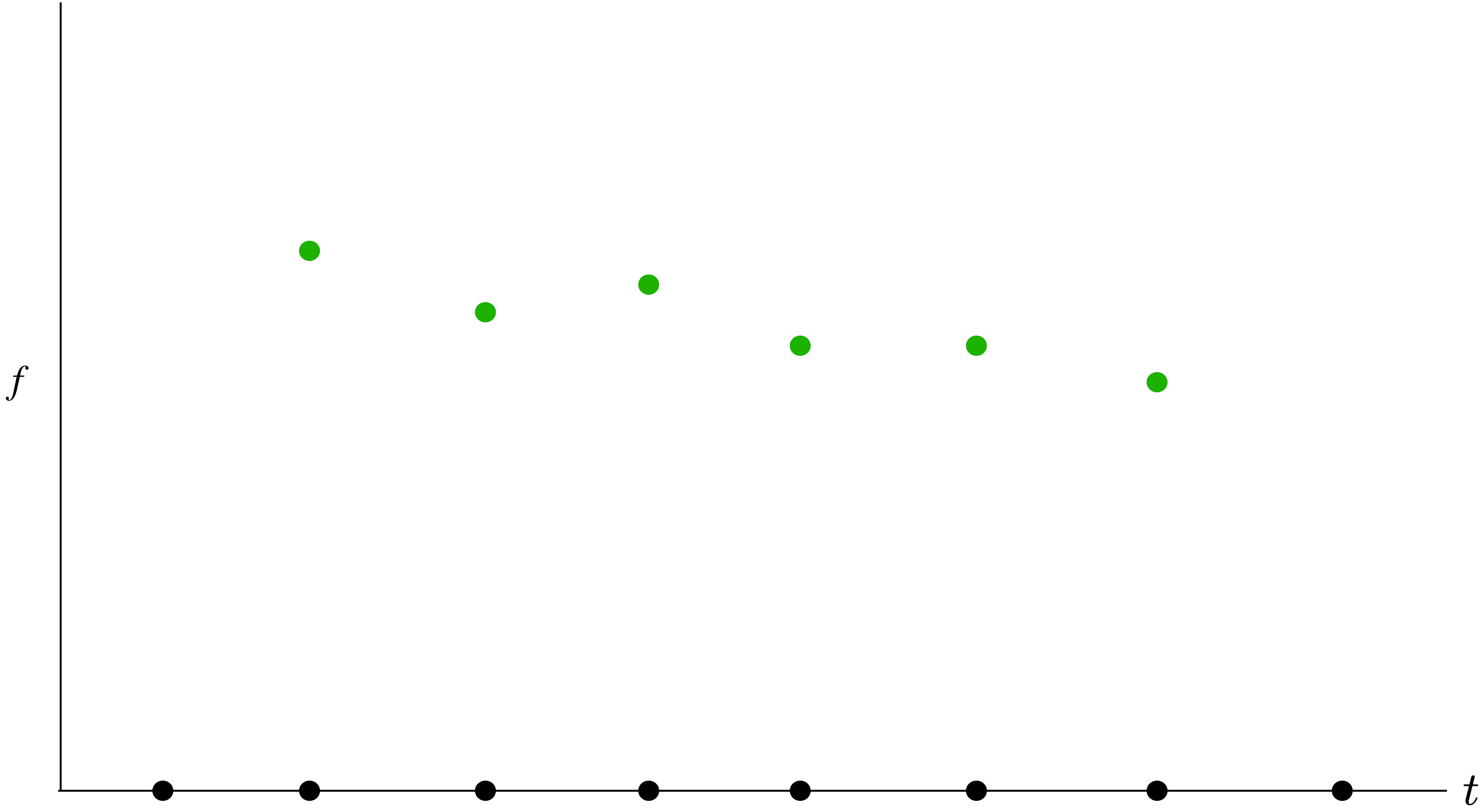
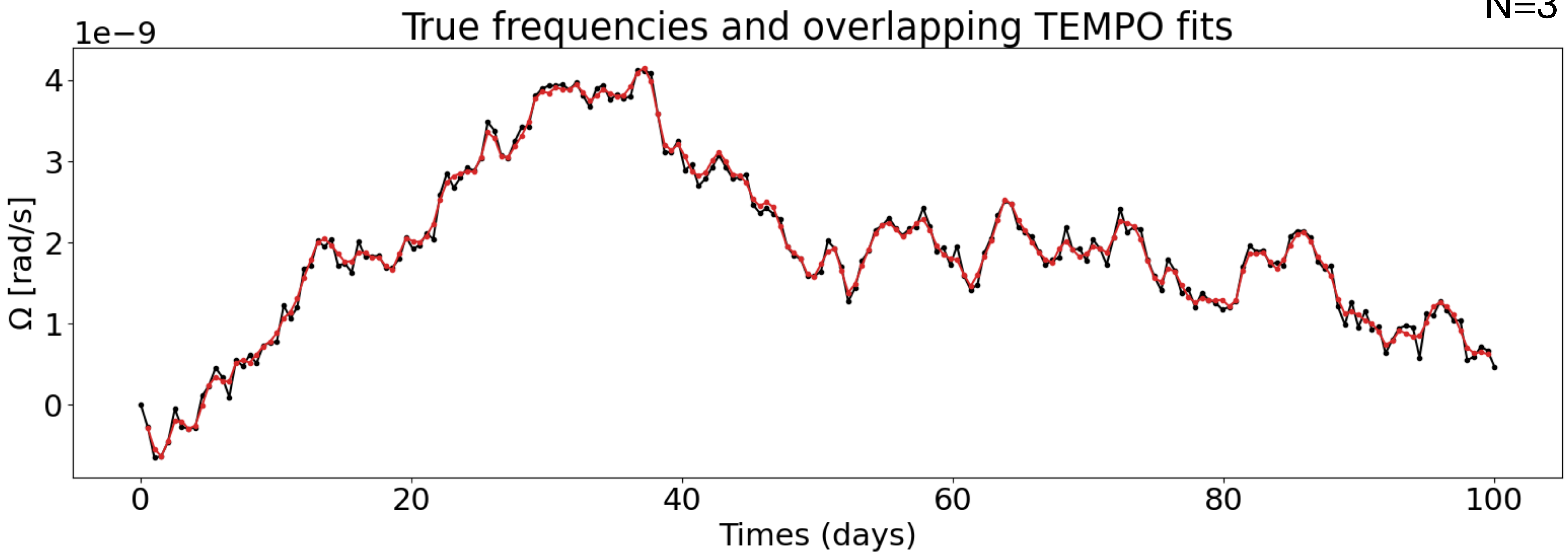


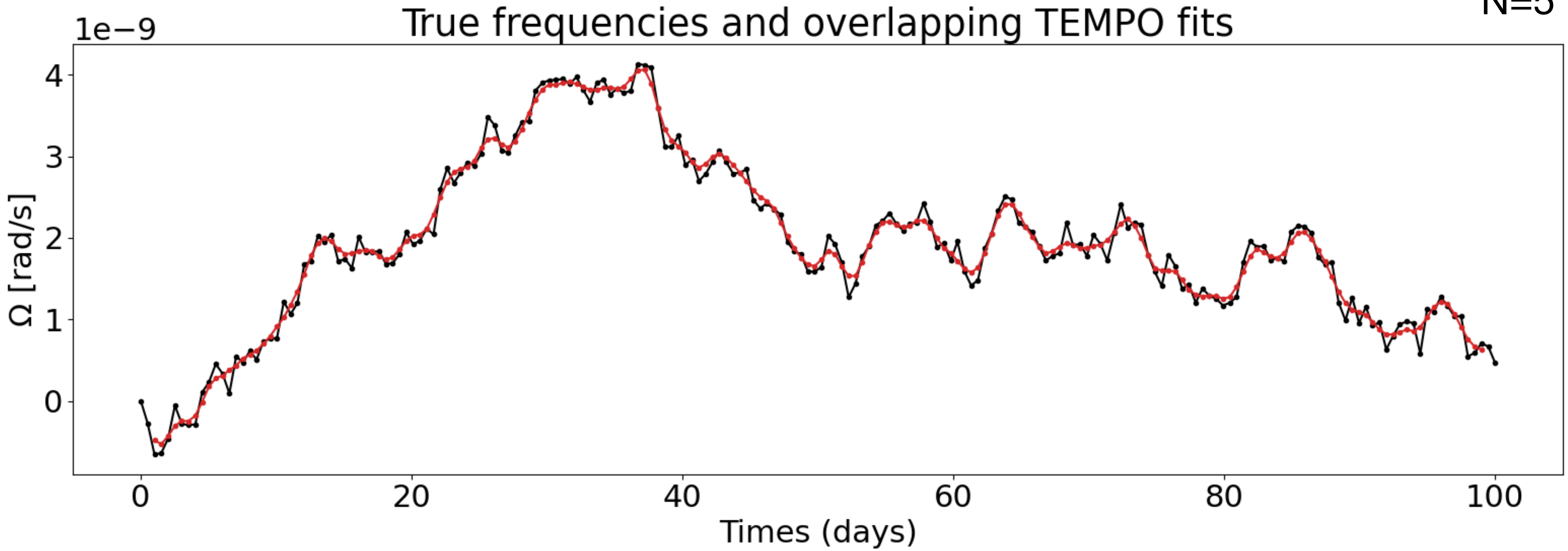
N=3



Fitting frequencies to TOAs smoothes out the data. It averages out the frequency over the period of fitting.

This removes details of the timing noise.

N=5



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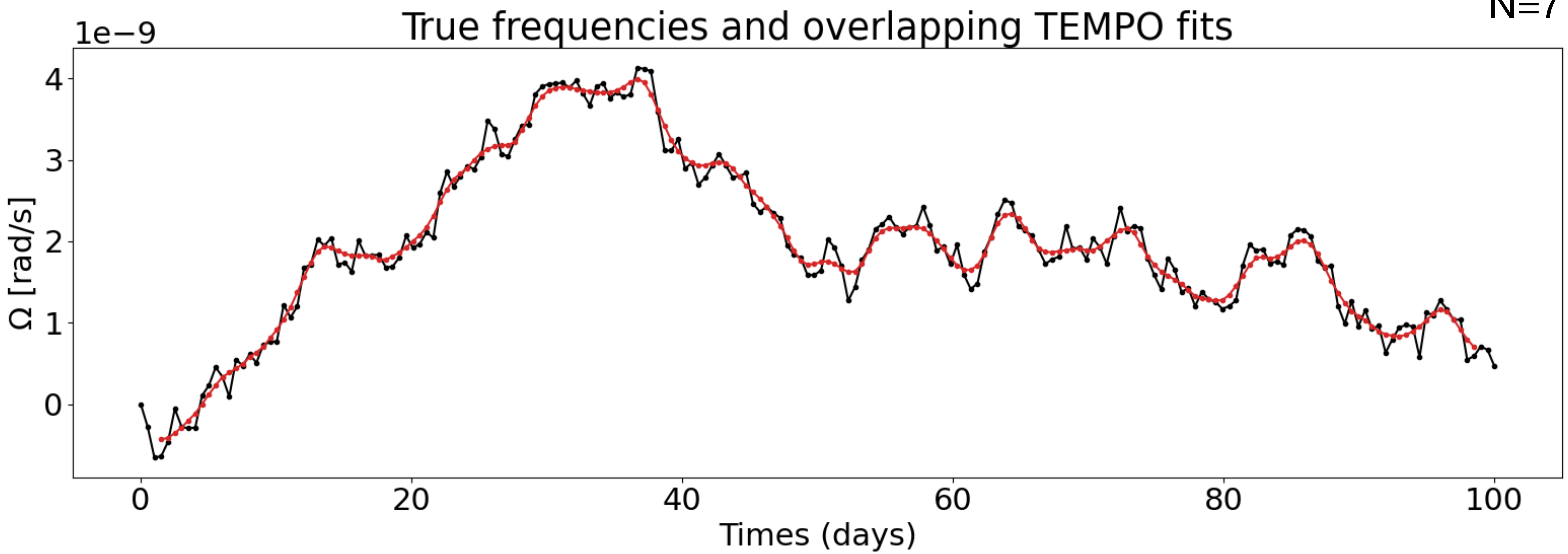
N=7



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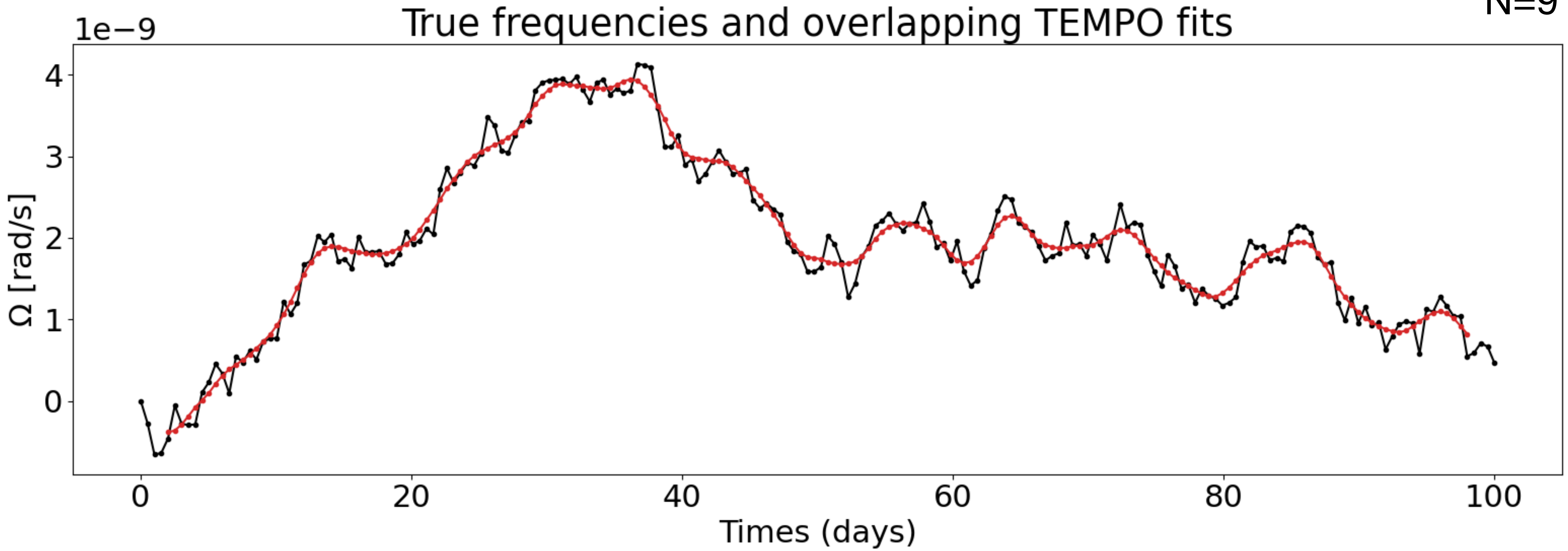
N=9



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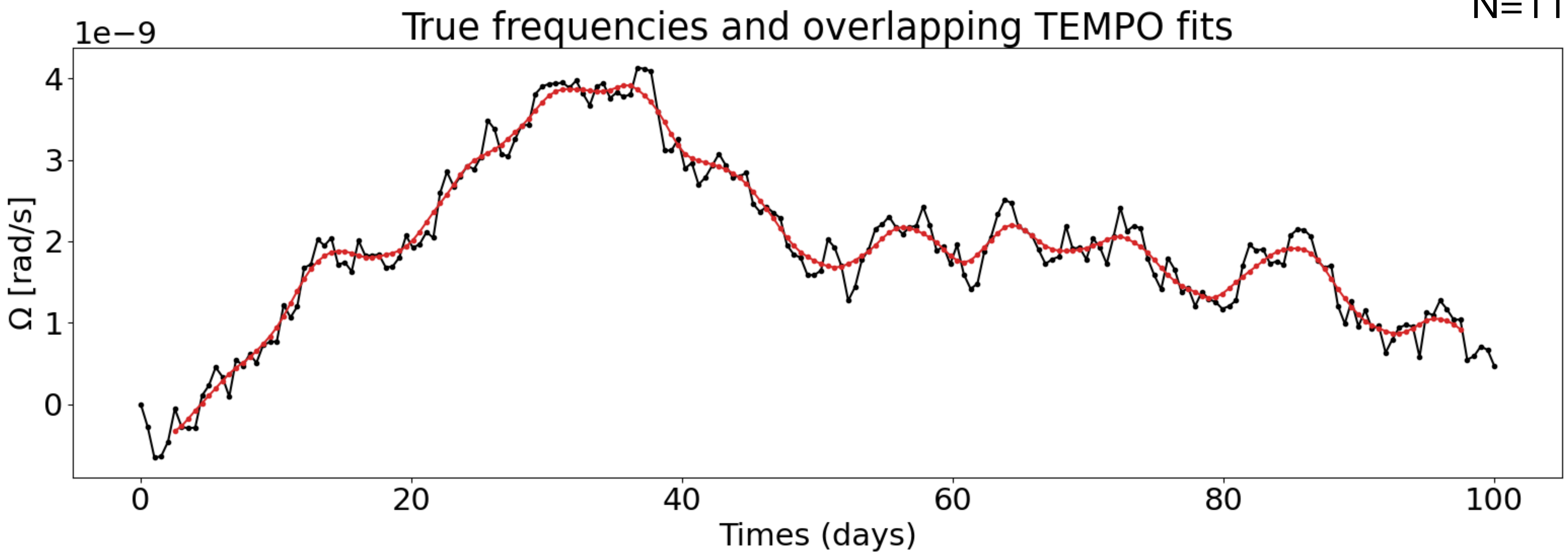
N=11



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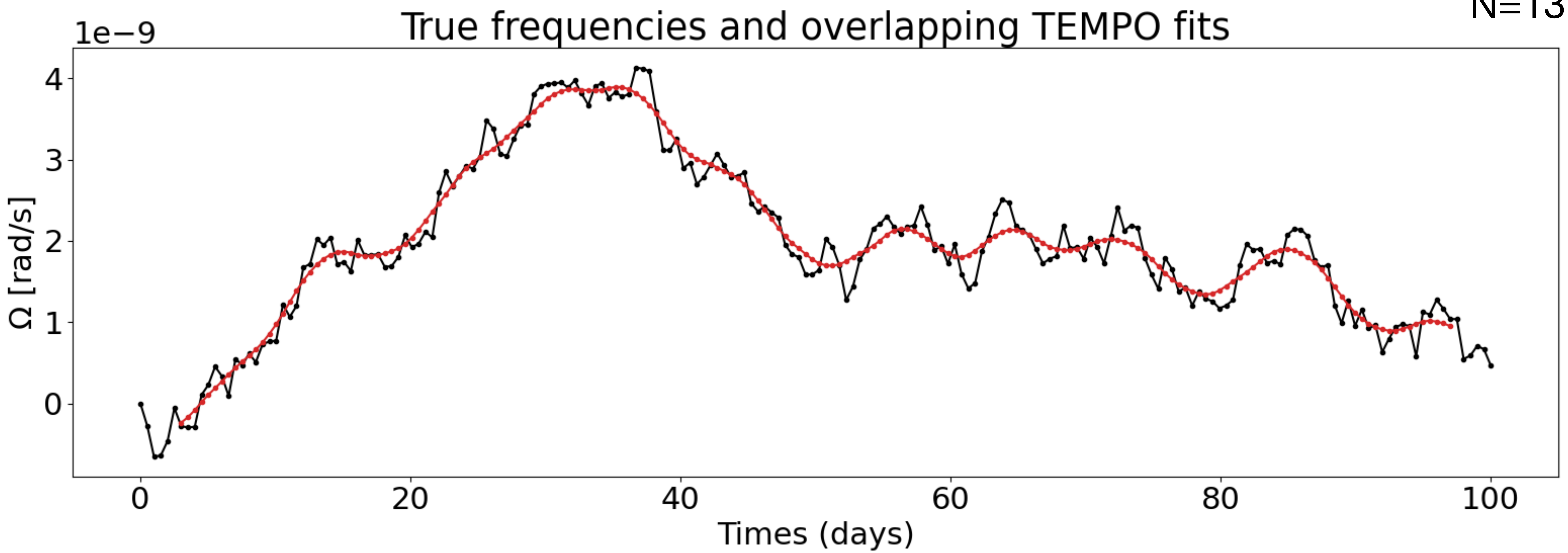
N=13



Fitting frequencies to TOAs smoothes out the data. It averages out the frequency over the period of fitting.

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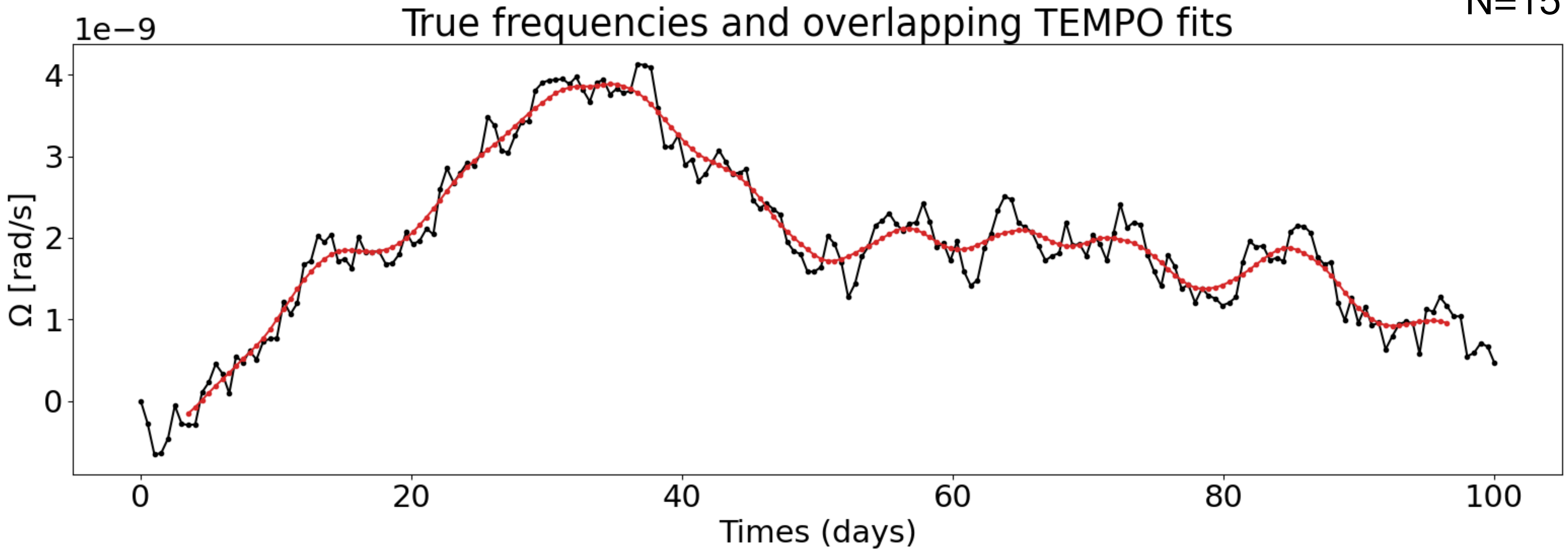
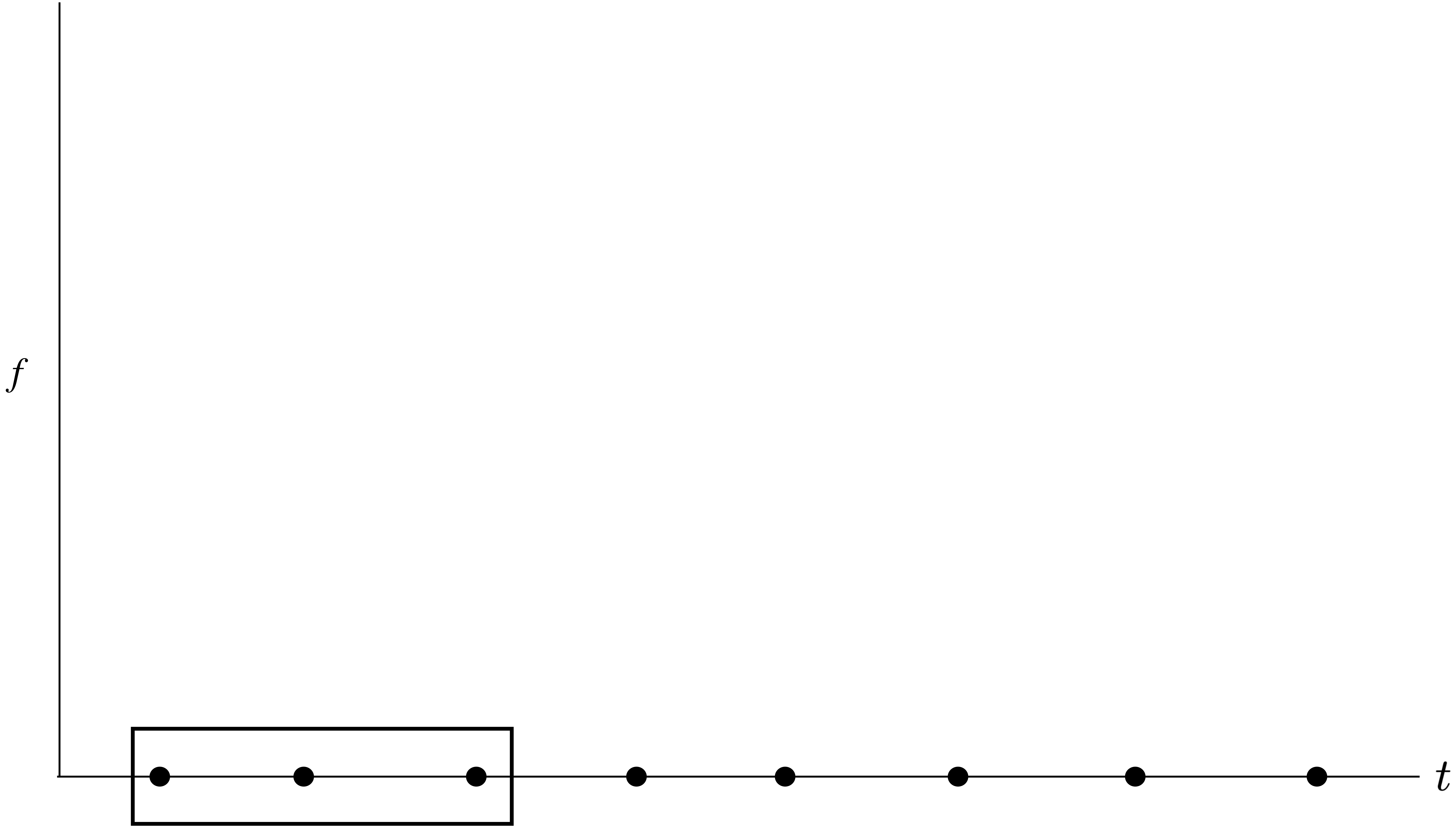
N=15



Fitting frequencies to TOAs smoothes out the data. It averages out the frequency over the period of fitting.

This removes details of the timing noise.

Fitting to non-overlapping sets of TOAs avoids this issue



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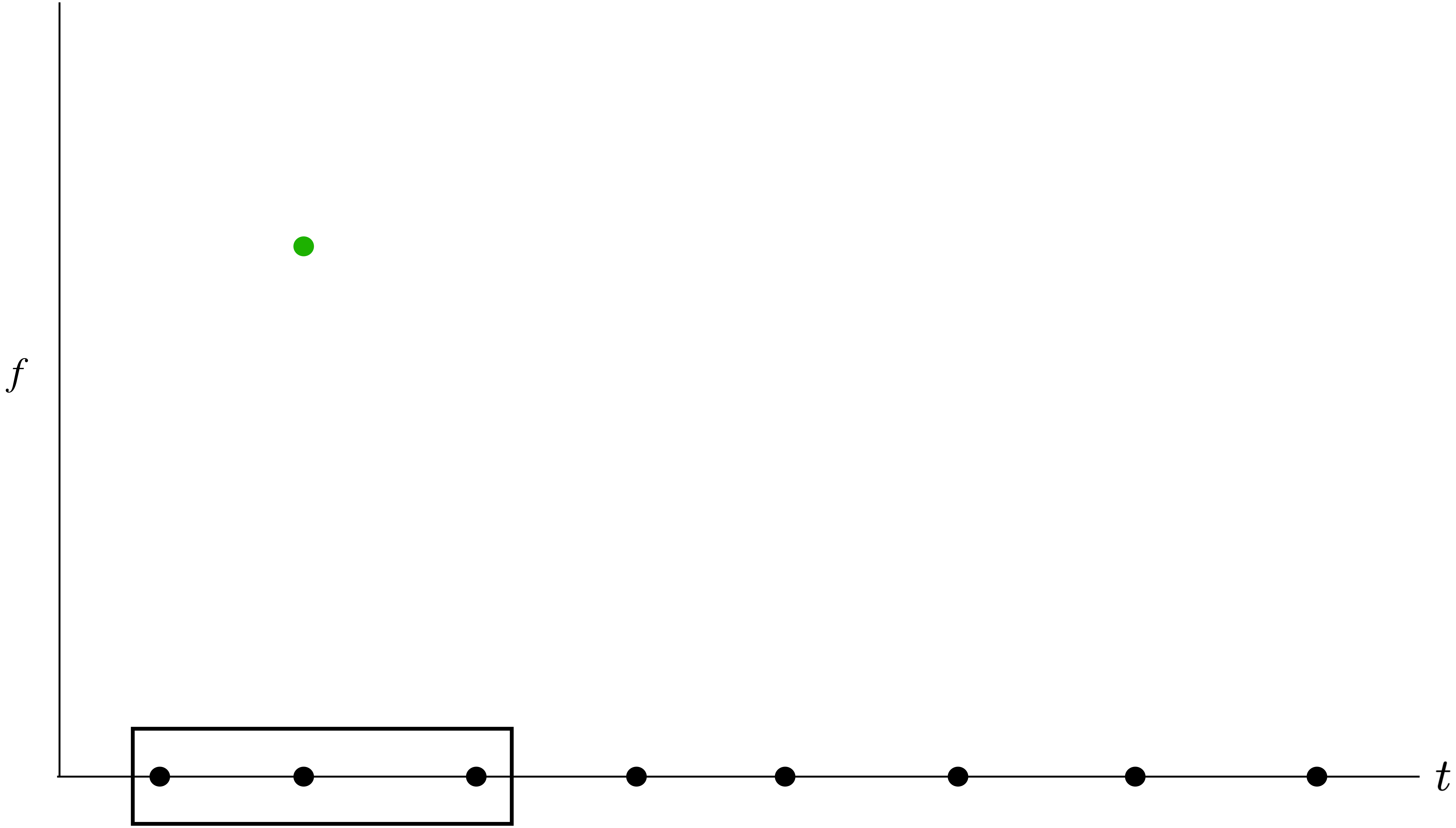

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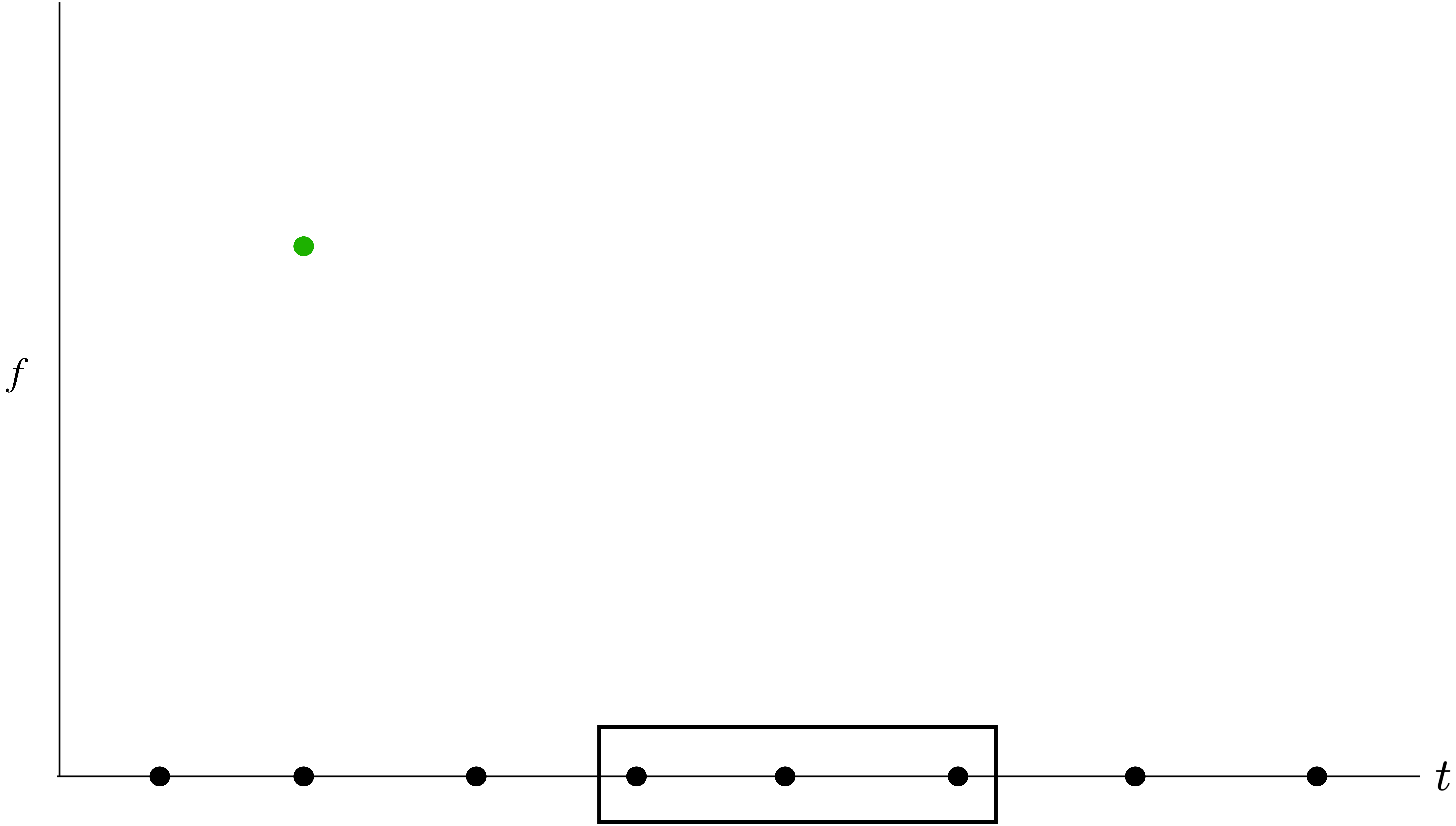

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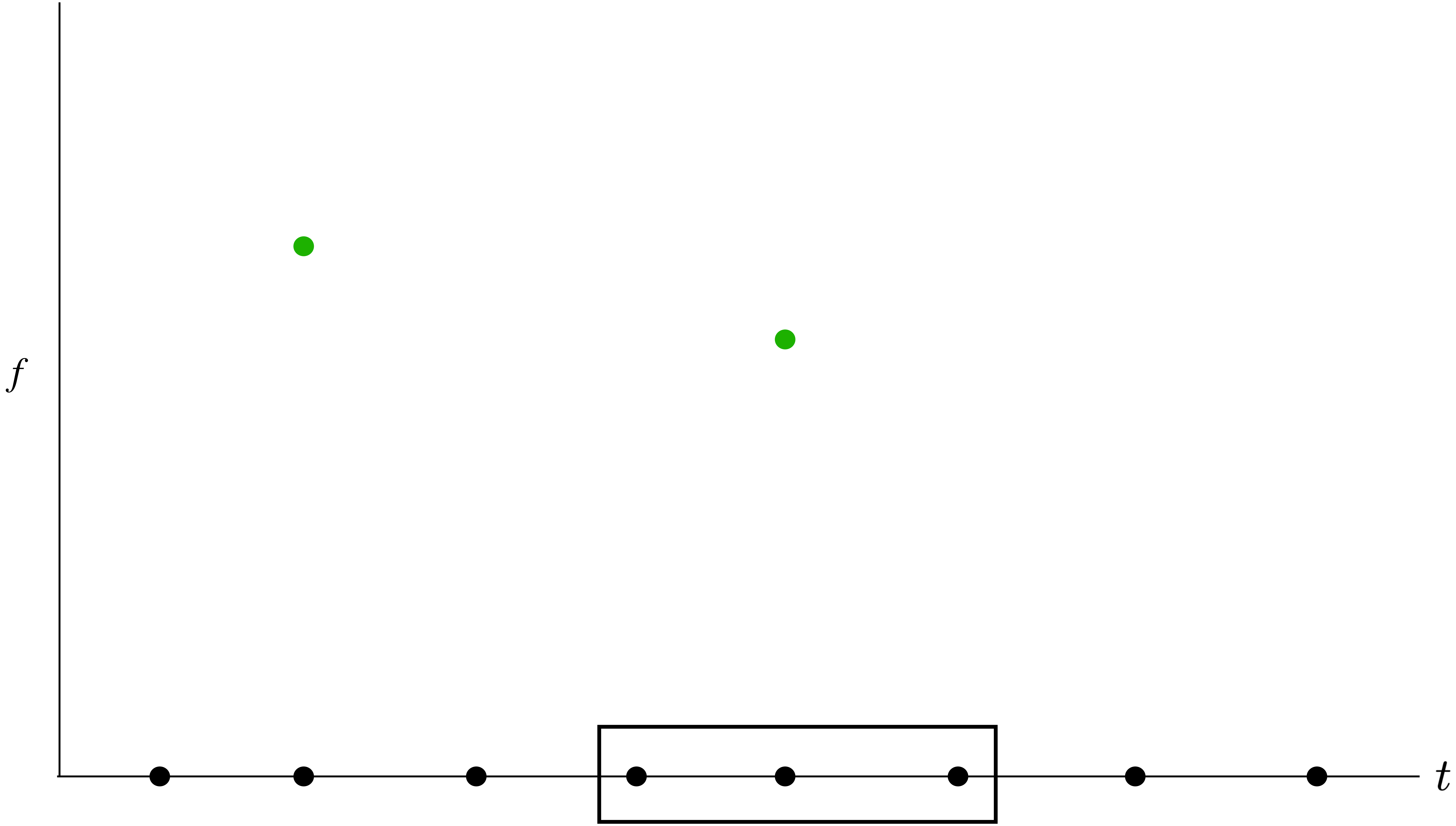

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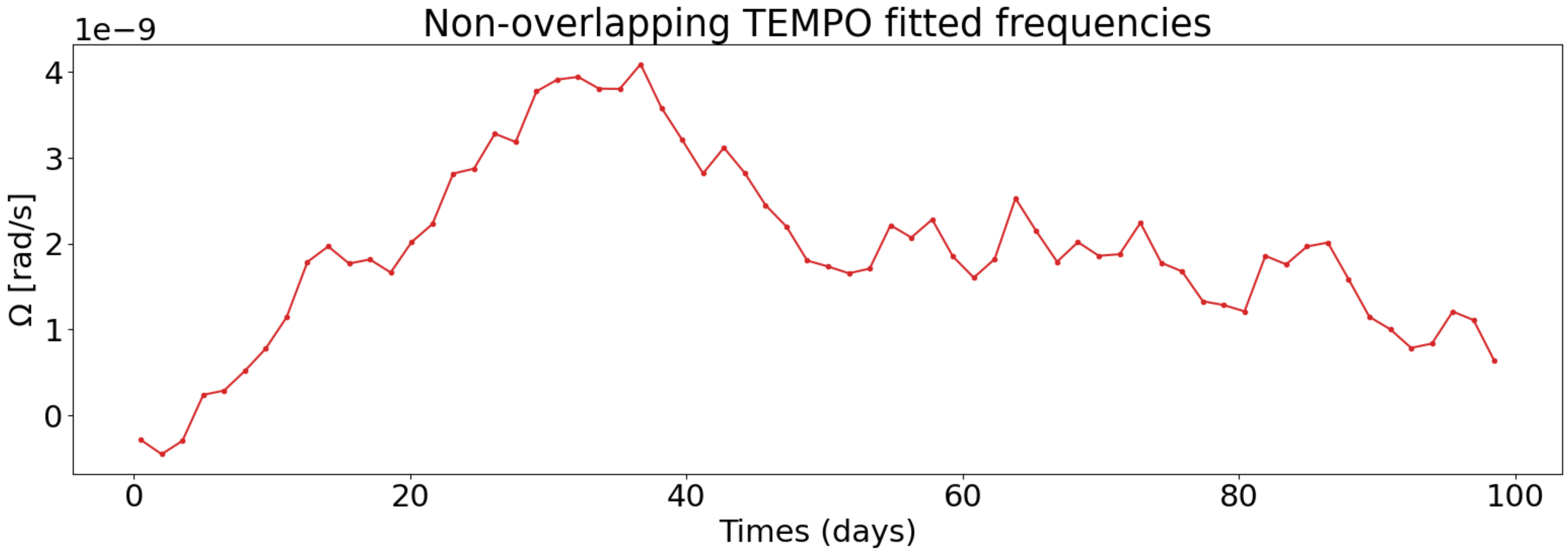
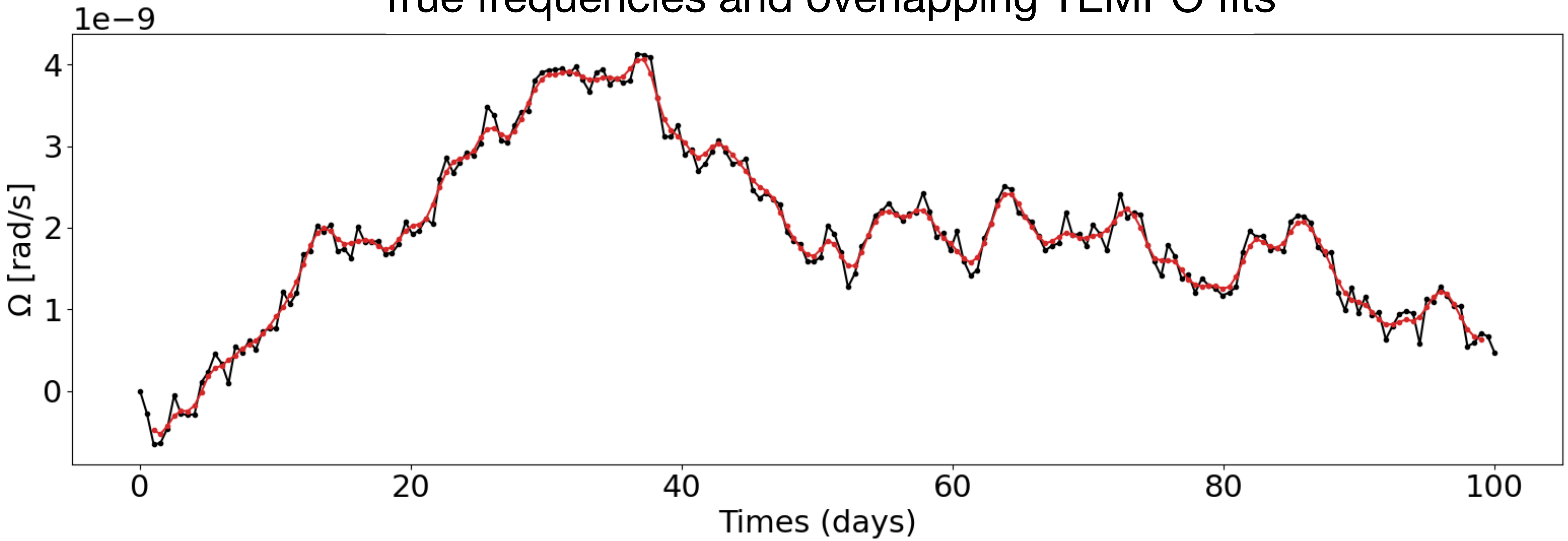
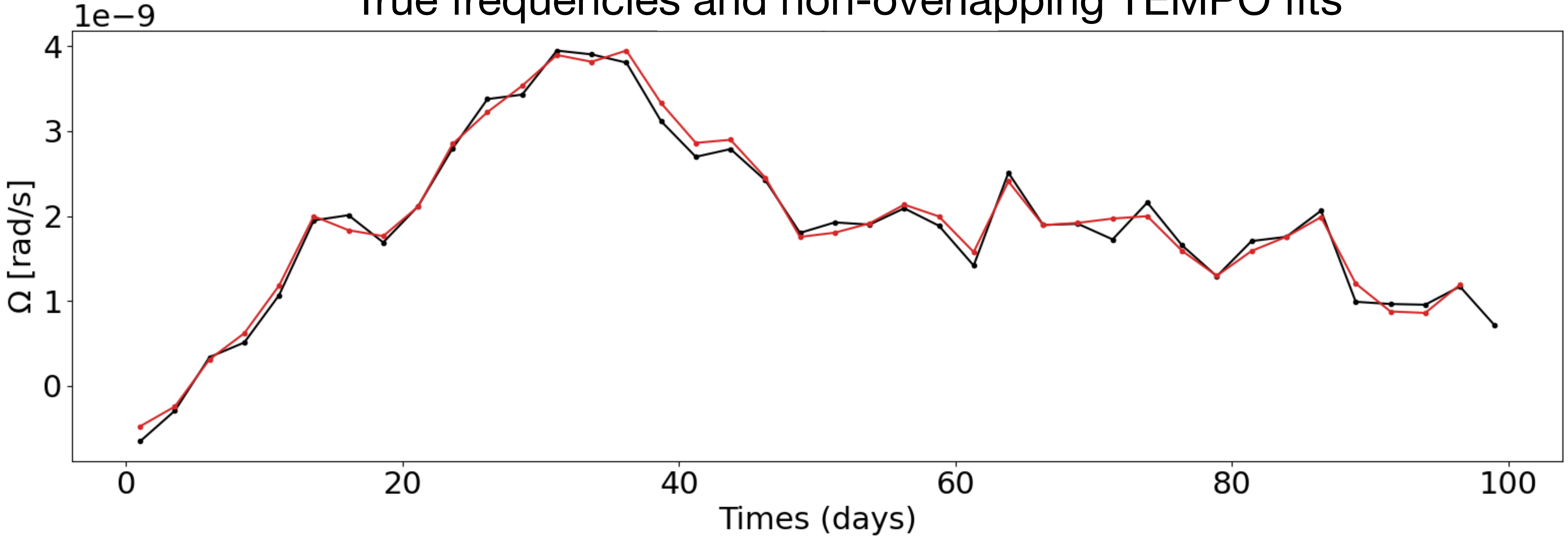


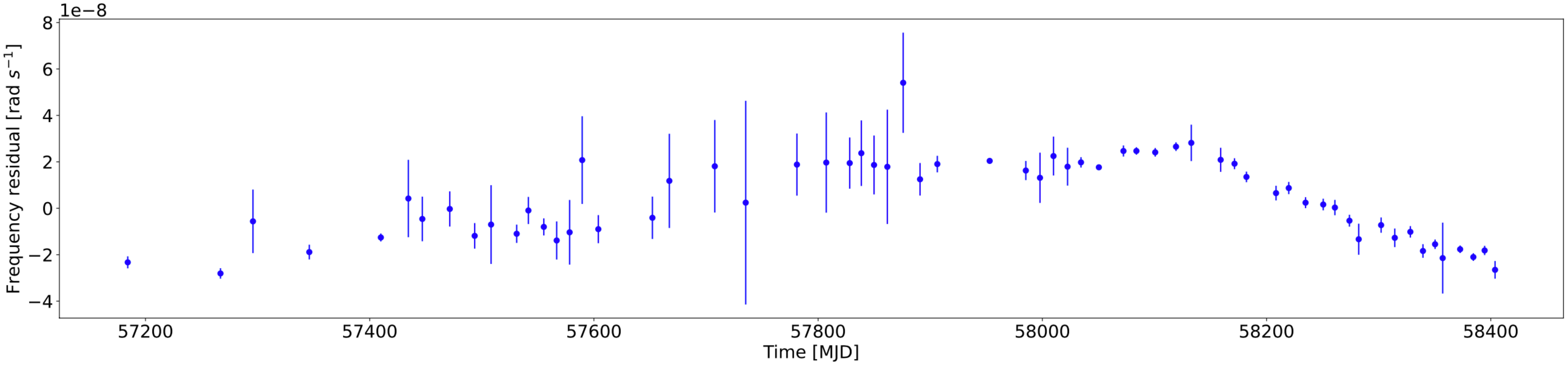
True frequencies and overlapping TEMPO fits



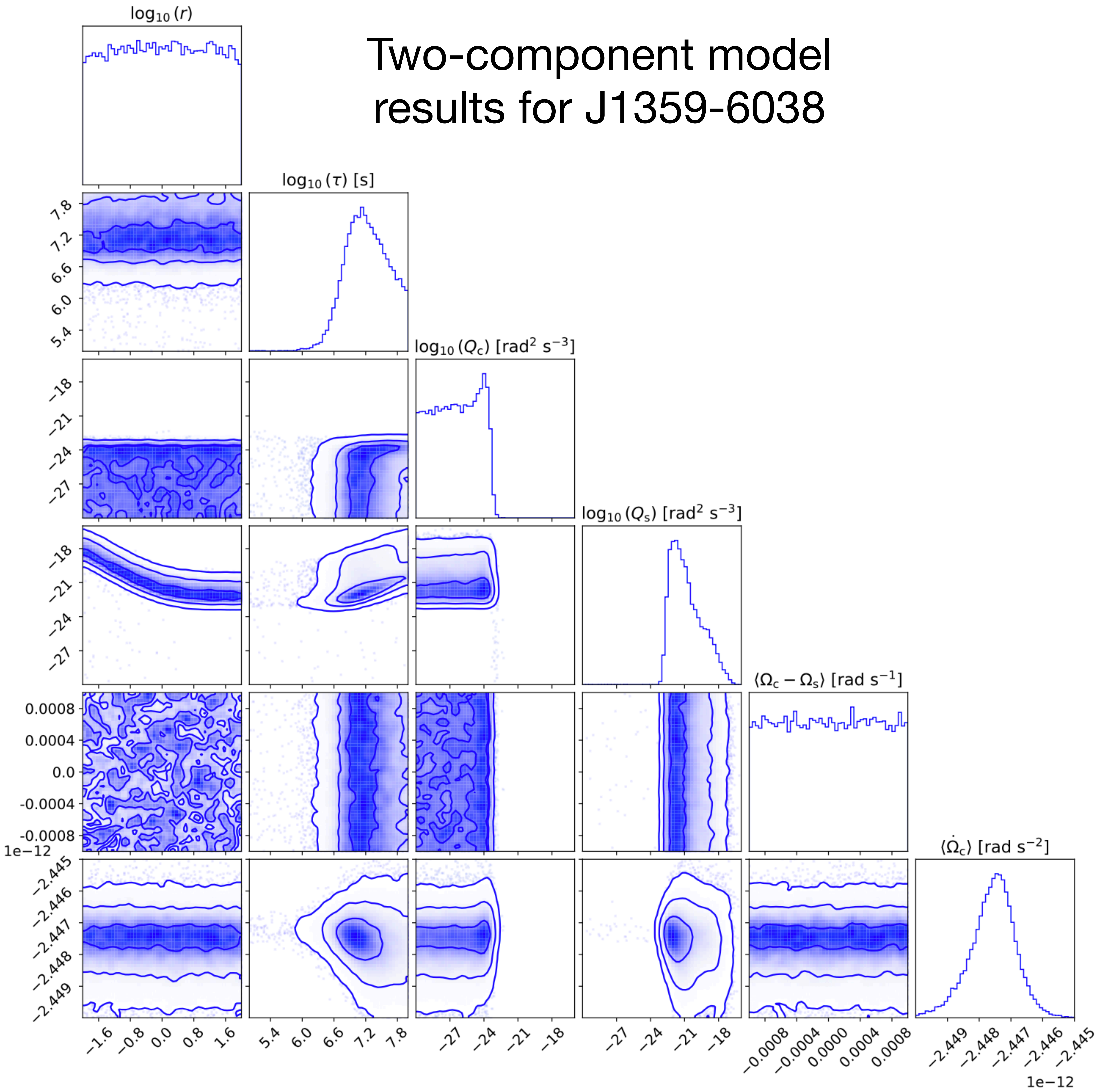
True frequencies and non-overlapping TEMPO fits



J1359-6038 frequency data. Non-overlapping sets of TOAs.

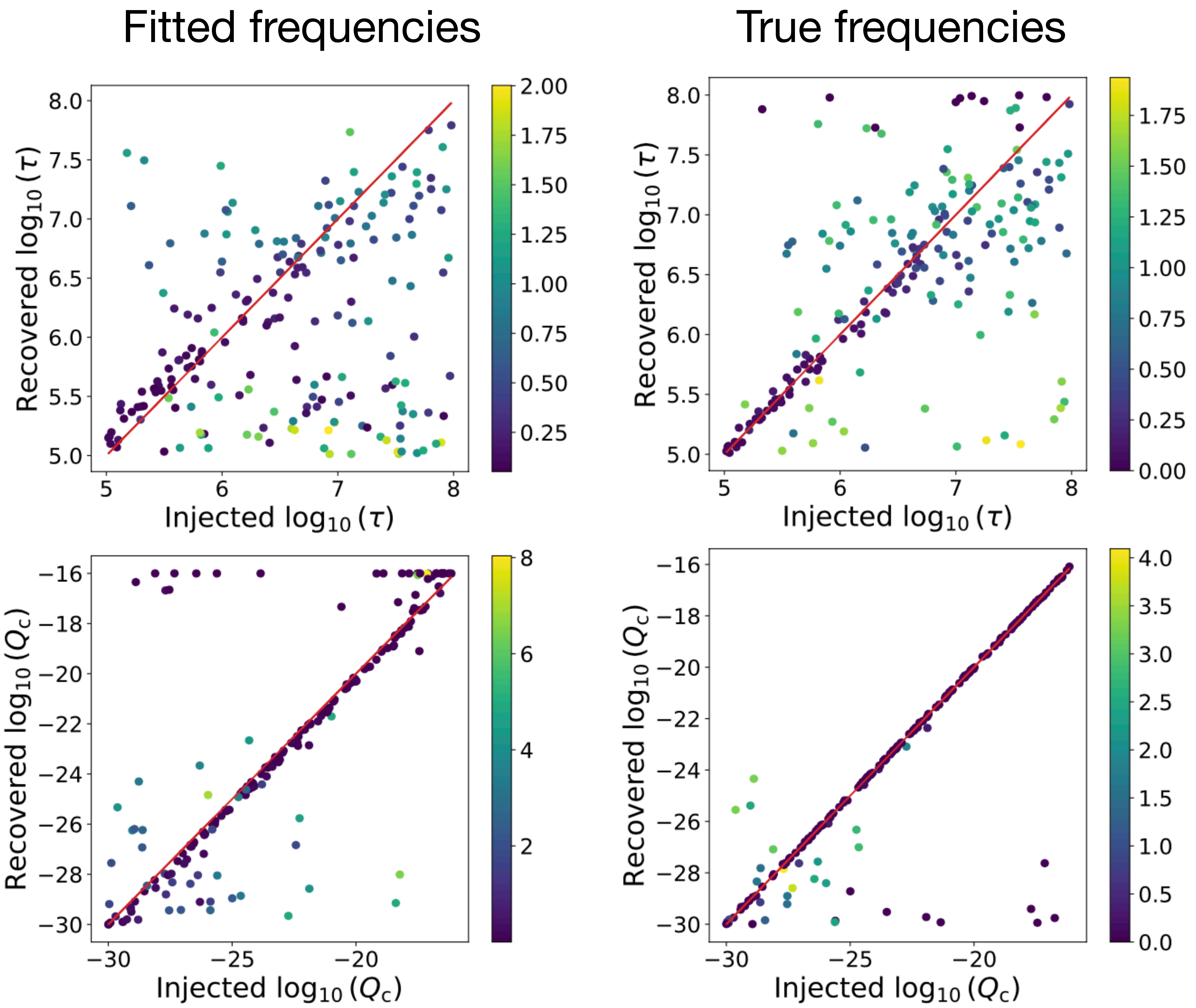


Two-component model results for J1359-6038



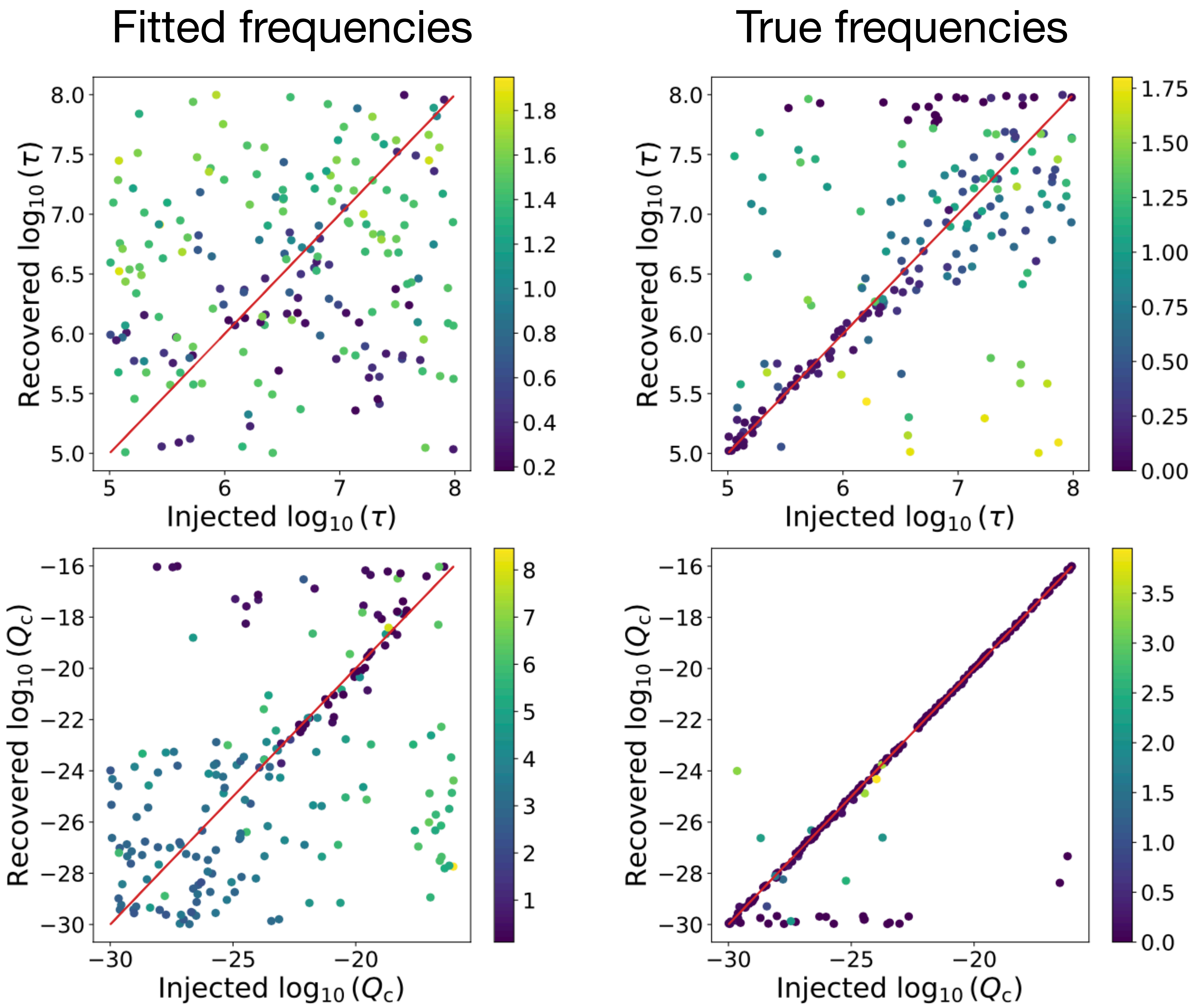
We tested the method for converting from TOAs to frequencies on simulations.

These results are quite successful.



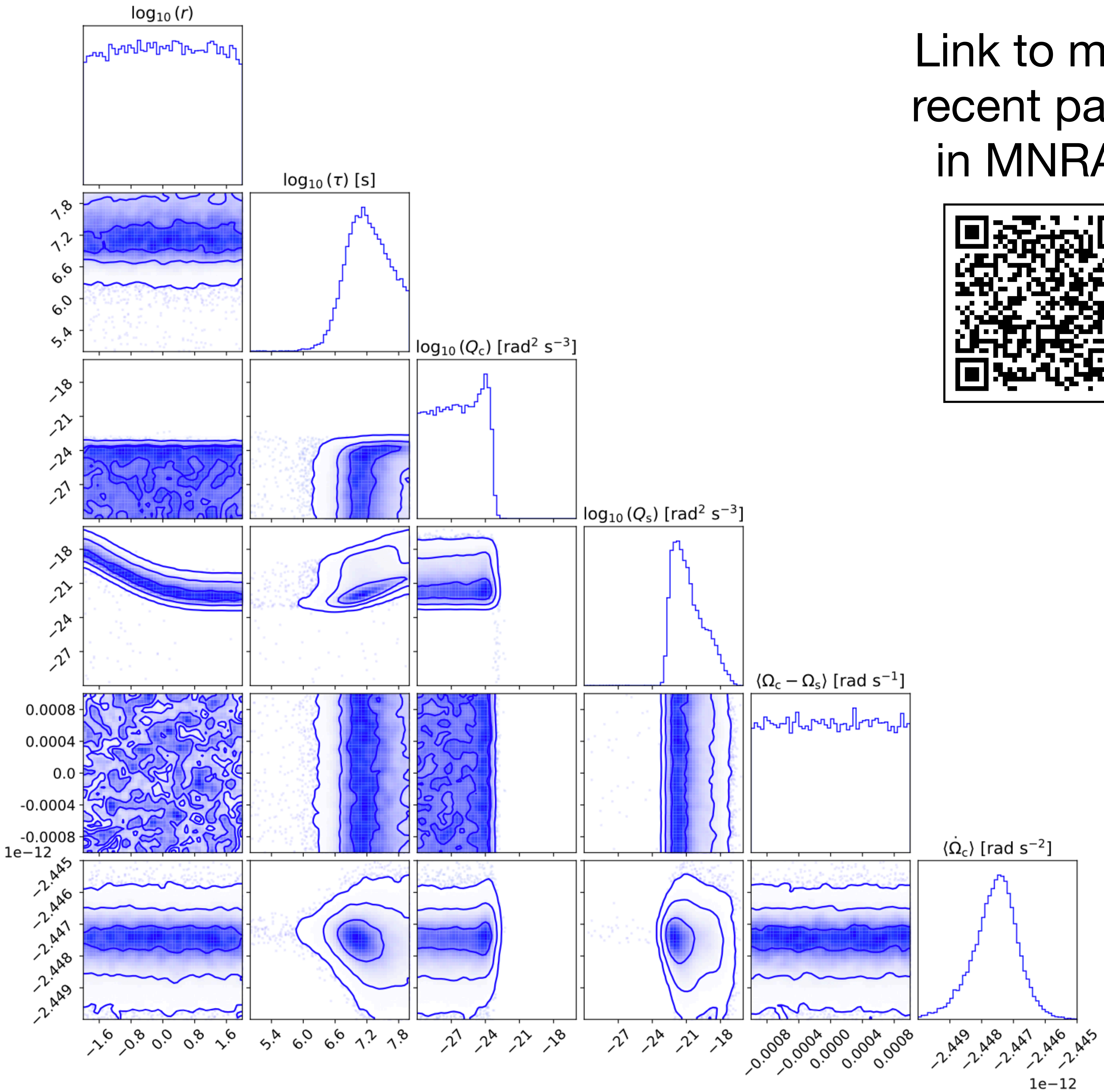
With lower quality data it can still be difficult.

But no significant bias is introduced.



Conclusion

- The Kalman filter method has been successfully demonstrated on simulations.
- Two-component model parameters were successfully recovered from timing noise for a real pulsar.



Link to most recent paper in MNRAS

End of Talk