

# BinaryWeave: A New Semicoherent Pipeline for Detecting a CW Signal From Scorpius X-1

Implementing Semicoherent F-stat templates on optimal lattices

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# Sources of CW-Signals: a Quick Reminder

- (Rapidly) spinning neutron stars with mass-quadrupolar deformations => equatorial ellipticity ( $\epsilon$ )
- Various non-radial oscillation modes, e.g., r-mode, g-mode, f-mode, in old and newly born neutron stars
- Ideal test beds:
  - ➔ spinning neutron stars in “messy environments”, e.g., NS in accreting binaries, LMXB systems
  - ➔ newly born neutron star that has yet to settle down to its long-term structures, e.g., supernova remnants
  - ➔ Unknown sources of special interests, e.g., galactic centre, globular clusters, etc.

Here I will specifically focus on Sco X-1, a known accreting NS in LMXB system

# Scorpius X-1: the Brightest Extra-Solar X-Ray Source in the Sky

- Scorpius X-1 (Sco X-1) is the brightest extra-solar X-ray sources in the sky
- A low-mass X-ray binary (LMXB) system with a companion with mass  $\sim 0.42 M_{\text{sun}}$
- X-ray and optical spectra from Sco X-1 suggests it harbours a neutron star as the primary object
- High X-ray luminosity  $\Rightarrow$  proxy for high mass-accretion rate  $\Rightarrow$  plausible large non-axisymmetric deformation
- Torque balance scenario: accretion induced spin-up torque = spin-down torque combined by all the dissipative mechanisms
- Certain astrophysical properties and spin-distributions of neutron stars advocates for strong CW emission as one of the most natural braking mechanisms

# Sco X-1 Source Properties

- Although the brightest and persistent X-ray emitter, NO pulsation is seen from Sco X-1 [Galaudage et al., MNRAS 509, 1745 (2022)]
- Optical and radio observations have measured different orbital parameters to a varied degrees of accuracies
- Eccentricity is well constrained:  $e \leq 0.0132$

Galloway et al., ApJ 781:14 (2014);  
Cherepashchuk et al., MNRAS 508, 1389 (2021);  
Killestein et. al., MNRAS 520, 5317 (2023)

TABLE I. Scorpius X-1: system parameters.

Sco X-1 parameter	Value	Uncertainty
Period	68023.70 sec	0.04 sec
Orbital semimajor axis	1.44 sec	0.18 sec
Time of ascension	897753994	100 sec
Orbital eccentricity	$< 0.068$	$3\sigma$
Right Ascension	$16^{\text{h}}19^{\text{m}}55^{\text{s}}.067$	$0''.06$
Declination	$-15^{\circ}38'25''.02$	$0''.06$
System inclination	$44^{\circ}$	$6^{\circ}$
Companion mass	$0.42M_{\text{Sol}}$	
X-ray flux	$3.9 \times 10^{-10} \text{ Wm}^{-2}$	

Ref: Messenger et al.,  
PRD 92, 023006 (2015)

**Note:** these observations are old now, and a new set of refined source parameter space has been reported in T. L. Killestein et. al., MNRAS 520, 5317–5330 (2023)

# Searching for a CW-Signal From Sco X-1

- **Problem at hand: detecting a CW-signal from Sco X-1**
- The source emits quasi-monochromatic continuous gravitational waves in its rest frame
  - ➔ However, its spin-frequency is completely unknown
- Being in a stellar binary system, the CW-signal goes through significant doppler modulations
  - ➔ We need to search over the orbital parameters of the binary system

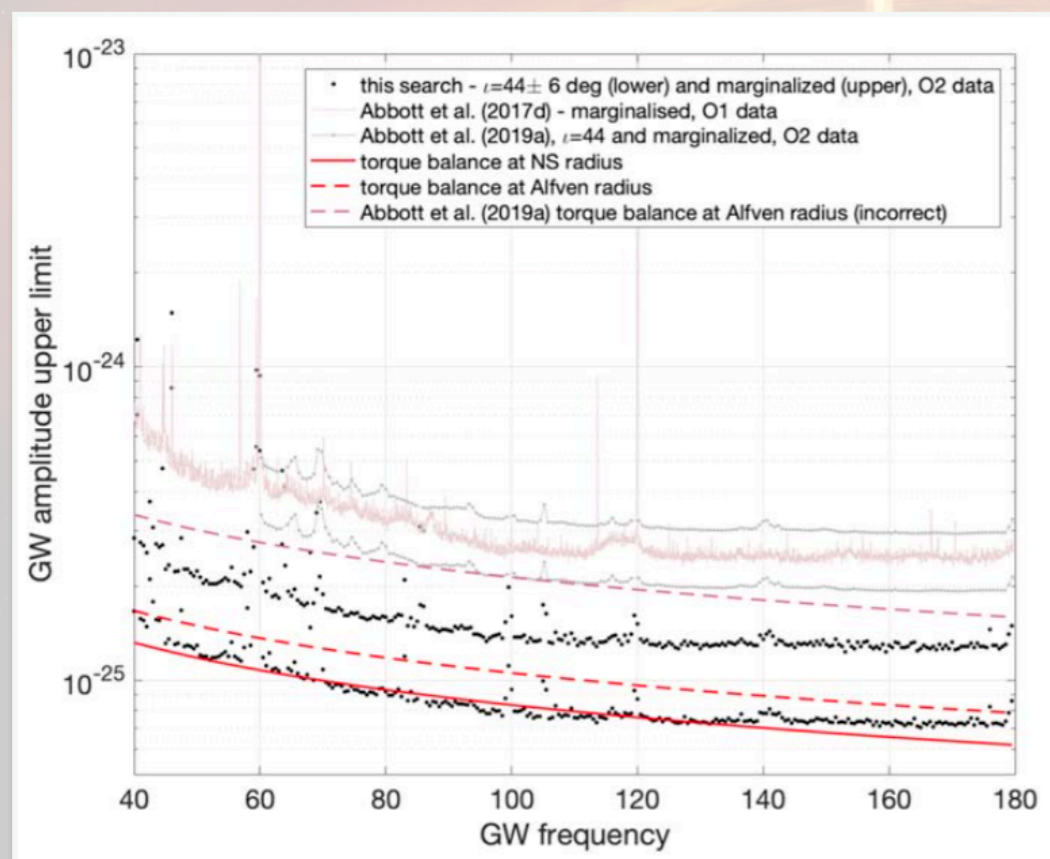
# Sco X-1 Search Results

- Sco X-1 has been searched extensively in GW detectors, including Advanced-LIGO, Advanced-VIRGO, KAGRA over a couple of decades
- However, only recently we have been able to beat the torque-balance limit in the low-frequency regime ( $< 200$  Hz)

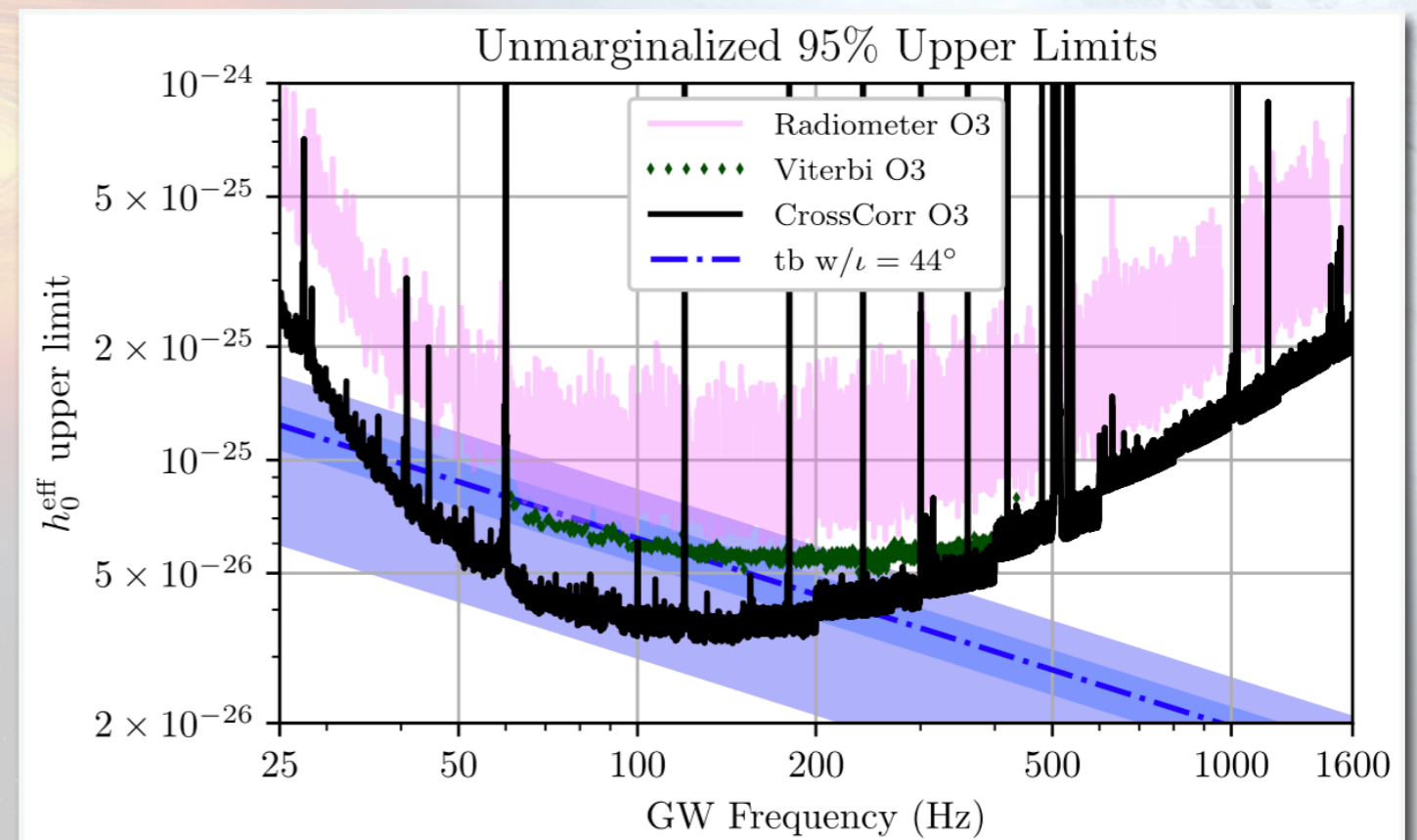
B. Abbott et al., PRD 76, 082001 (2007); J. Aasi et al., PRD 91, 062008 (2015); B. P. Abbott et al., PRD 100, 122002 (2019); Y. Zhang et al., ApJL 906:L14 (2021); R. Abbott et al., ApJL 941:L30 (2022)

- Recent searches with updated source parameters of *Killestein et al. (2023)*

Whelan et al., ApJ, Vol. 949, Issue 2, id.117 (2023); Vargas & Melatos, arXiv:2310.19183



Ref: Y. Zhang et al., ApJL 906:L14 (2021)

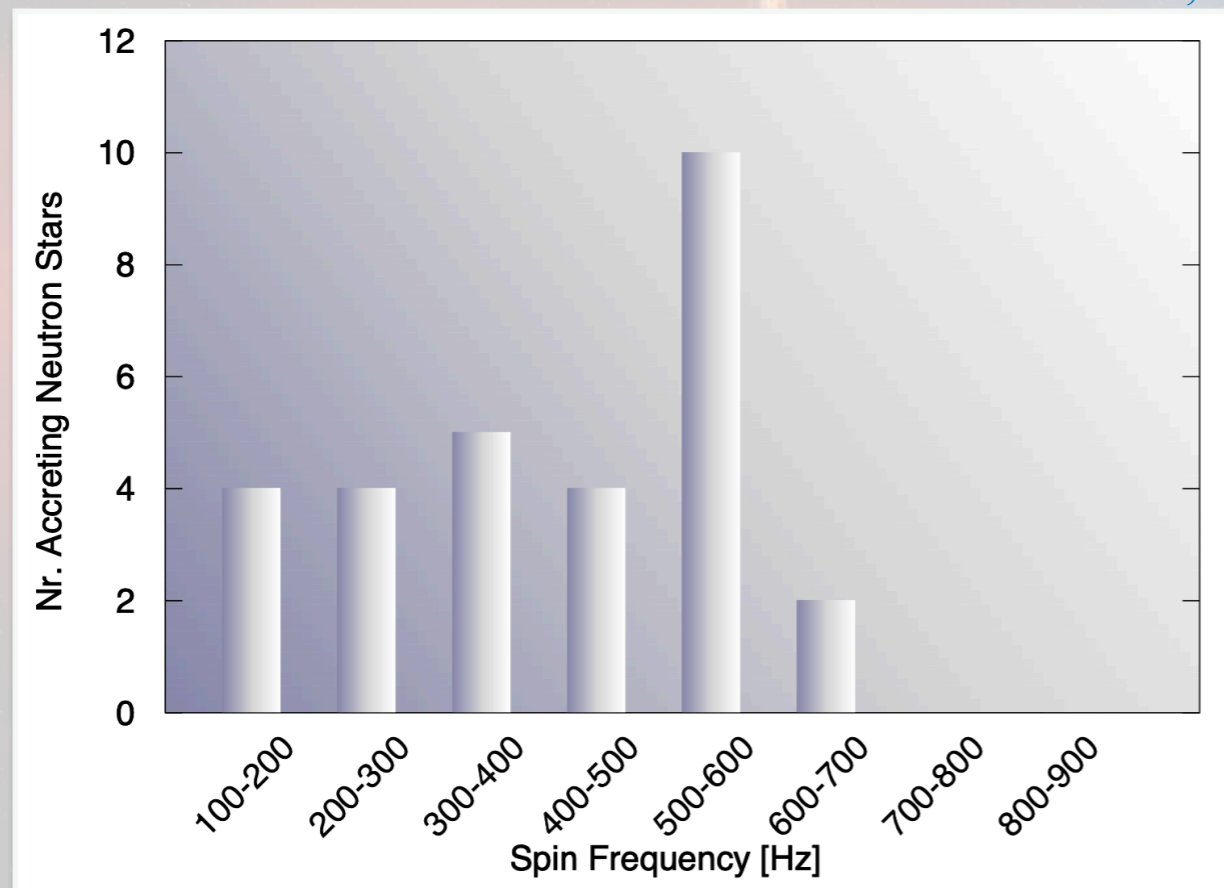


Ref: R. Abbott et al., ApJL 941:L30 (2022)

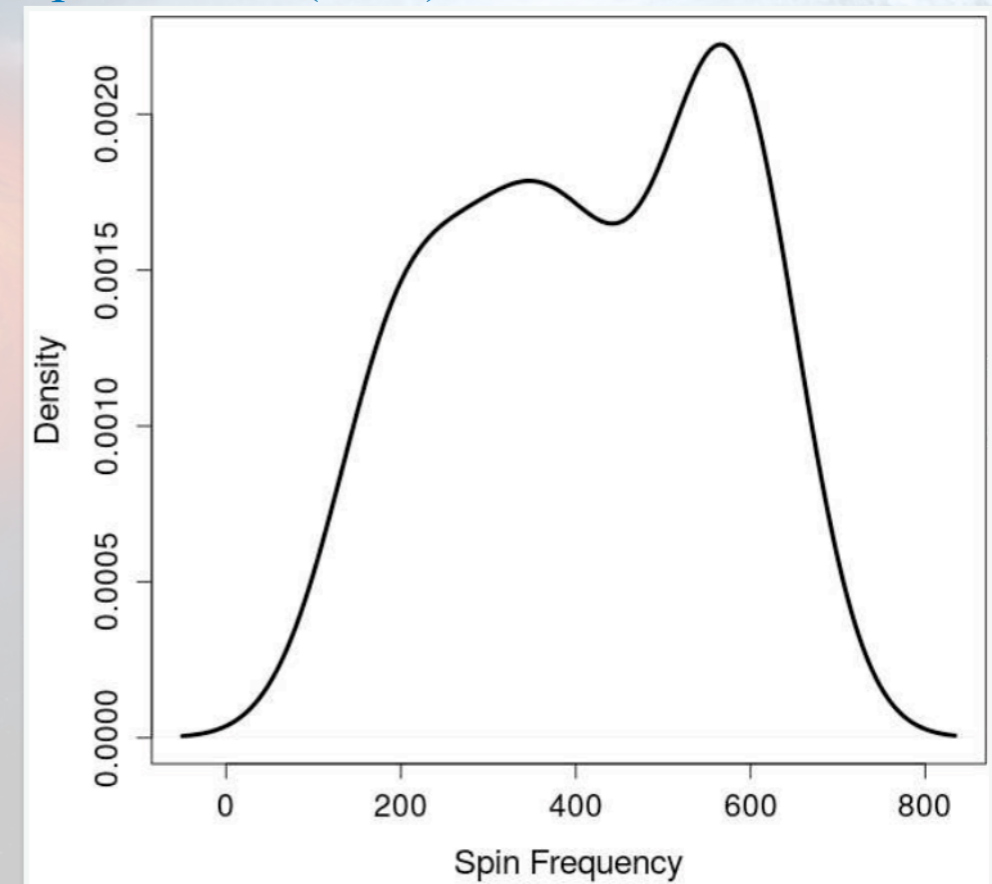
# Spin Frequency of Accreting NSs

- Accreting neutron stars (in LMXBs) are generally fast spinning objects; frequency in [200, 700 Hz]  
*D. Chakrabarty, AIPC Proc., Vol. 1068, pp. 67-74 (2008); A. Patruno, et al., ApJ 850:106 (2017)*
- Accretion transfer (+ve) angular momentum to the NSs, acts as the primary mechanism for spin-up
- Sco X-1 is one of the highest accreting NS LMXB systems; it likely to host a rapidly spinning neutron star, possibly in the range of **~ 300 — 700 Hz**

Ref: A. Patruno, et al., ApJ 850:106 (2017)



Histogram of accreting neutron stars comprising all known AMXPs and NXPs.



KDE estimates of all known accreting neutron stars (AMXPs + NXPs).

# Searching for a CW-Signal From Sco X-1 [Revisited ...]

- **Problem at hand: detecting a CW-signal from Sco X-1**
- The source emits quasi-monochromatic continuous gravitational waves in its rest frame
  - ➔ However, its spin-frequency is completely unknown
- Being in a stellar binary system, the CW-signal goes through significant doppler modulations
  - ➔ We need to search over the orbital parameters of the binary system

**The target parameter space becomes enormous  
due to limited observational constraints!**





**A New Search Pipeline for Sco X-1:  
BINARYWEAVE**

# Overview: BinaryWeave

- This is a **semi-coherent** CW search pipeline for signals from a spinning neutron star in **binary system with known sky-position**
- The primary target is **Sco X-1** over a wide range of frequency band and orbital parameter space
- However, it can be used for directed searches from other binary systems with known sky-position (including **other LMXBs**)
- This pipeline is developed following the method in [Leaci & Prix, PRD 91, 102003 \(2015\)](#)
- The pipeline has been implemented in the “**WEAVE**-infrastructure” initially developed by K. Wette and R. Prix [[K. Wette et al., PRD 97, 123016 \(2018\)](#)] (see: [[K. Wette at LVC-meeting, Glasgow \(2016\)](#); DCC: [<LIGO-G1601794-v2](#)])

# Basic Structure

- The entire observation time is **split** into **N** number of **segments**
- Each segments is searched with match-filtering the data against a bank of templates of phase/doppler-parameters (denoted by,  $\lambda$ )
- Results in well-known coherent  $\mathcal{F}$ -statistic for each of the **N** segments by maximising over the four amplitude-parameters (denoted by,  $\mathcal{A}$ )  
[JKS, PRD 58, 063001 \(1998\); R. Prix, PRD 75, 023004 \(2007\)](#)
- **Sum** over the  $\mathcal{F}$ -**statistic** values from those **N-segments incoherently** to get the final **semi-coherent  $\mathcal{F}$ -statistic** distribution
- Search over the source parameter space ( $P$ ): orbital-parameters (**ASINI**, **PORB**, **TASC**) along with **FREQ** (CW-frequency), etc., ...

# Weave Modus Operandi

- Tile (near) optimal covering lattice  $A_n^*$  or usual  $Z_n^*$  lattice grids in D-dim search parameter space ( $P$ ) for each coherent-segment [[R. Prix, PRD 75, 023004 \(2007\)](#), [R. Prix, LVC CW F2F \(Ref: 8\)](#)]
- Perform coherent  $\mathcal{F}$ -**statistic** searches at each of the lattice points in  $P$
- Sum over the  $\mathcal{F}$ -**statistic** values from those N-segments incoherently to get the final semi-coherent  $\mathcal{F}$ -**statistic** distribution
  - While summing one can opt for either [nearest-neighbor interpolation](#) for each of the coherent segments [[K. Wette, PRD 90, 122010 \(2014\)](#)]
  - OR exactly at the same lattice points in parameter space ([non-interpolating](#))

Developer: K. Wette, R. Prix

**BINARYWEAVE** presently incorporates only the [non-interpolating](#) searches.

# Optimal Covering Lattice

- One of the primary goals is to put templates on  $A_n^*$  lattice-grid
- $A_n^*$  is optimal/near-optimal covering lattice for  $D = 2, \dots, 16$  dimensions [ref: 4]

Compared to  $Z_n^*$ , the efficiency of coverage for  $A_n^*$  is:

Dimension	3	4	5	6	7
Efficiency	1.9	2.8	4.3	6.8	10.9

- This results in saving computational cost to search a signal over wide parameter space

# Constant Metric Requirement for $A_n^*$ Lattice Tiling

In the long segment limit ( $T_{\text{obs}} \gg P_{\text{orb}}$ ) the coherent metric is defined as :

$$\tilde{g}_{ff}^{\text{LS}} = \pi^2 \frac{\Delta T^2}{3}$$

$$\tilde{g}_{a_p a_p}^{\text{LS}} = 2\pi^2 f^2$$

$$\tilde{g}_{\Omega\Omega}^{\text{LS}} = 2\pi^2 f^2 a_p^2 \left( \frac{\Delta T^2}{12} + \Delta_{ma}^2 \right)$$

$$\tilde{g}_{t_{asc} t_{asc}}^{\text{LS}} = 2\pi^2 f^2 (a_p \Omega)^2$$

$$\tilde{g}_{\kappa\kappa}^{\text{LS}} = \frac{\pi^2}{2} f^2 a_p^2$$

$$\tilde{g}_{\eta\eta}^{\text{LS}} = \frac{\pi^2}{2} f^2 a_p^2$$

where,  $\Delta_{ma} = t_{\text{mid}} - t_{\text{asc}}$   
and  $\Delta T = \text{segment length}$

[Leaci & Prix, PRD 91, 102003 (2015)]

$$\tilde{g}_{\Omega t_{asc}}^{\text{LS}} = \tilde{g}_{t_{asc} \Omega}^{\text{LS}} = 2\pi^2 f^2 a_p^2 \Omega \Delta_{ma}$$

# Implement a New Coordinate System for Lattice Tiling

- Old (i.e., observer/user) set of coordinates are

$$\lambda := \{a_p, \Omega, t_{asc}, \kappa, \eta\}$$

- We get a new set of coordinates for lattice/internal param-space

$$\lambda_{int} := \{a_p, v_p, d_{asc}, \kappa_p, \eta_p\}$$

$$a_p = a_p$$

$$v_p = a_p \times \Omega = 2\pi(a_p/P_{orb})$$

$$d_{asc} = a_p \times \Omega \times t_{asc} = v_p \times t_{asc}$$

$$\kappa_p = a_p \times \kappa$$

$$\eta_p = a_p \times \eta$$

The coordinate transformation functions

[AM, Prix & Wette, PRD  
107, 062005 (2023)]

# The Metric in the New Lattice Coordinate

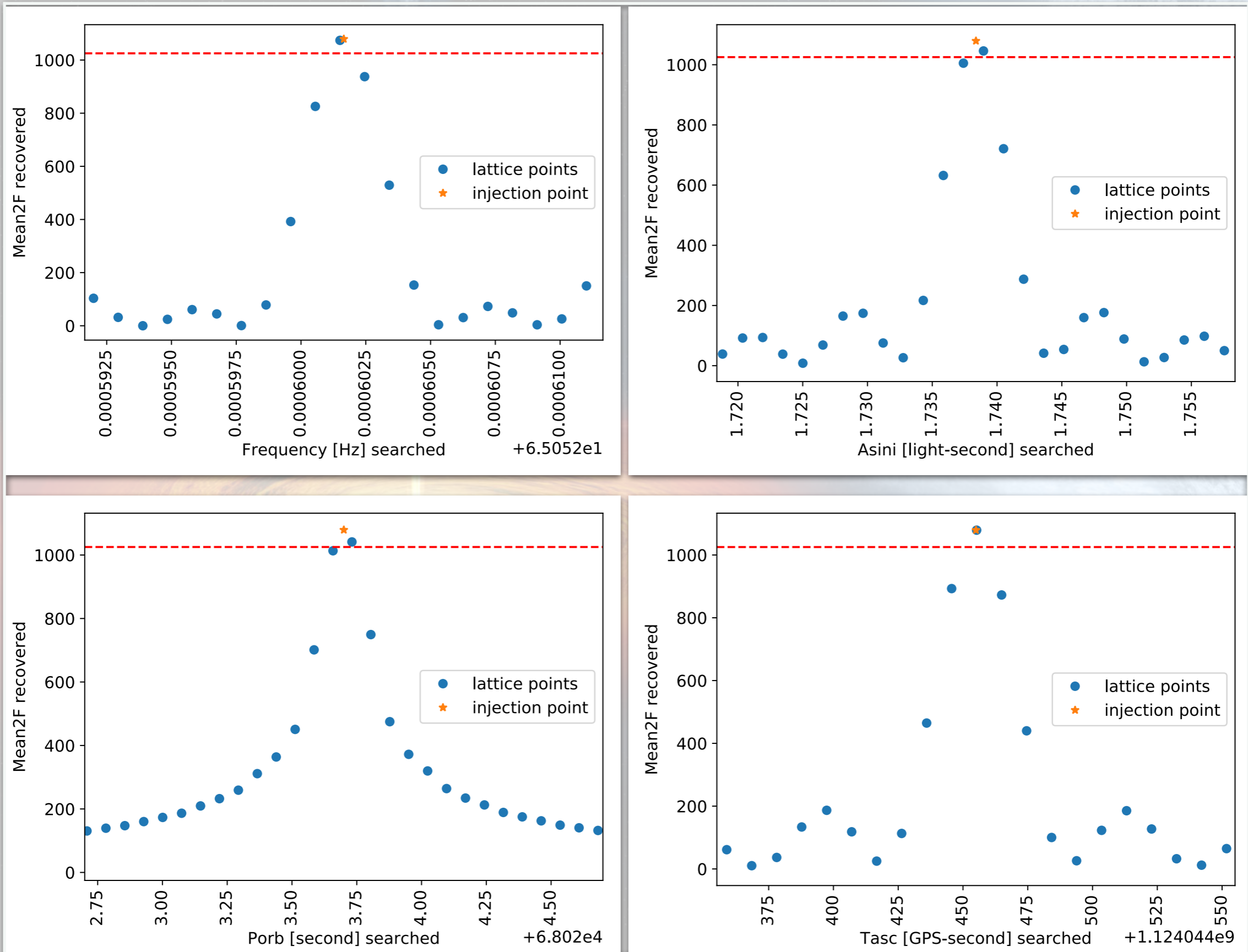
Corresponding non-zero terms in the new form of metric are

$$\begin{aligned}\tilde{g}_{ff}^{\text{LS}} &= \pi^2 \frac{\Delta T^2}{3} \\ \tilde{g}_{a_p a_p}^{\text{LS}} &= 2\pi^2 f^2 \\ \tilde{g}_{v_p v_p}^{\text{LS}} &= 2\pi^2 f^2 \left( \frac{\Delta T^2}{12} + \Delta_{ma}^2 \right) \\ \tilde{g}_{d_{asc} d_{asc}}^{\text{LS}} &= 2\pi^2 f^2 \\ \tilde{g}_{\kappa\kappa}^{\text{LS}} &= \frac{\pi^2}{2} f^2 \\ \tilde{g}_{\eta\eta}^{\text{LS}} &= \frac{\pi^2}{2} f^2 \\ \tilde{g}_{v_p d_{asc}}^{\text{LS}} &= \tilde{g}_{d_{asc} v_p}^{\text{LS}} = 2\pi^2 f^2 \Delta_{ma}\end{aligned}$$

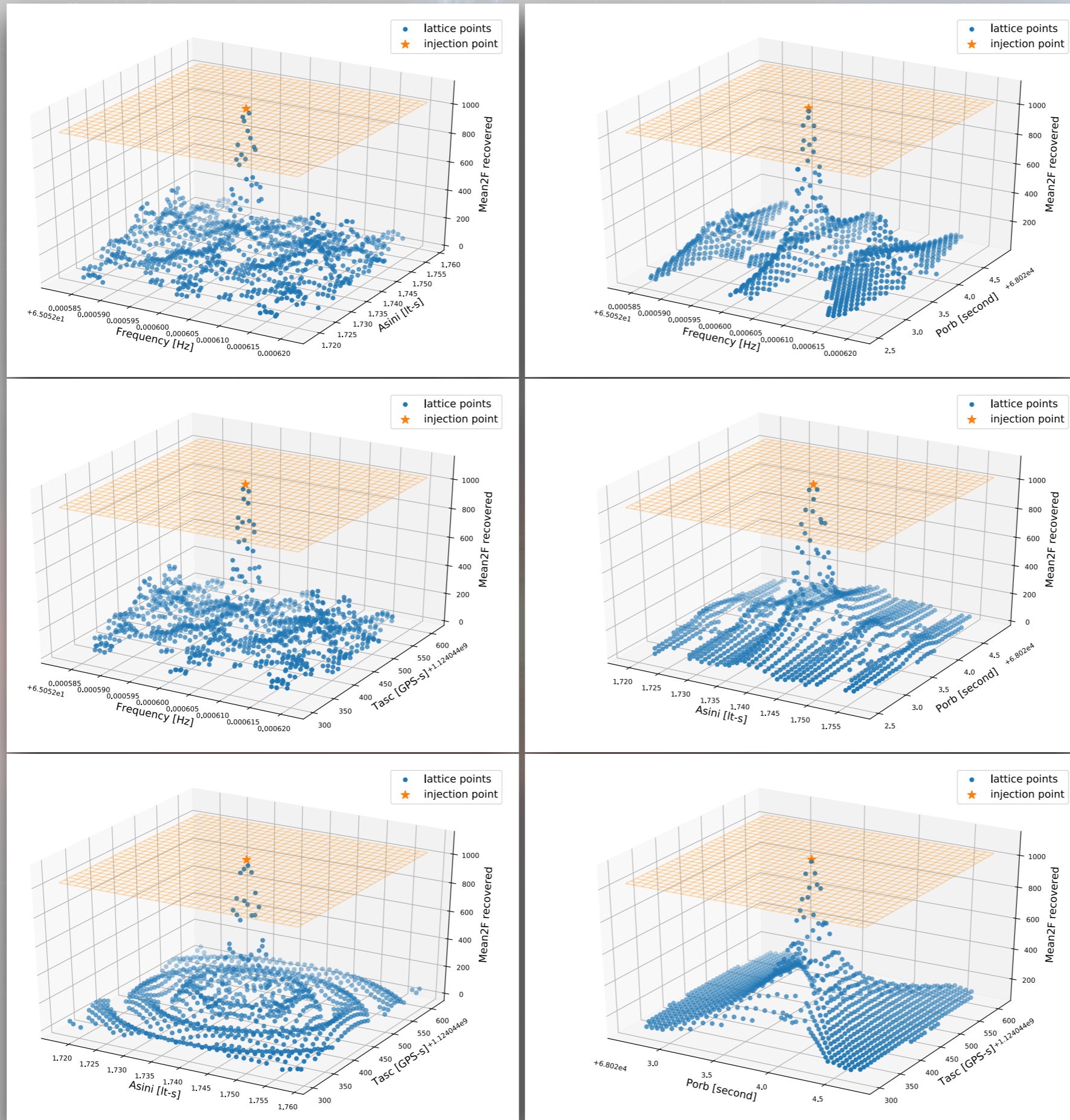
- Each of the metric coefficient is **nearly-constant** now.
- Internally we use  $\lambda_{int} := \{a_p, v_p, d_{asc}, \kappa_p, \eta_p\}$  coordinates to perform lattice-tiling
- For the remaining parameters (f and  $\Delta_{ma}$ ) we put templates in a conservative way
- We set  $f = f_{\max}$  and  $\Delta_{ma} = \max(\Delta_{ma})$  over the search range
- Good approximation when,  $\Delta f \ll f$  and  $\Delta T_{asc} \ll \Delta_{ma}$



# Injection-Recovery: 1-D Template Banks



# Injection-Recovery: 2-D Template Banks



[AM, Prix & Wette, PRD  
107, 062005 (2023)]

# Mismatch Distribution: Semi-Coherent Case

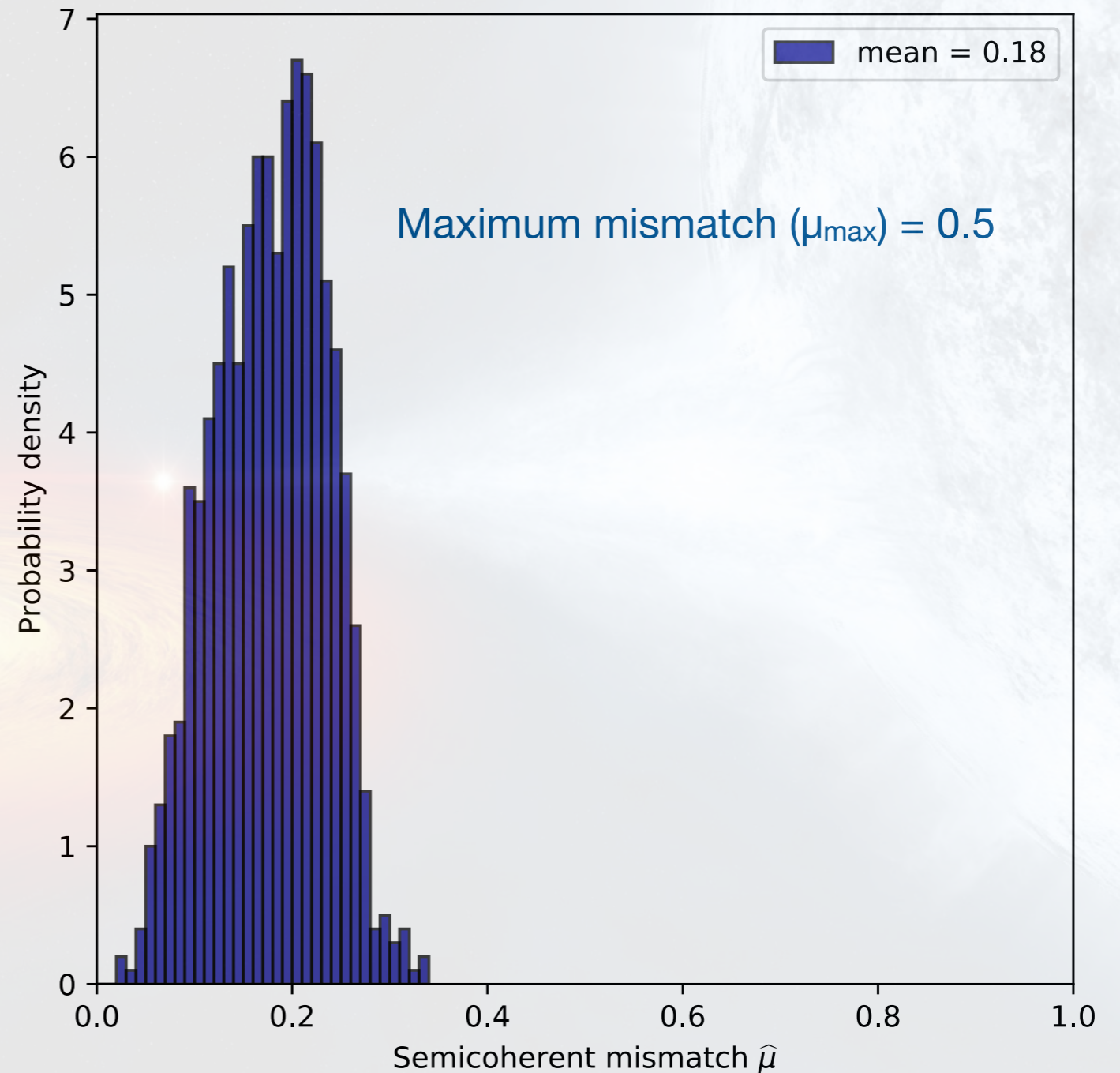
Mismatch ( $\mu$ ) is defined as:

$$\mu = \frac{\rho^2(\mathcal{A}, \lambda_s; \lambda_s) - \rho^2(\mathcal{A}, \lambda_s; \lambda)}{\rho^2(\mathcal{A}, \lambda_s; \lambda_s)}$$

$T_{\text{obs}} = 30$  days,  $T_{\text{seg}} = 1$  day  
 1000 randomly drawn samples  
 for injection-recovery test over  
 large parameter space:  $\mathcal{P}_0$

$\mathcal{P}_0$

**FREQ:** 10 — 700 Hz  
**ASINI:** 0.3 — 3.5 lt-sec  
**PORB:**  $68023.7 \pm 0.2$  sec  
**TASC:**  $1124044455 \pm 1000$  sec



Semi-coherent mismatch distribution for the 4D template bank searching {FREQ, ASINI, PORB, TASC} for an injected signal

[MPW, PRD 107, 062005 (2023)]

# Results From Two Realistic Search Setups

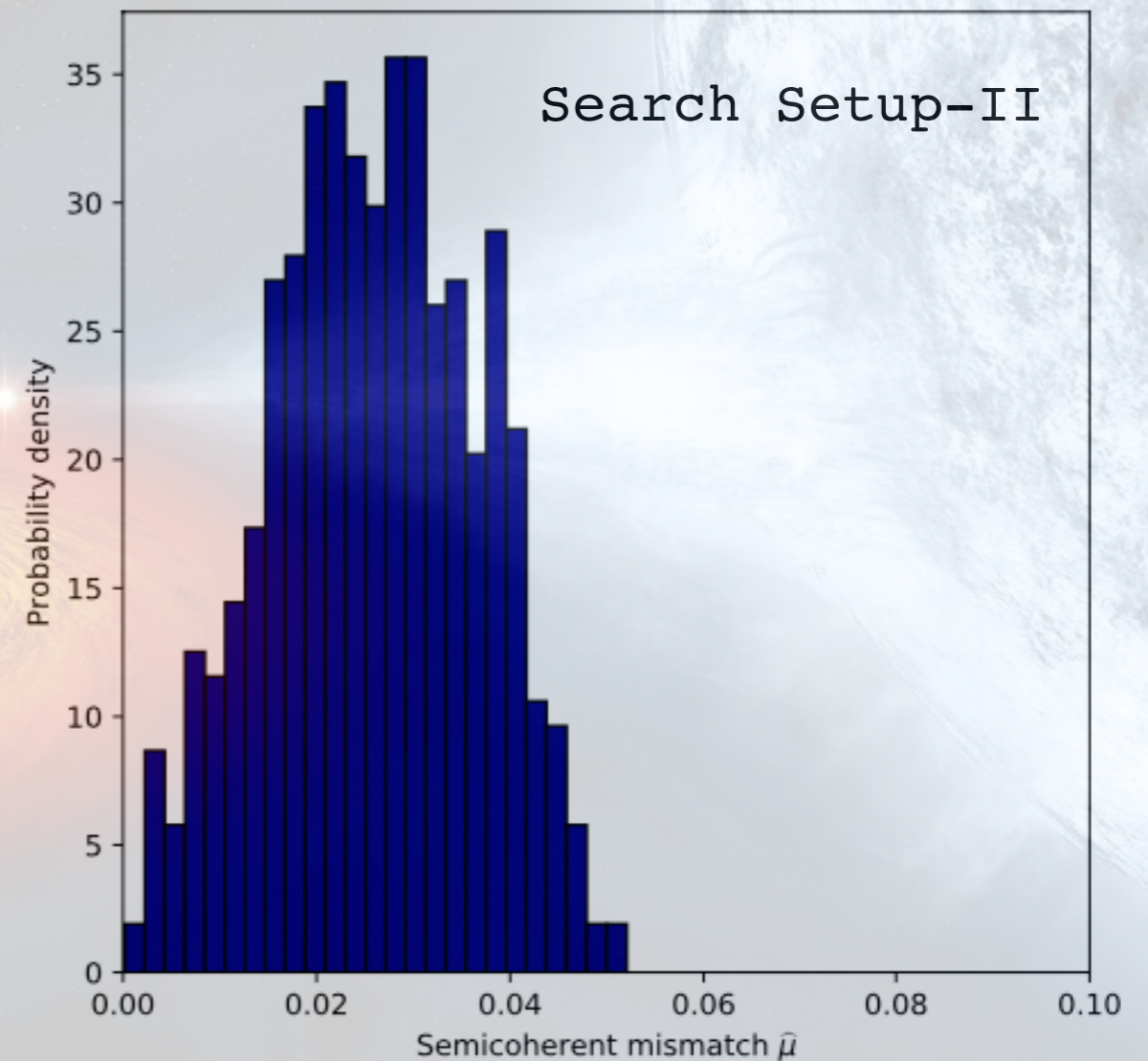
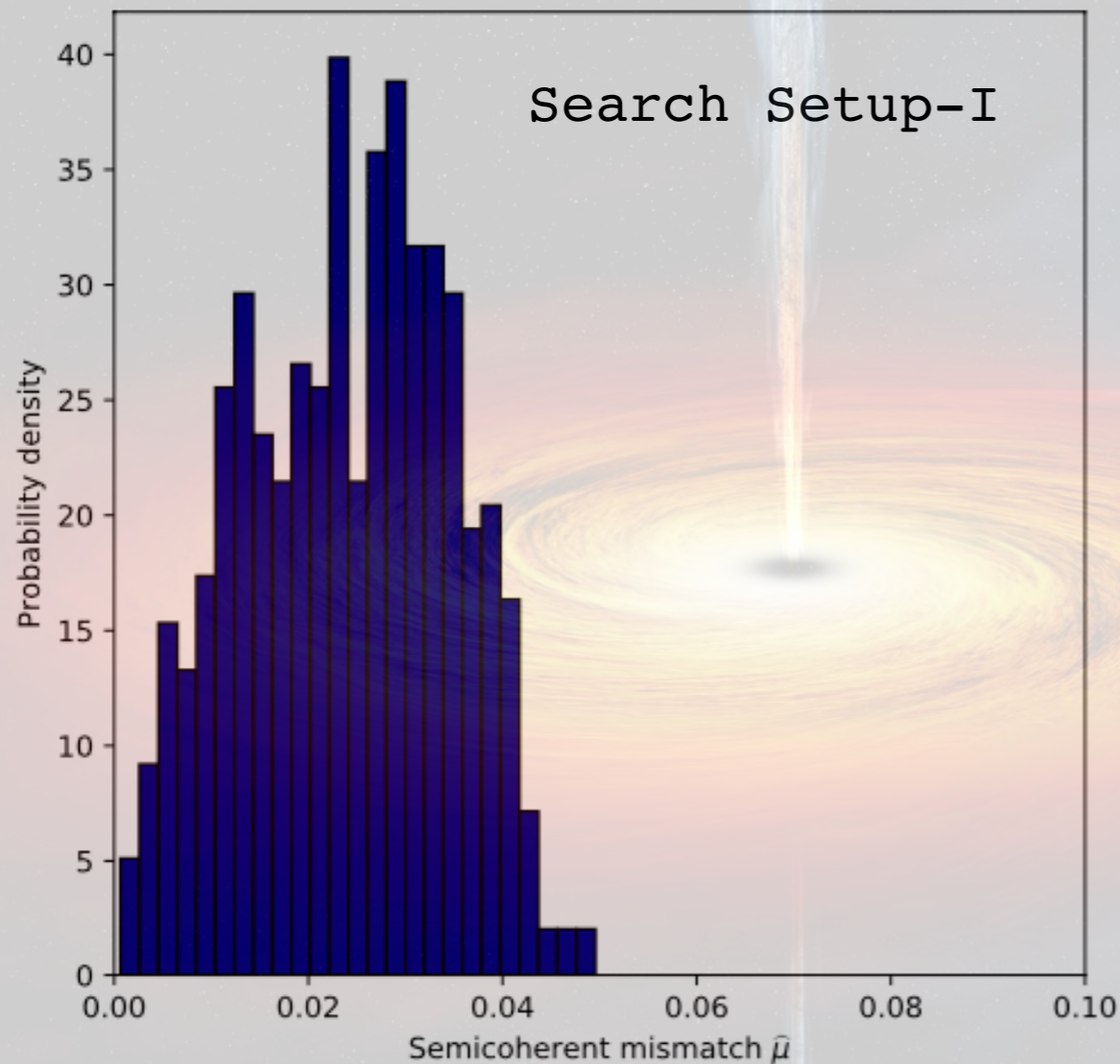
- Example of `BINARYWEAVE` pipeline characteristics and timing model for two search setups:

Table I

Search setup	$T_{\text{obs}}$ [months]	$\Delta T$ [days]	$N$	$\mu_{\text{max}}$
search setup-I	6	1	180	0.031
search setup-II	12	3	120	0.056

# Mismatch Distribution: Small Mismatch Test

- ❖ Two sets of 500 randomly drawn injection-recovery samples for small mismatch maximum mismatch ( $\mu_{\max}$ ) = 0.05:



Semicoherent mismatch distribution for the 4D template bank searching  $\{\text{FREQ}, \text{ASINI}, \text{PERIOD}, \text{TASC}\}$  for an injected signal

# Number of Templates

Number of templates ( $= \mathcal{N}$ ) can be calculated as:

$$\mathcal{N} = \theta_n \mu_{\max}^{-n/2} \int_{\mathcal{P}} \sqrt{\det g(\lambda)} d^n \lambda,$$

where,  $\theta_n = \begin{cases} 2^{-n} n^{n/2} & \text{for } \mathbb{Z}_n, \\ \sqrt{n+1} \left[ \frac{n(n+2)}{12(n+1)} \right]^{n/2} & \text{for } A_n^*. \end{cases}$

$n$ : number of dimension

$\mu_{\max}$ : maximum mismatch

Total number of templates for the 4D search  
of {FREQ, ASINI, PERIOD, TASC} is:

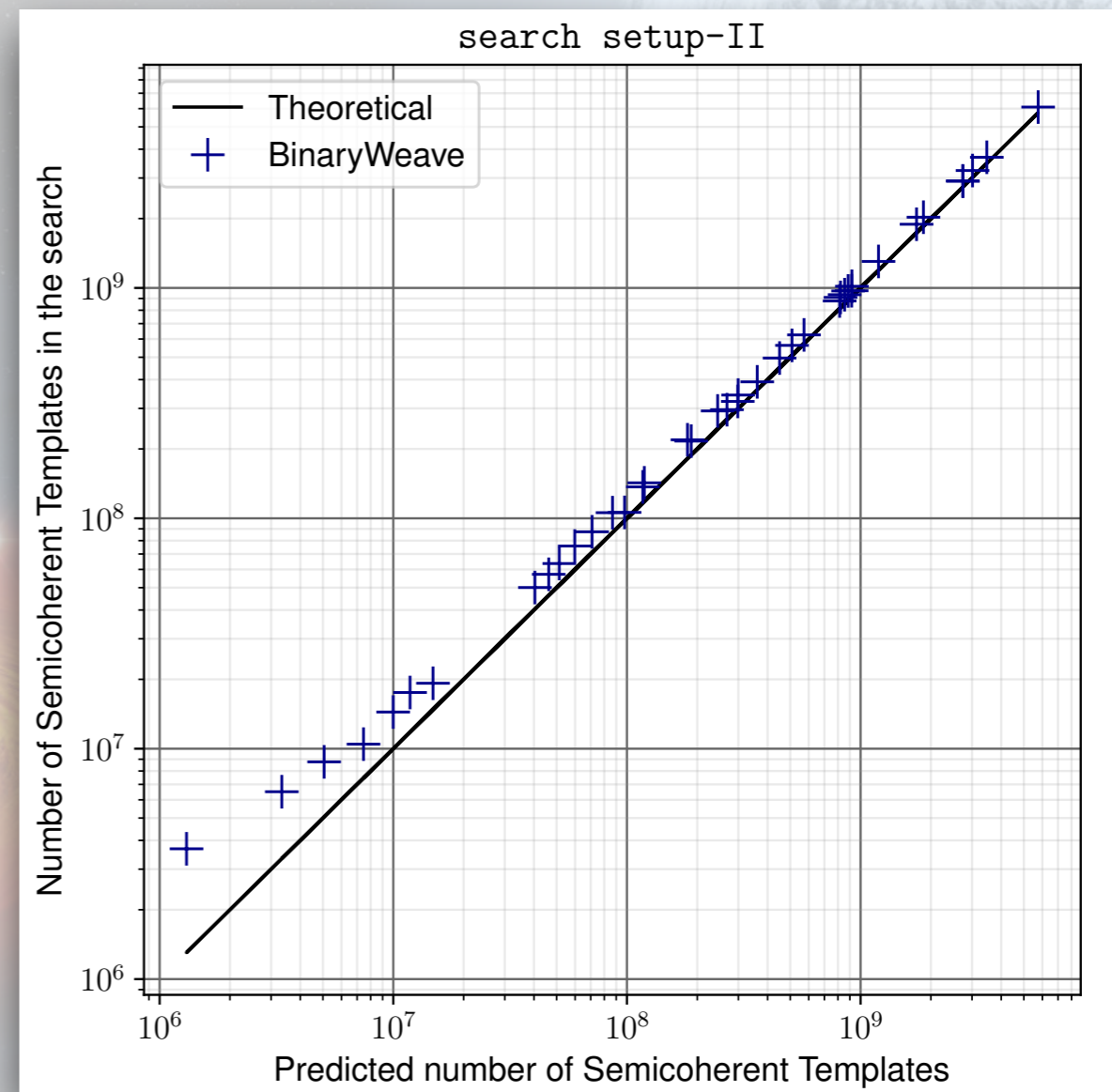
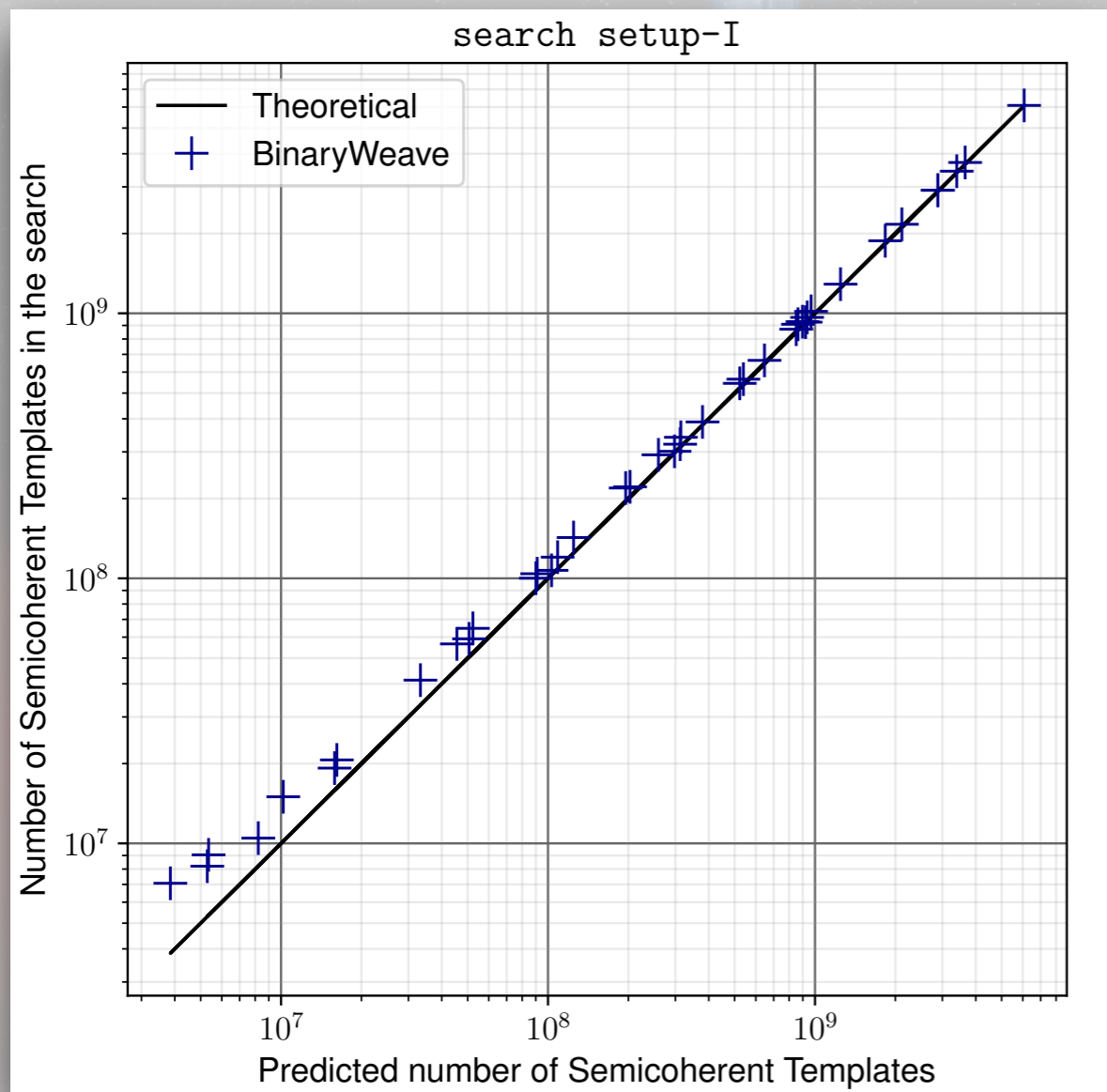
$$\hat{\mathcal{N}}_{4D} = \frac{\theta_4}{\mu_{\max}^2} \frac{\pi^4 \gamma \Delta T^2}{36\sqrt{2}} (f_{\max}^4 - f_{\min}^4) (a_{p,\max}^3 - a_{p,\min}^3) \\ \times (\Omega_{\max}^2 - \Omega_{\min}^2) (t_{\text{asc},\max} - t_{\text{asc},\min}),$$

with,

$$\gamma = \sqrt{1 + 12 \frac{(\overline{\Delta}_{\text{ma}}^2 - \overline{\Delta}_{\text{ma}}^2)}{\Delta T^2}}.$$

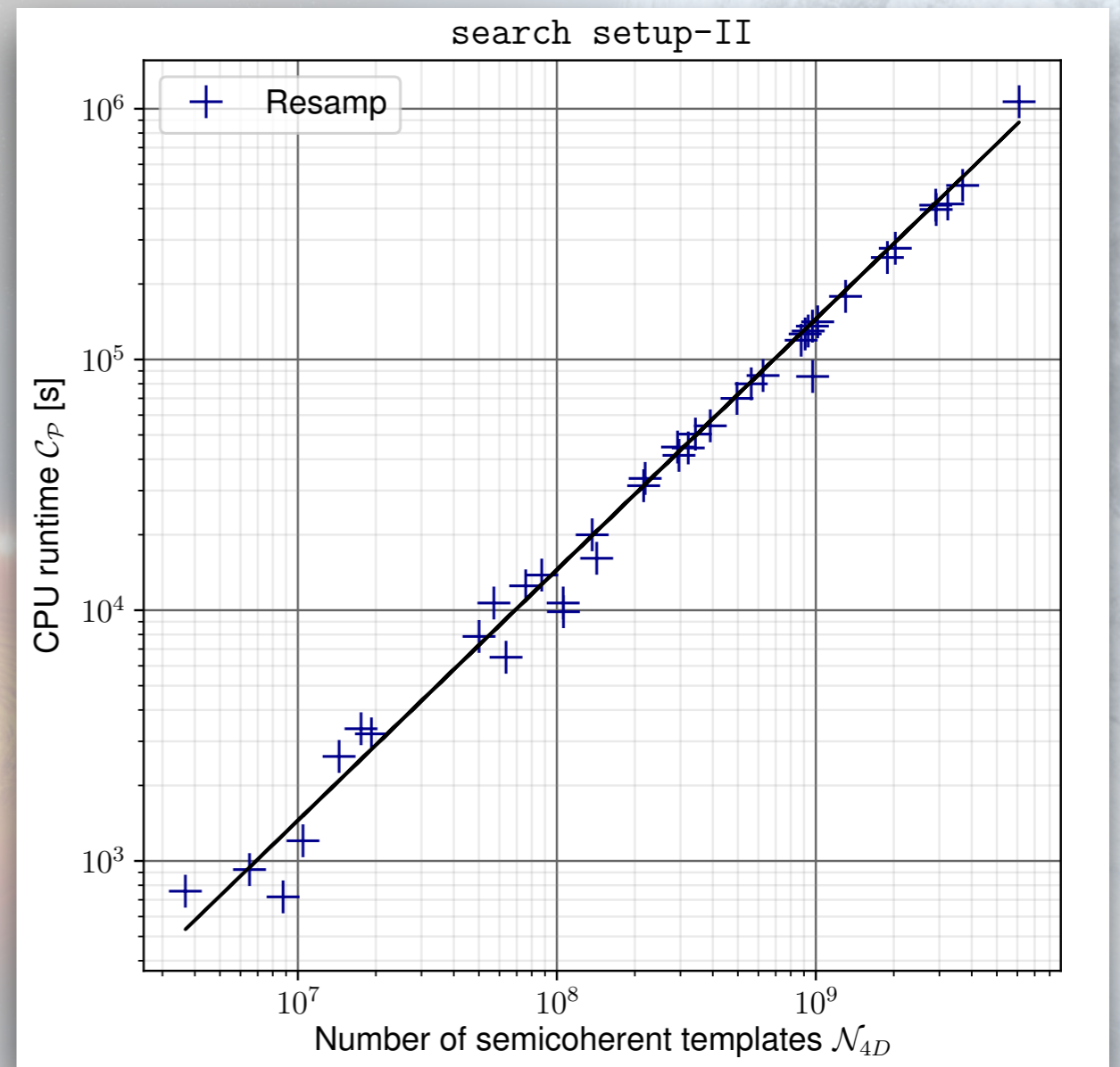
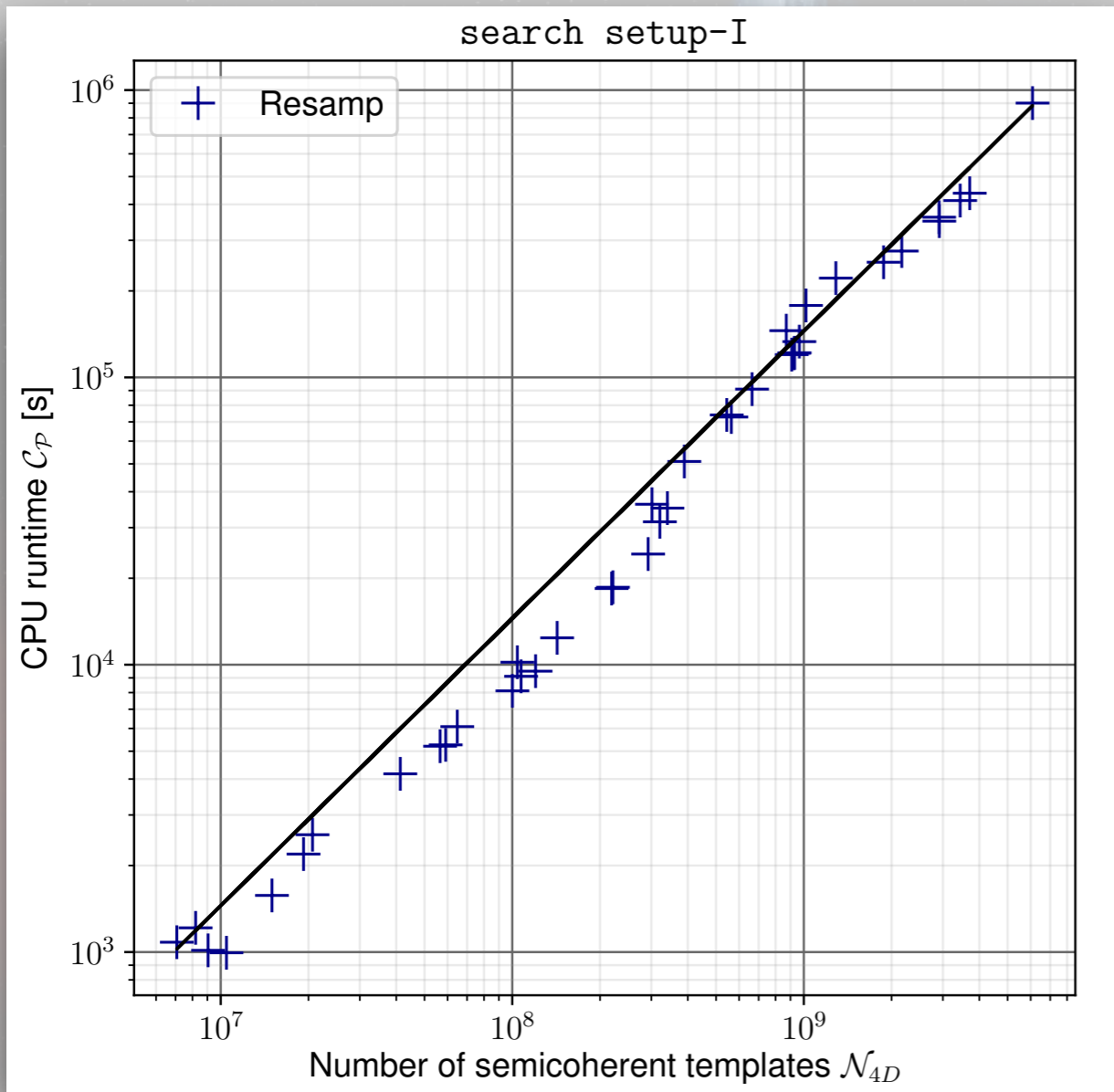
[Leaci & Prix, PRD 91, 102003 (2015)]

# BinaryWeave: Template Bank Size



Number of semicoherent templates  $N_{4D}$  constructed by BinaryWeave versus with the theoretical predictions. Each point '+' corresponds to a simulated 4D-box search around a randomly chosen parameter-space location.

# BinaryWeave: Timing Model



CPU run-time  $C_P$  per search box as a function of the number of (semi-coherent) templates  $N_{4D}$  for that box, for **SEARCH SETUP-I** (left plot) and **SEARCH SETUP-II** (right plot), defined in Table. II. The points '+' mark the measured **BINARYWEAVE** run times, while the solid line indicates the effective cost model prediction, using an effective cost per template of  $C_t = 0.145\text{ms}$ .



# Sensitivity Depths

- Sensitivity Depths (with per-template false-alarm probability ‘ $p_{fa}$ ’ and detection probability ‘ $p_{det}$ ’) is defined as:

$$\mathcal{D}_{p_{fa}}^{p_{det}} \equiv \frac{\sqrt{S_n}}{h_{p_{fa}}^{p_{det}}}$$

- Sensitivity Depths for 6 different search setups at ‘ $p_{fa}$ ’ =  $10^{-10}$  are:

Table II

Search setup	$T_{obs}$ [months]	$\Delta T$ [days]	$N$	$\mu_{max}$	$\mathcal{D}_{p_{fa}}^{90\%}$ [ $1/\sqrt{\text{Hz}}$ ]	$\mathcal{D}_{p_{fa}}^{95\%}$ [ $1/\sqrt{\text{Hz}}$ ]	$\mathcal{D}_{p_{fa}}^{99\%}$ [ $1/\sqrt{\text{Hz}}$ ]
search setup-I	6	1	180	0.031	77	72	60
search setup-II	12	3	120	0.056	116	107	91
search setup-III	6	3	60	0.025	96	89	75
search setup-IV	12	1	360	0.025	93	86	73
search setup-V	6	10	18	0.025	120	111	94
search setup-VI	12	10	36	0.025	150	138	117

# Sensitivity Depths at Different Computational Costs

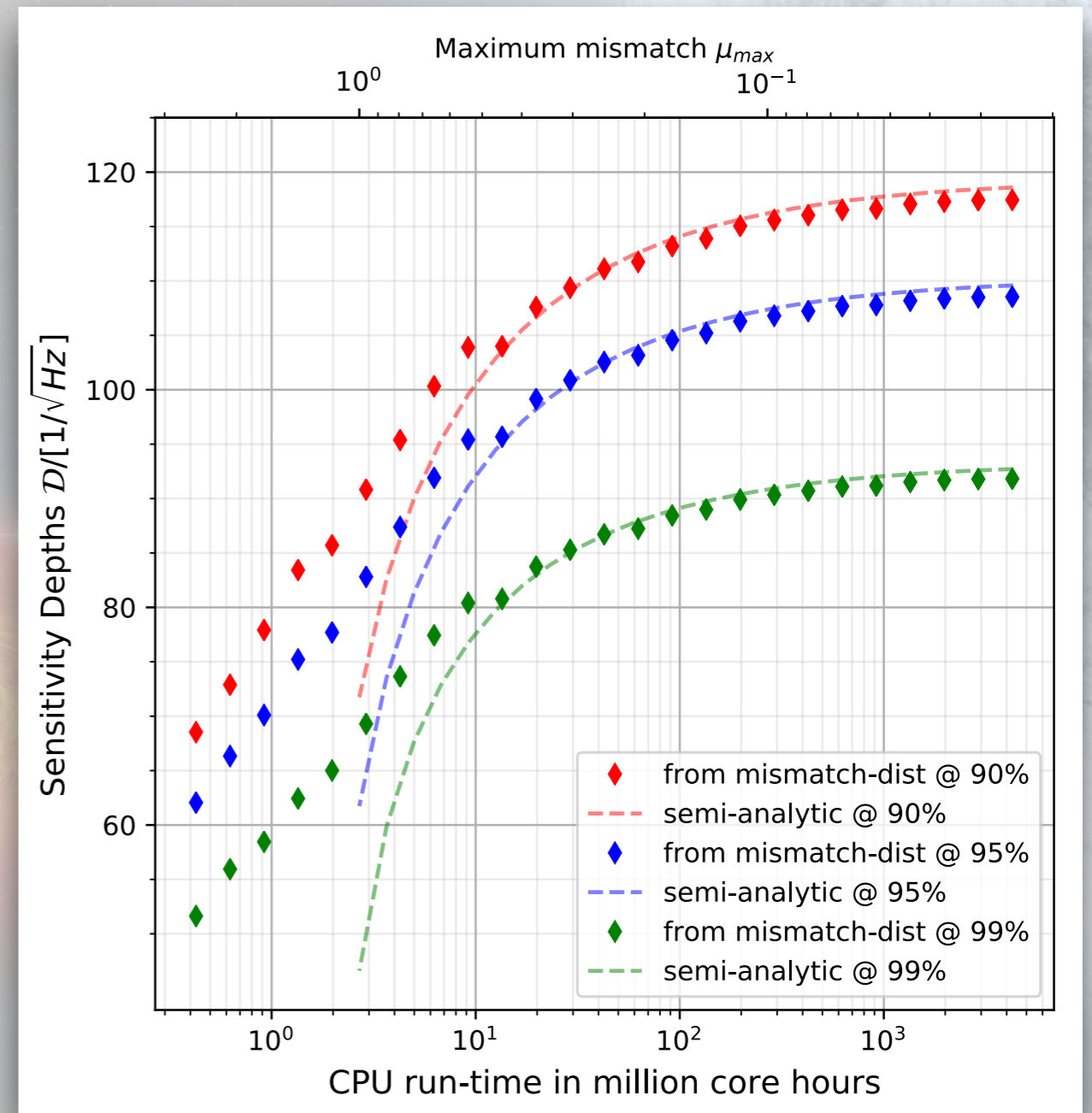
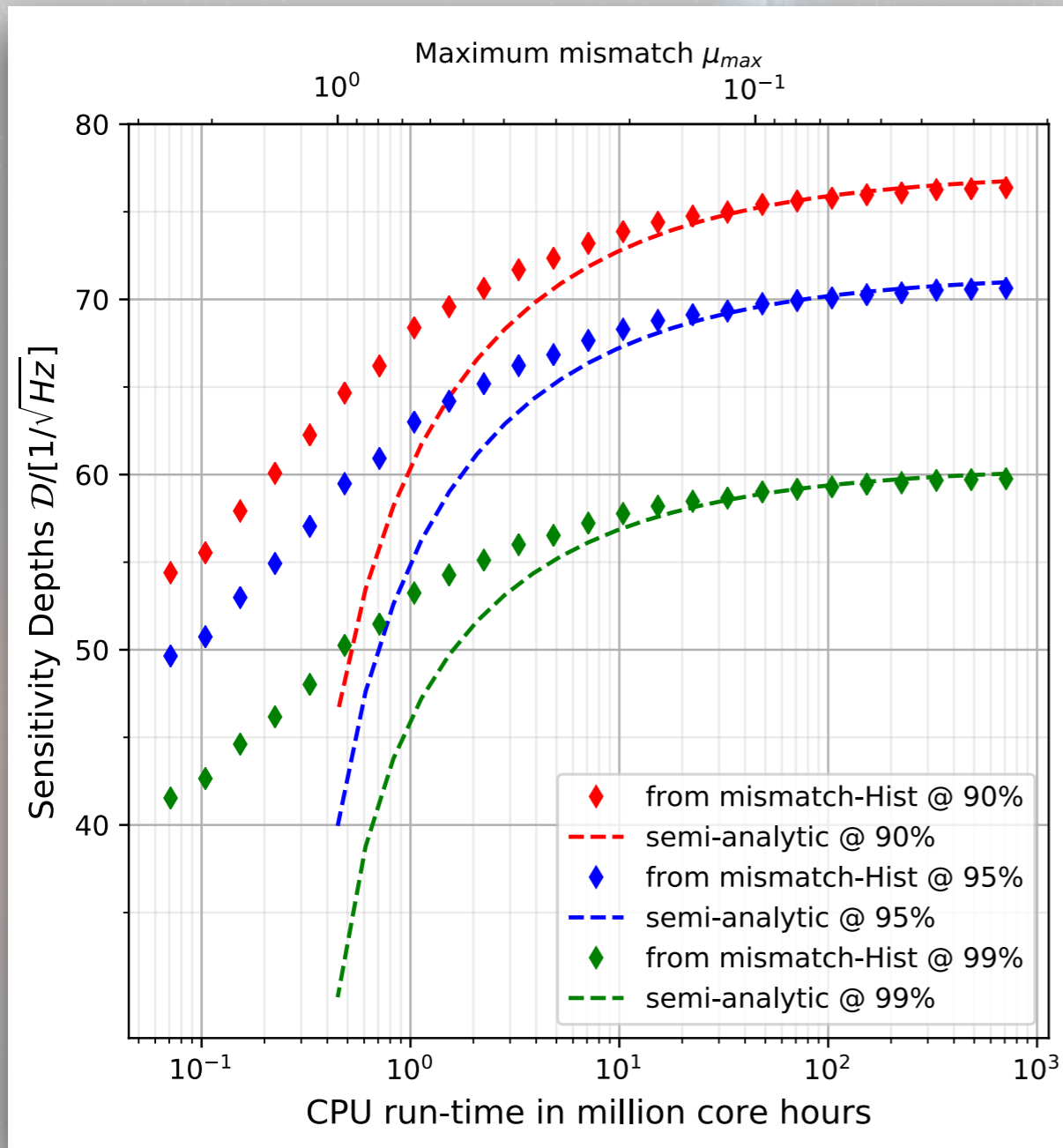


Fig: Lower-limit of search sensitivity as a function of computational cost is shown here. The left panel corresponds to **SEARCH SETUP-I** ( $T_{\text{obs}} = 180$  days,  $T_{\text{seg}} = 1$  day) and the right-panel corresponds to **SEARCH SETUP-II** ( $T_{\text{obs}} = 360$  days,  $T_{\text{seg}} = 3$  days).



**What `BINARYWEAVE` Can Say About  
High Frequency/Larger Parameter  
Space Search?**

# Observational Scenarios: Different Parameter Spaces

Table III

Search space $\mathcal{P}$	$f$ [Hz]	$a_p$ [ls]	$P_{\text{orb}}$ [s]	$t_{\text{asc}}$ [GPS s]	Reference(s)/comment(s)
$\mathcal{P}_0$	10–700	0.3–3.5	$68023.7 \pm 0.2$	$1124044455.0 \pm 1000$	BINARYWEAVE test range
$\mathcal{P}_1$	20–500	1.26–1.62	$68023.70496 \pm 0.0432$	$897753994 \pm 100$	Leaci and Prix [36]
$\mathcal{P}_2$	60–650	1.45–3.25	$68023.86048 \pm 0.0432$	$974416624 \pm 50$	Abbott <i>et al.</i> [28]
$\mathcal{P}_3$	40–180	1.45–3.25	$68023.86 \pm 0.12$	$1178556229 \pm 417$	Zhang <i>et al.</i> [29]
$\mathcal{P}_4$	600–700				different ranges in frequency with broad range in $a_p$
$\mathcal{P}_5$	1000–1100				
$\mathcal{P}_6$	1400–1500	1.45–3.25	$68023.70496 \pm 0.0432$	$974416624 \pm 100$	
$\mathcal{P}_7$	20–250				
$\mathcal{P}_8$	20–1000				
$\mathcal{P}_9$	20–1500				
$\mathcal{P}_{10}$	600–700				different ranges in frequency with narrow range in $a_p$
$\mathcal{P}_{11}$	1000–1100				
$\mathcal{P}_{12}$	1400–1500	1.40–1.50	$68023.70496 \pm 0.0432$	$974416624 \pm 100$	
$\mathcal{P}_{13}$	20–500				
$\mathcal{P}_{14}$	20–1000				
$\mathcal{P}_{15}$	20–1500				
$\mathcal{P}_{16}$	600–700				different ranges in frequency with well-constrained $a_p$
$\mathcal{P}_{17}$	1000–1100				
$\mathcal{P}_{18}$	1400–1500	1.44–1.45	$68023.70496 \pm 0.0432$	$974416624 \pm 100$	
$\mathcal{P}_{19}$	20–500				
$\mathcal{P}_{20}$	20–1000				
$\mathcal{P}_{21}$	20–1500				

Different parameter space search regions considered for Sco X-1.  $\mathcal{P}_0$  has been used in this study as a test range for various Monte-Carlo tests of **BINARYWEAVE**.  $\mathcal{P}_{1-3}$  represent observational constraints considered in recent CW searches and studies. In addition, various combinations of parameter-ranges are considered,  $\mathcal{P}_{4-21}$ , in order to explore the impact of improved observation constraints and reduced search ranges.

# Computational Costs for Different Parameter Space

Table IV

	(I,3D)	(I,4D)	(II,3D)	(II,4D)
$\mathcal{P}_1$	3.18	23.51	3.93	43.23
$\mathcal{P}_2$	28.50	466.48	35.22	857.69
$\mathcal{P}_3$	5.00	63.40	6.17	116.57
$\mathcal{P}_4$	26.38	577.57	32.60	1061.95
$\mathcal{P}_5$	68.76	2425.79	84.96	4460.17
$\mathcal{P}_6$	131.09	6381.48	161.97	11733.30
$\mathcal{P}_7$	3.24	20.42	4.01	37.54
$\mathcal{P}_8$	207.74	5226.87	256.69	9610.37
$\mathcal{P}_9$	701.14	26461.02	866.33	48652.49
$\mathcal{P}_{10}$	0.90	11.65	1.12	21.42
$\mathcal{P}_{11}$	2.36	48.94	2.91	89.97
$\mathcal{P}_{12}$	4.49	128.73	5.55	236.70
$\mathcal{P}_{13}$	0.11	0.41	0.14	0.76
$\mathcal{P}_{14}$	7.12	105.44	8.80	193.87
$\mathcal{P}_{15}$	24.03	533.80	29.70	981.46
$\mathcal{P}_{16}$	0.09	1.16	0.11	2.13
$\mathcal{P}_{17}$	0.23	4.86	0.29	8.93
$\mathcal{P}_{18}$	0.45	12.78	0.55	23.50
$\mathcal{P}_{19}$	0.01	0.04	0.01	0.08
$\mathcal{P}_{20}$	0.71	10.47	0.88	19.25
$\mathcal{P}_{21}$	2.40	52.99	2.96	97.43

- Computing-cost estimates in million core hours [Mh] for different parameter spaces  $\mathcal{P}_n$  ( $n = 1, 2, \dots, n$ ) defined in Table. III.
- We consider two setups, **SEARCH SETUP-I** and **SEARCH SETUP-II** of Table I, assuming either a 3D or 4D template-bank.

3D search: **FREQ, ASINI, TASC**

4D search: **FREQ, ASINI, PORB, TASC**

Table I

Search setup	$T_{\text{obs}}$ [months]	$\Delta T$ [days]	$N$	$\mu_{\text{max}}$
search setup-I	6	1	180	0.031
search setup-II	12	3	120	0.056

# Prospective Future Direction

- Communicate with **EM-observations for better constraints** on orbital parameters and spin-frequency of NS in Sco X-1: e.g. X-ray/Optical/IR/radio observers?
- Thoroughly searching for X-ray pulsation from Sco X-1 (*detection will be the game-changer!*) => challenging but worthwhile [[Galudage et al., MNRAS 509, 1745 \(2022\)](#)]
- Convince the EM-observer to make an updated observation of  $\mathbf{P}_{\text{ORB}}$  and  $\mathbf{T}_{\text{ASC}}$  near the middle of an observing run to maximise the benefits
  - ➡ It is worth exploring if long-term ( $\sim 5\text{-}10$  yrs) phase-evolution of Sco X-1 binary orbit can provide stricter constraints on  $\mathbf{P}_{\text{ORB}}$  and  $\mathbf{T}_{\text{ASC}}$
- Perhaps communicating with larger community to regarding tighter constraints on **ASINI**
  - ➡ It will need deep observations in optical/IR/radio bands dedicated for this purpose; it will be critical for a breakthrough!
- Possibility of implementing GPU-based computation of  $\mathcal{F}$ -statistic (e.g., **CUDA**, **OPENCL**?) [[Wette et al., PRD 103, 083020](#)]
- Spin-wandering effect due to stochastic accretion rate [[AM, Messenger & Riles, PRD 97, 043016 \(2018\)](#)] has been neglected in this study; worth incorporating Viterbi-like summing of segments for  $\mathcal{F}$ -statistic [[Melatos et al., PRD 104, 042003 \(2021\)](#)]

# References Important for BINARYWEAVE

1. P. Leaci & R. Prix, *Phys. Rev. D* 91, 102003 (2015)
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3. K. Wette, *Phys. Rev. D* 90, 122010 (2014)
4. K. Wette, S. Walsh, R. Prix, M. A. Papa, *Phys. Rev. D* 97, 123016 (2018)
5. P. Jaranowski, A. Krolak, and B. F. Schutz, *Phys. Rev. D* 58, 063001 (1998) [JKS]
6. R. Prix, *Phys. Rev. D* 75, 023004 (2007)
7. R. Prix, *Classical Quantum Gravity* 24, S481 (2007)
8. CW F2F presentation by R. Prix at LVC-meeting, Budapest (2015); DCC: <[LIGO-G1501145-v2](#)>
9. K. Wette, *Phys. Rev. D* 90, 122010 (2014)
10. A. Mukherjee, R. Prix, and K. Wette, *Phys. Rev. D* 107, 062005 (2023) [MPW]

A composite image of space phenomena. On the left, a black hole is depicted with a bright yellow and orange accretion disk and a blue jet extending upwards. In the center, a bright star with a lens flare is visible. On the right, a large, blue, textured nebula or gas cloud is shown. The background is a dark field of stars.

**THANK YOU!**





# Backup Slides

# Optimal Covering Lattice OR Optimal Detection Lattice?

- Recently in a series of papers, Allen et. al. pointed out that an optimal covering lattice is NOT necessarily an optimal detection lattice
- The quantity that maximises detection probability is the optimal lattice quantiser [B. Allen, PRD (2021), B. Allen and E. Agrell, Ann. der Phys. (2021), B. Allen and A. Shoom, PRD (2021)]
- The quantiser constant  $\mathcal{G}$  is the second moment, i.e., average squared distance from the nearest templates [B. Allen, PRD (2021)]
- However, it turned out that the advantage of optimal detection lattices (as pointed out by Allen+) offer only marginal improvements [B. Allen and A. Shoom, PRD (2021)]
  - ➔  $A_n^*$  lattices seem to be near-optimal choice for  $n = 3 - 8$  dim