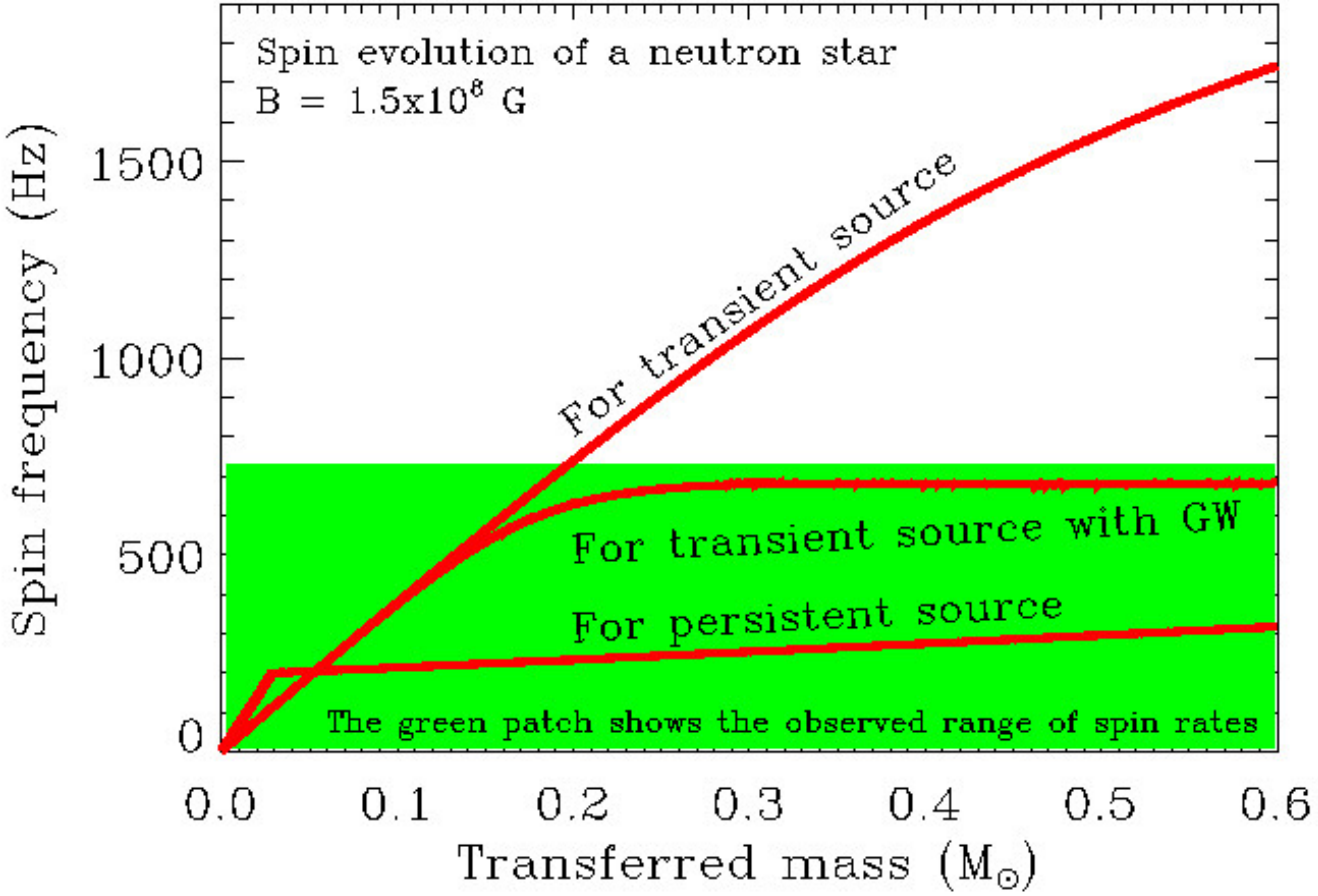
Computation of spin evolution of millisecond pulsars: a way to probe continuous gravitational waves



Sudip Bhattacharyya

Department of Astronomy and Astrophysics

Tata Institute of Fundamental Research

Mumbai, India

Plan

1) Background: neutron star spin evolution

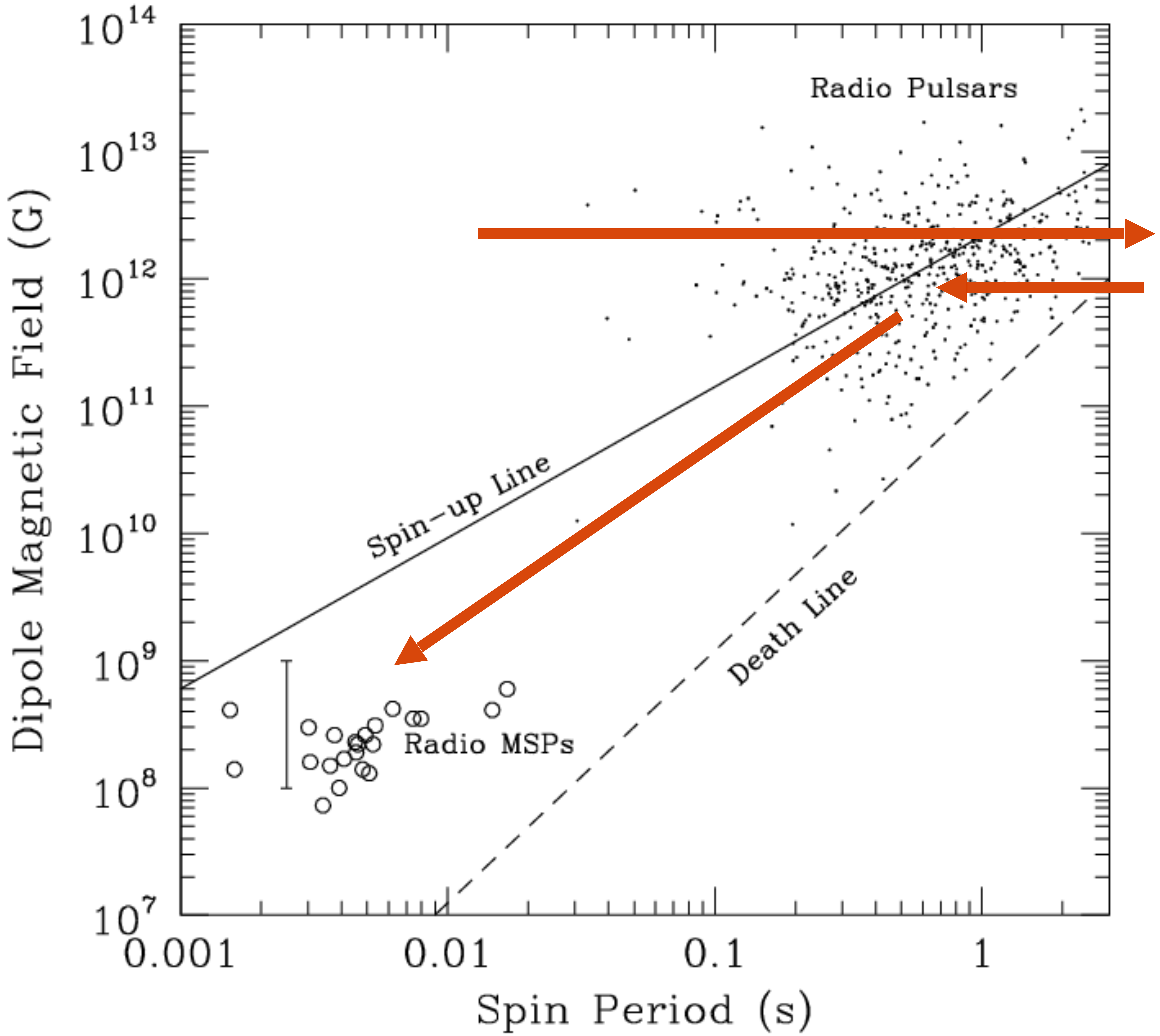
2) Standard picture of spin evolution

3) New picture: spin evolution for transient accretion

4) Implication of the new picture: a counter-intuitive scenario

5) Takeaway message

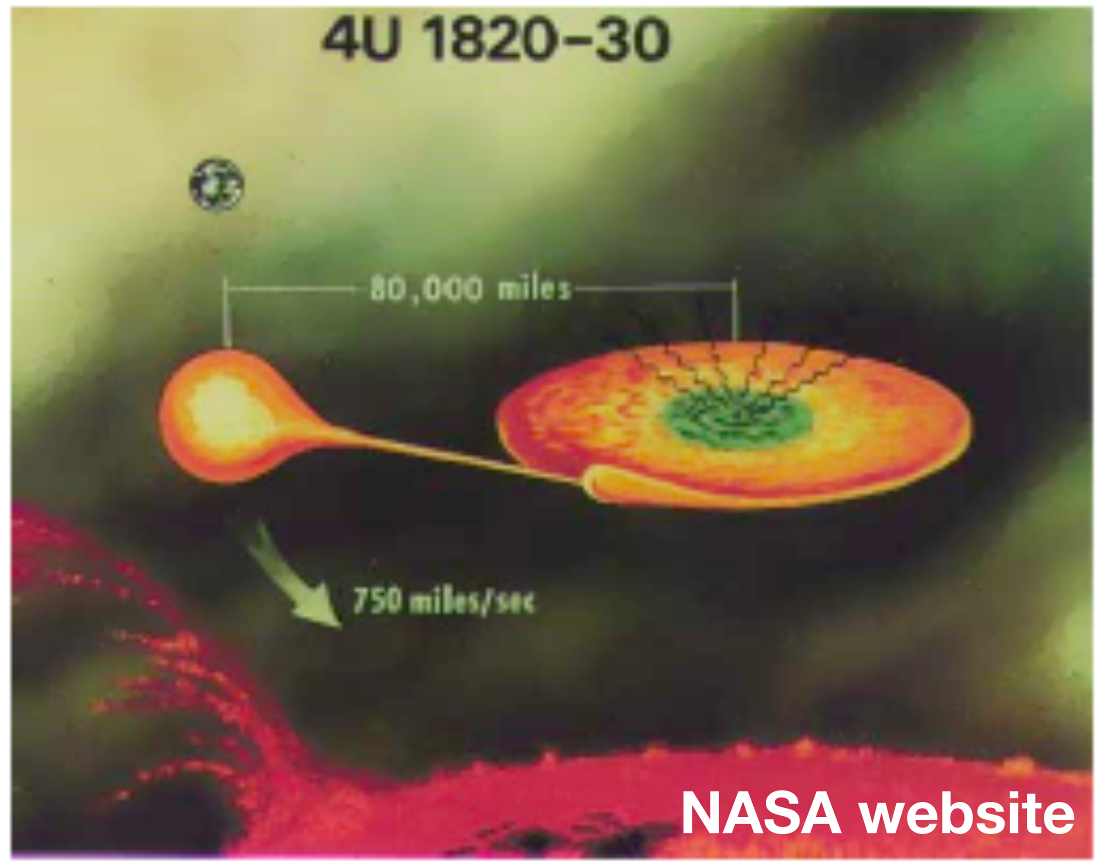
Background: neutron star spin evolution



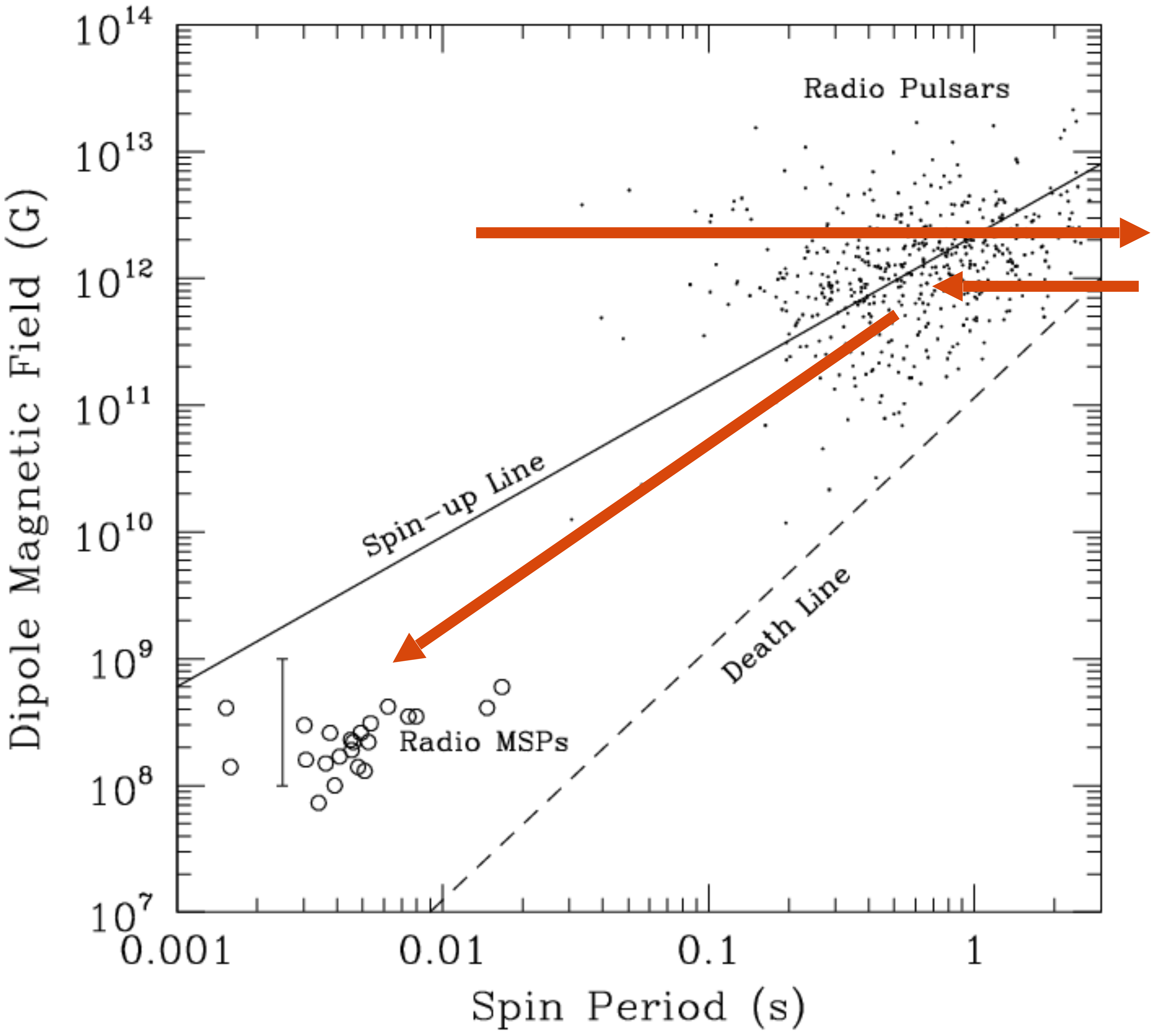
Courtesy: D. Chakrabarty

1. Neutron stars are typically born with $B \sim 10^{12}$ G, $P \sim 20$ ms. They spin down due to radiative loss of spin kinetic energy.

2. If accreting matter from a low-mass donor star (in the low-mass X-ray binary [LMXB] phase), then the accretion (for $\sim 10^9$ yr) decays the neutron star magnetic field to $B \sim 10^8$ G, and increases the stellar spin frequency (ν) to > 100 Hz.



Background: neutron star spin evolution



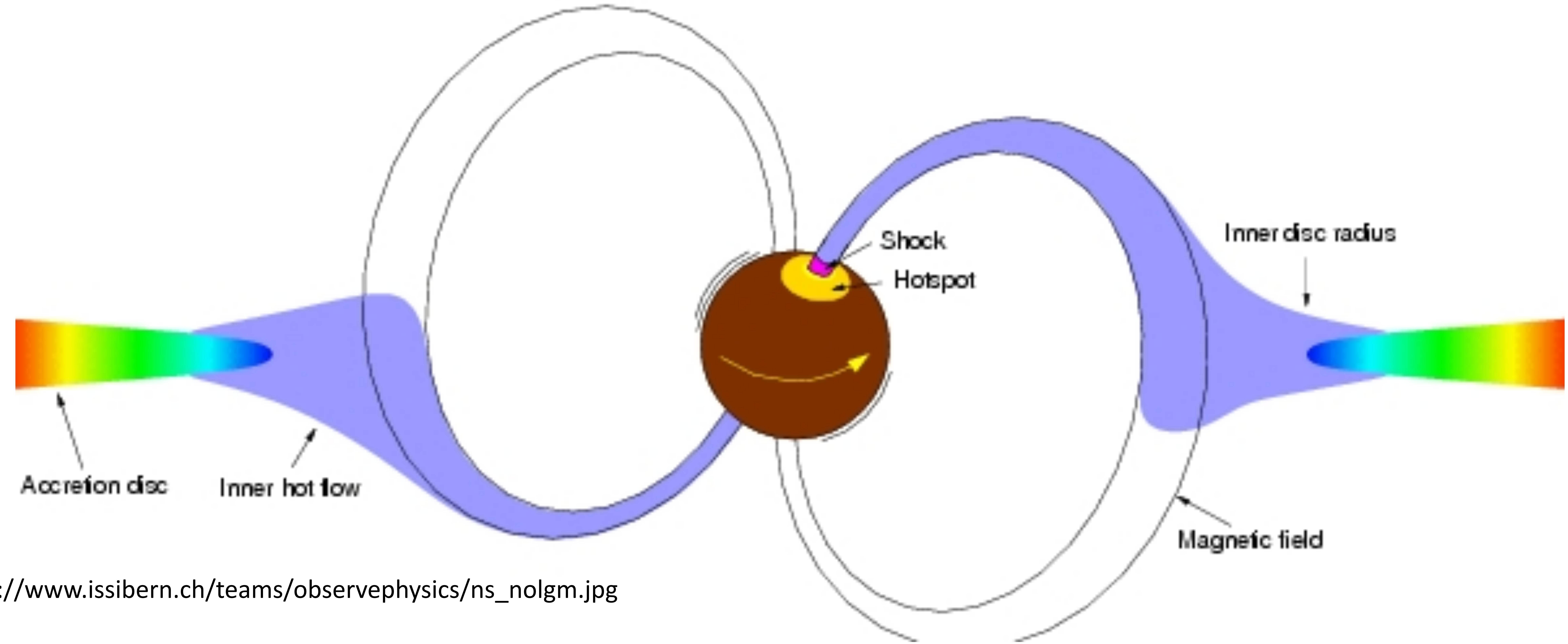
Courtesy: D. Chakrabarty

1. Neutron stars are typically born with $B \sim 10^{12}$ G, $P \sim 20$ ms. They spin down due to radiative loss of spin kinetic energy.

2. If accreting matter from a low-mass donor star (in the low-mass X-ray binary [LMXB] phase), then the accretion (for $\sim 10^9$ yr) decays the neutron star magnetic field to $B \sim 10^8$ G, and increases the stellar spin frequency (ν) to > 100 Hz.

3. At the end of the accretion phase, the donor star is exhausted or binary is detached, leaving a rapidly spinning neutron star (which could appear as a millisecond radio pulsar).

Background: neutron star spin evolution

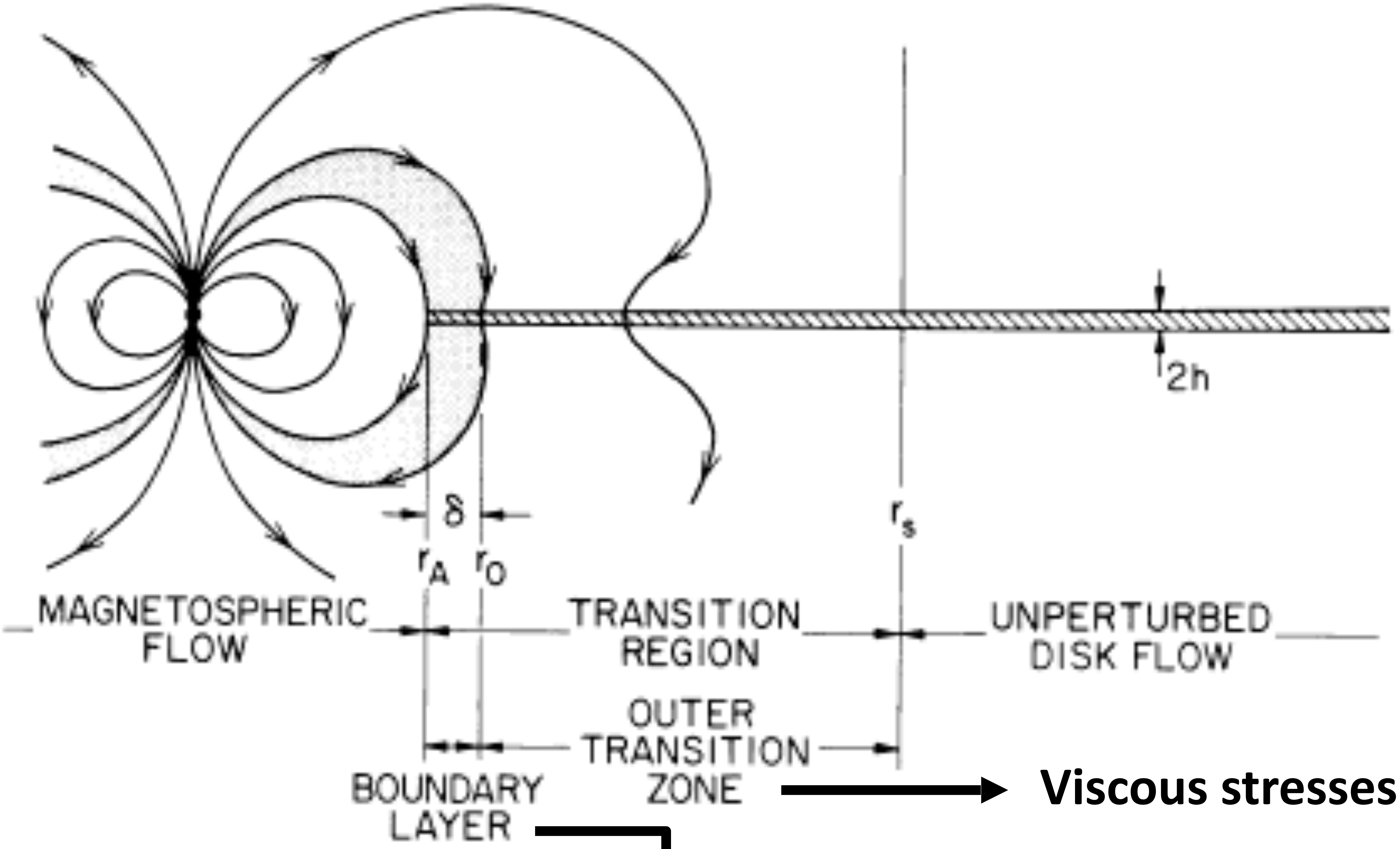


http://www.issibern.ch/teams/observephysics/ns_nolgm.jpg

For more than twenty LMXBs, magnetically channeled accretion flow onto polar caps makes hot spots and hence observable pulsations of X-ray intensity. These are accretion-powered millisecond X-ray pulsars.

Standard picture of spin evolution

Disk-magnetosphere interaction

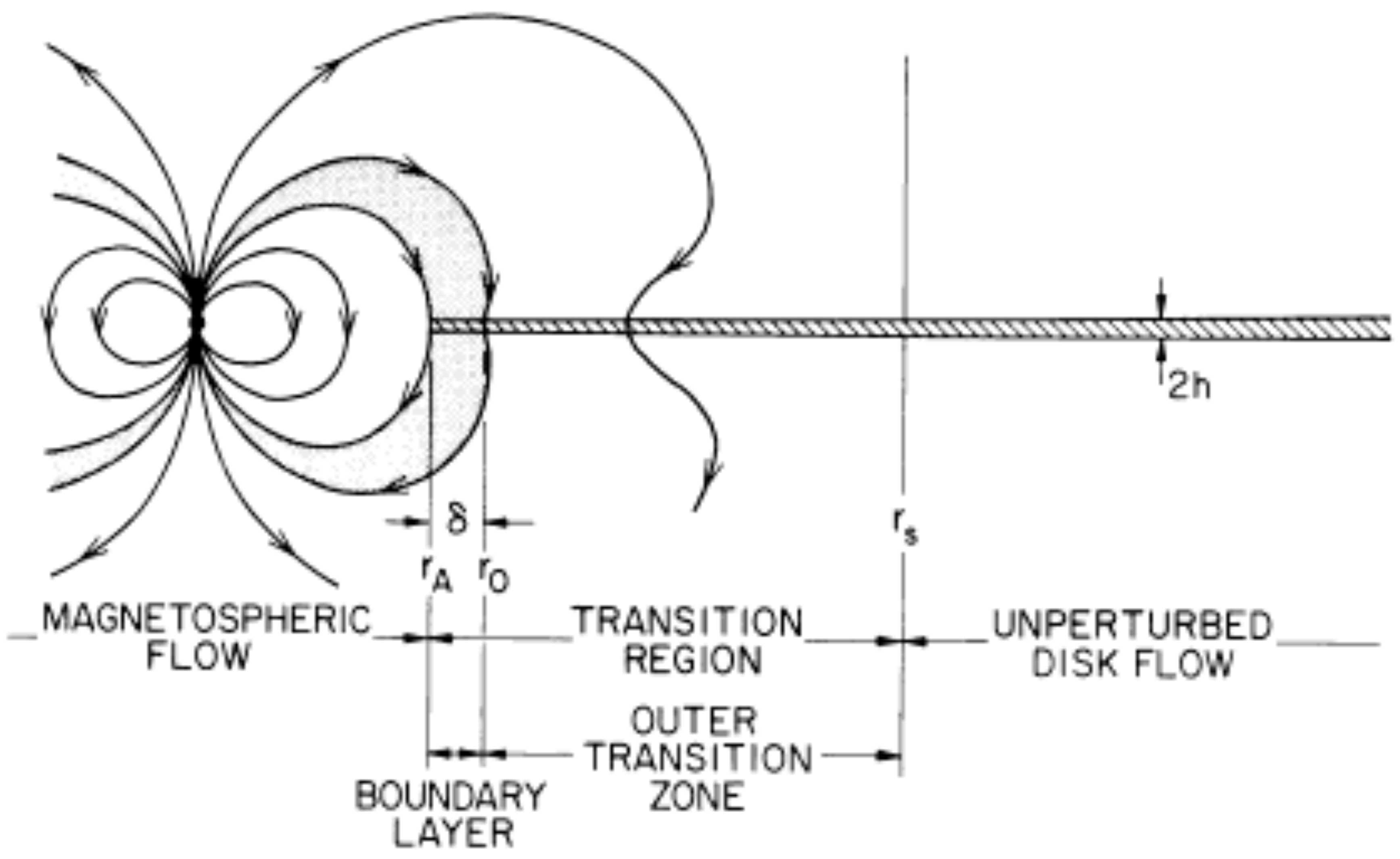


Viscous stresses dominate

Magnetic stresses dominate ($\delta << r_A$)

Ghosh and Lamb (1978)

Disk-magnetosphere interaction



Magnetospheric radius

$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, where the accretion disk stops. ($\xi \sim 0.3$-1.4).

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$, where the Keplerian spin frequency is equal to the neutron star spin frequency.

For $r_m < r_{co}$ ⇒ positive torque (for example, $\dot{M}.[GMr_m]^{0.5}$) ⇒ spin up ⇒ r_{co} decreases

For $r_m > r_{co}$ ⇒ negative torque (and propeller effect) ⇒ spin down ⇒ r_{co} increases

So a self-regulated mechanism operates:

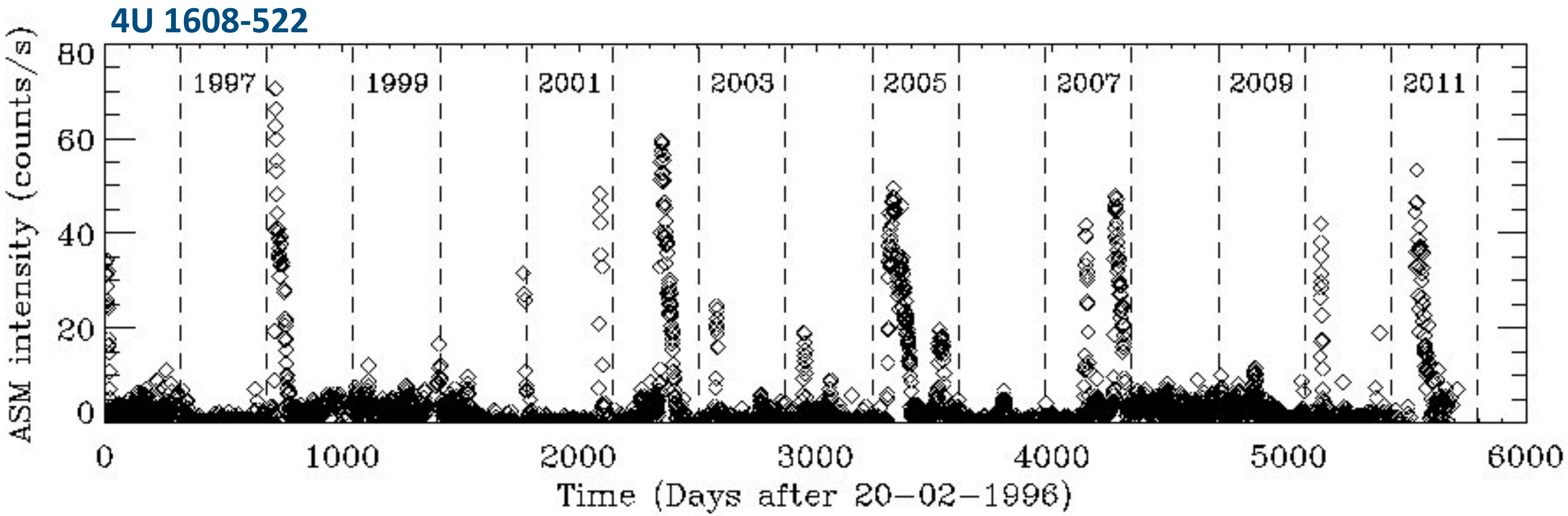
r_{co} tends to r_m. At $r_m = r_{co}$: no torque ⇒ spin equilibrium.

Spin equilibrium frequency $\nu_{eq} = 3000$ Hz . $\xi^{-3/2}.B_8^{-6/7}.R_6^{-18/7}.M^{5/7}.(\dot{M}/\dot{M}_{Edd})^{3/7}$

Neutron star's spin frequency ν evolves to become ν_{eq}

New picture: spin evolution for transient accretion

New Development: Effect of Transient Accretion



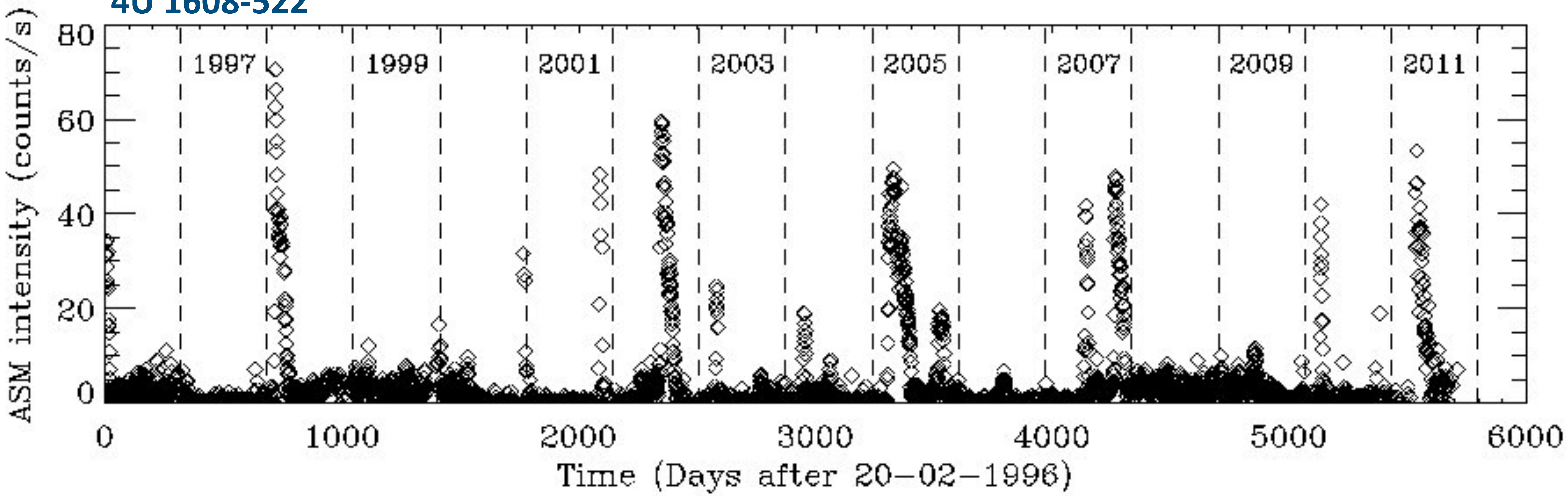
Most of the neutron star LMXBs are X-ray transient sources. Moreover, almost all the X-ray ms pulsars (in fact all known accreting ms pulsars) are transients. So the effect of transient accretion on the ms pulsar spin evolution should be considered.

But pulsar spin evolution is traditionally computed by assuming quasi-persistent accretion at the long-term average accretion rate.

SB & D. Chakrabarty, 2017, ApJ, 835, 4: How is spin evolution affected if transient accretion is treated explicitly?

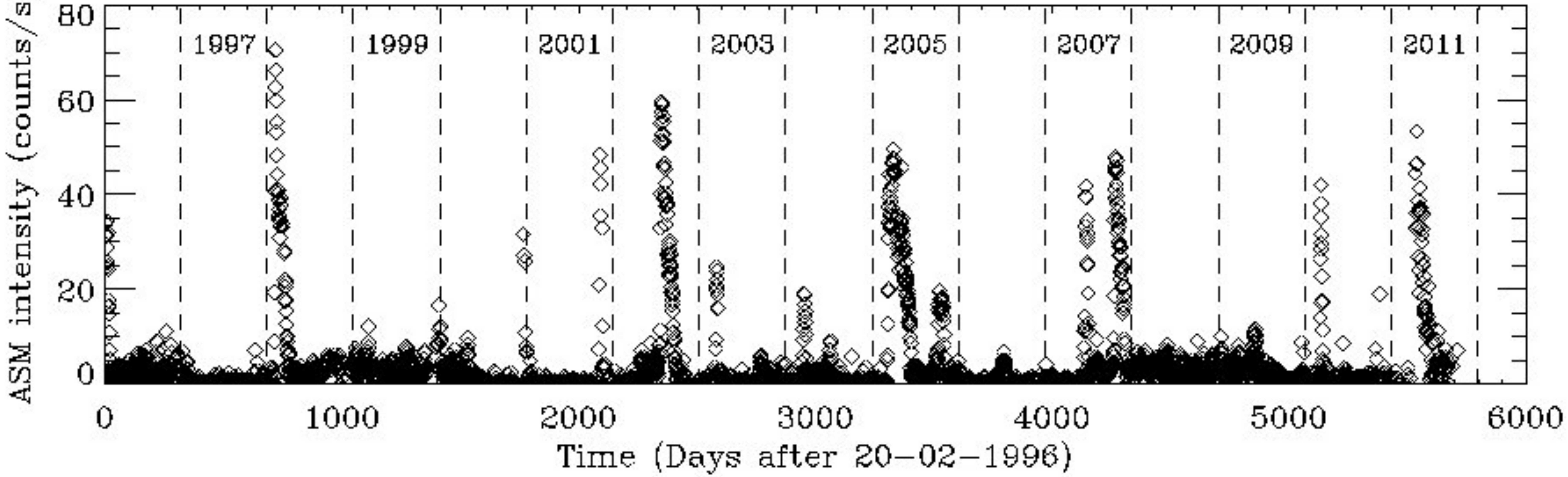
Transient neutron star LMXBs

4U 1608-522



Does transience make any difference in the spin equilibrium condition and frequency?

Transient neutron star LMXBs



Does transience make any difference in the spin equilibrium condition and frequency?

Yes, a crucial difference.

Because, for transients, r_m drastically changes, as $\dot{M}$ ($\propto r_m^{-7/2}$) evolves by several orders of magnitude in an outburst cycle. Therefore, except for one r_m value, the spin equilibrium condition ($r_m = r_{co}$) is not satisfied throughout the outburst.

So how do we estimate the spin equilibrium frequency for a transient source?

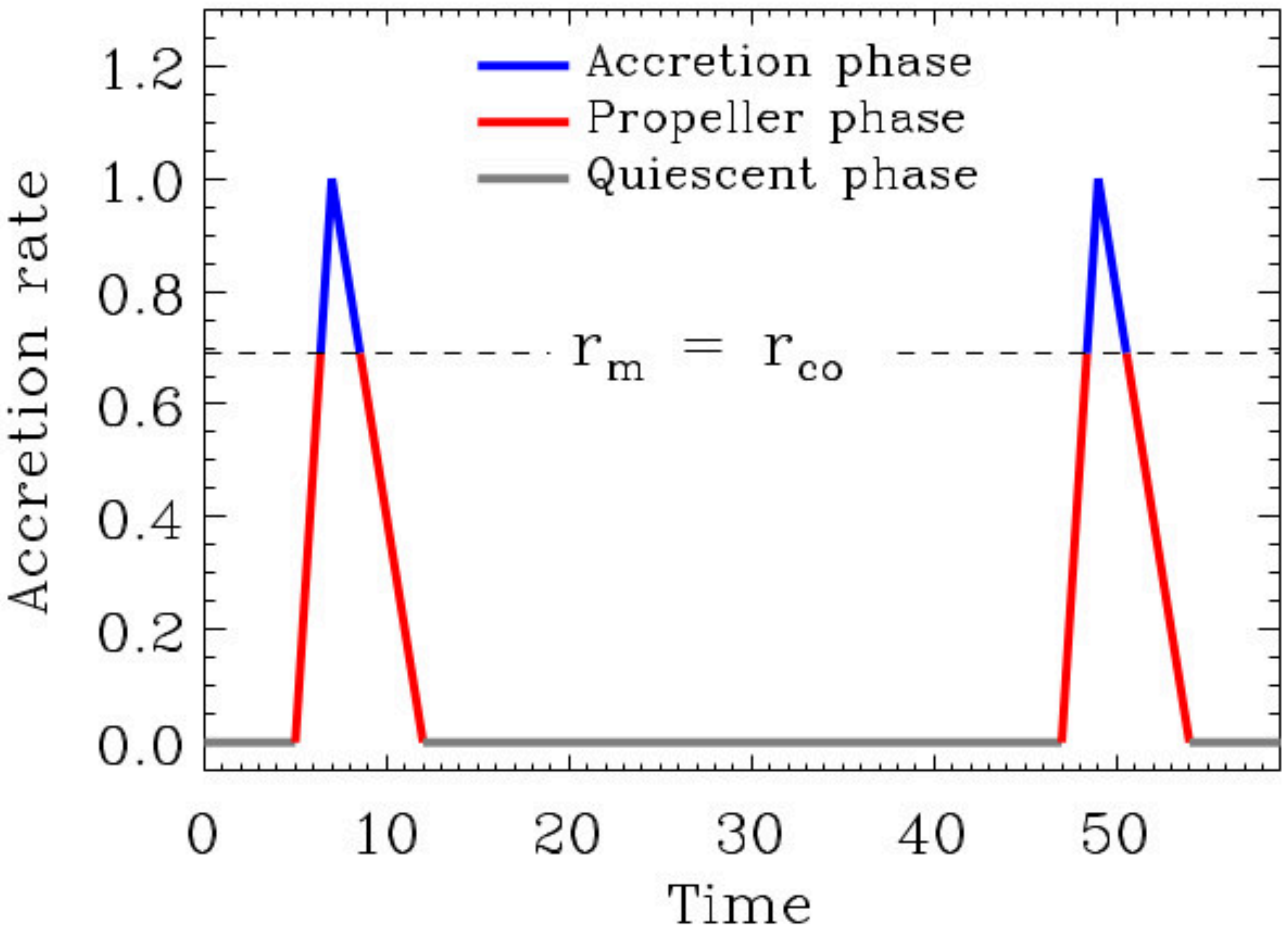
Numerical results

Transient neutron star LMXBs

We numerically compute the spin evolution of a neutron star through a series of outbursts for various sets of parameter values.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Series of outbursts: three phases of each outburst cycle



Magnetospheric radius

$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, **where the accretion disk stops.** ($\xi \sim 0.3\text{-}1.4$).

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$, **where the Keplerian spin frequency is equal to the neutron star spin frequency.**

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Three different regimes of accretion were theoretically identified.

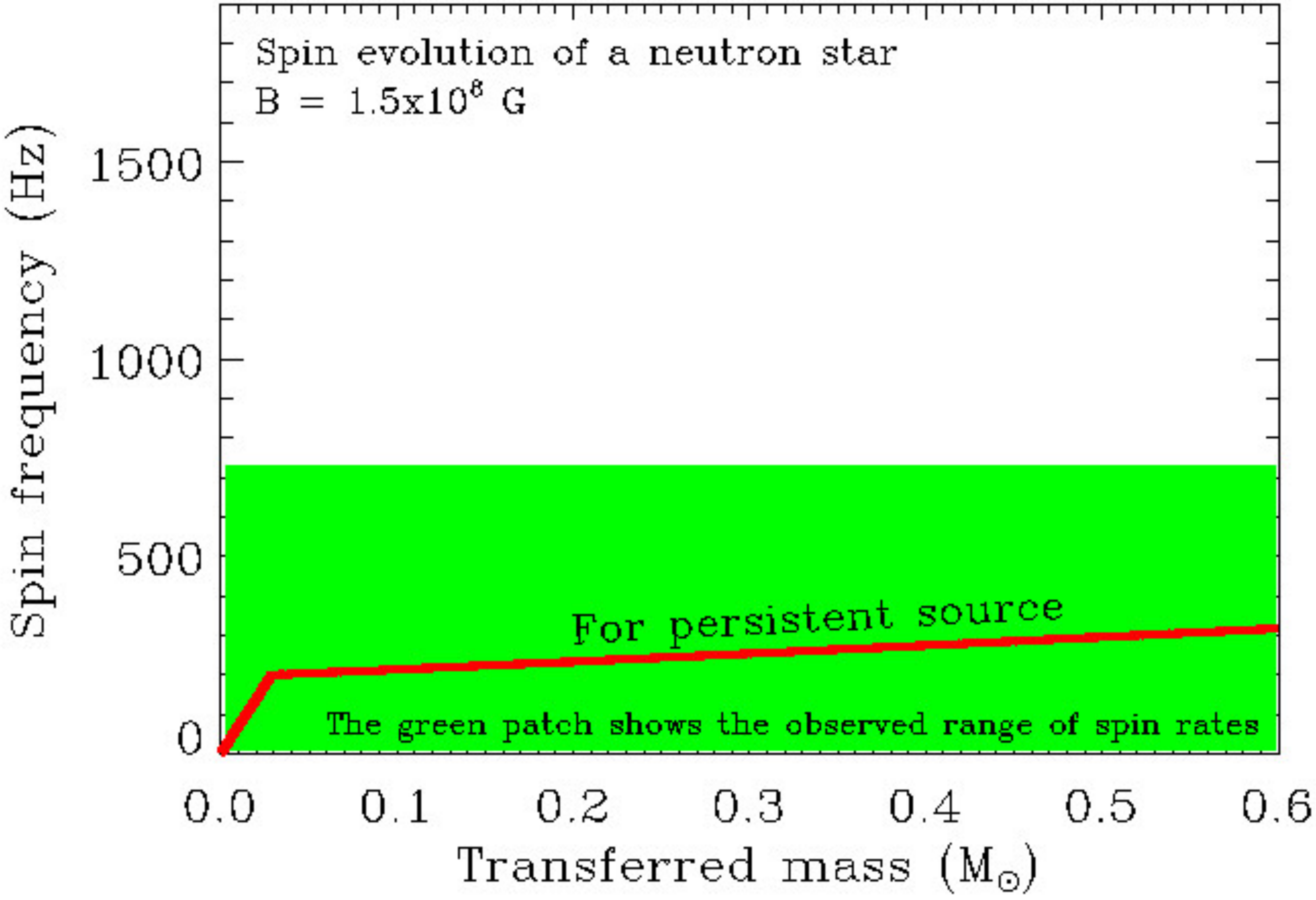
(1)Accretion phase ($r_m < r_{co}$), **positive torque on the neutron star.**

(2)Propeller phase ($r_{co} < r_m < r_{lc}$), **negative torque on the neutron star.**

(3)Quiescent phase, when accretion is stopped by the wind of the pulsar which is turned on.

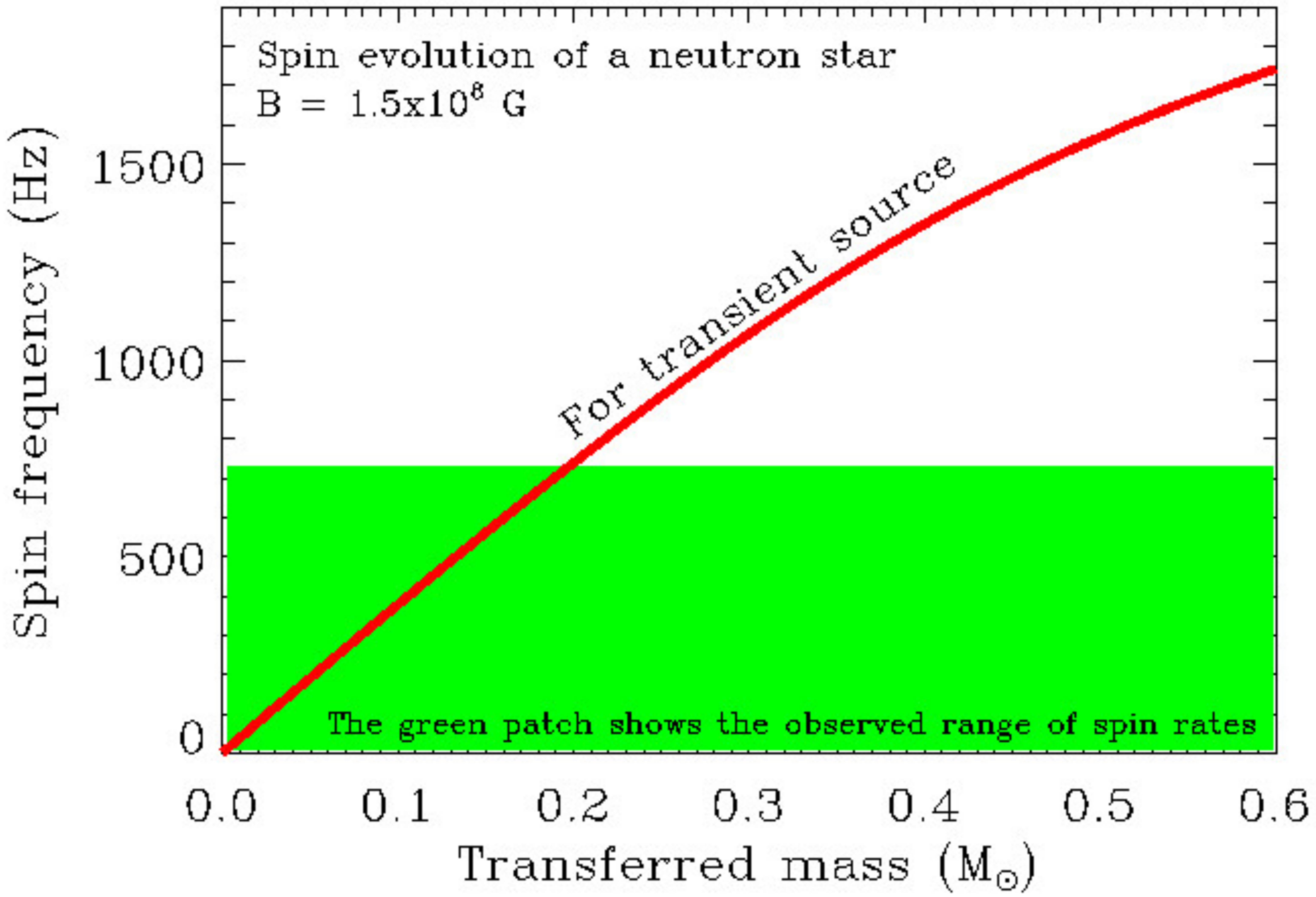
Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



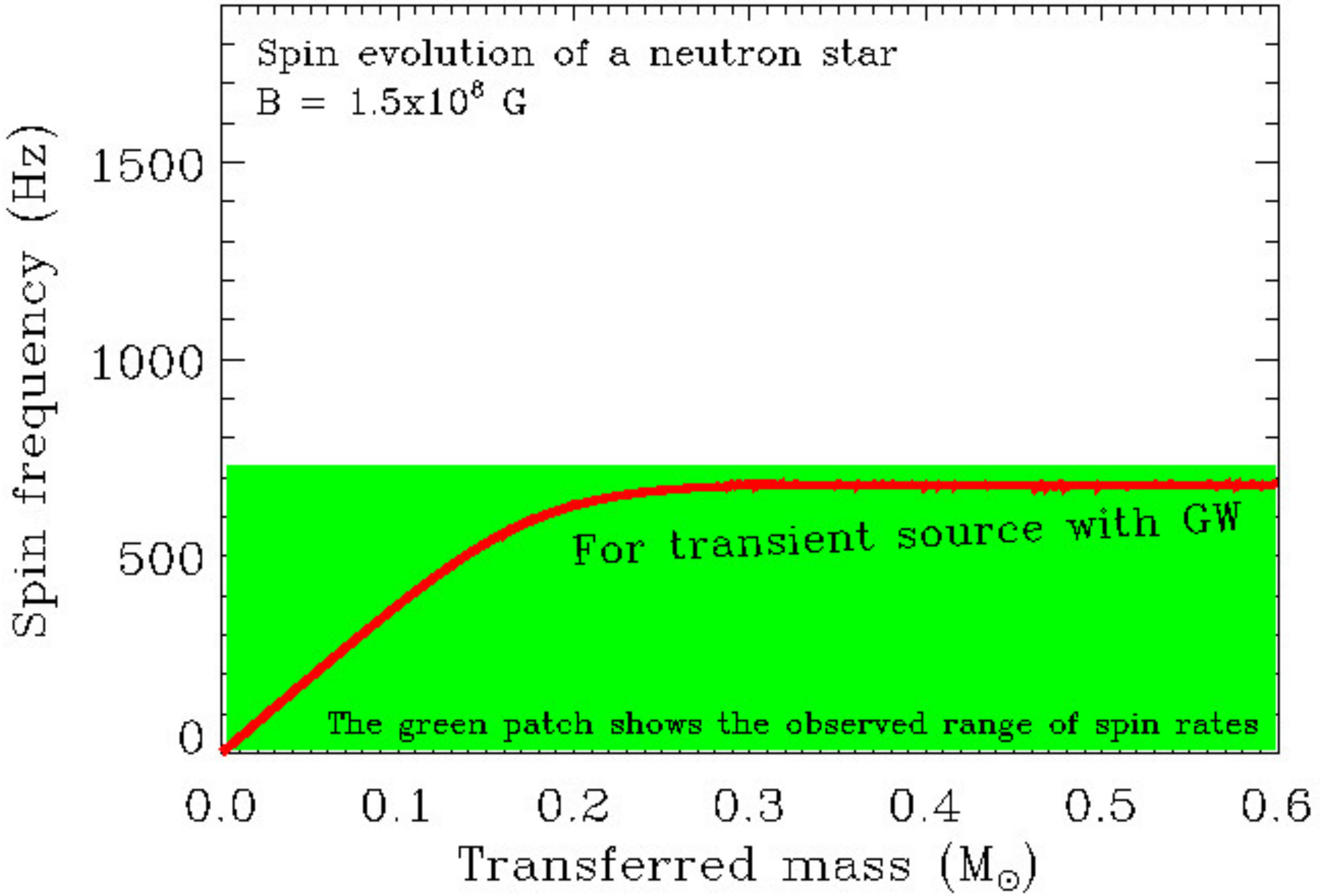
Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



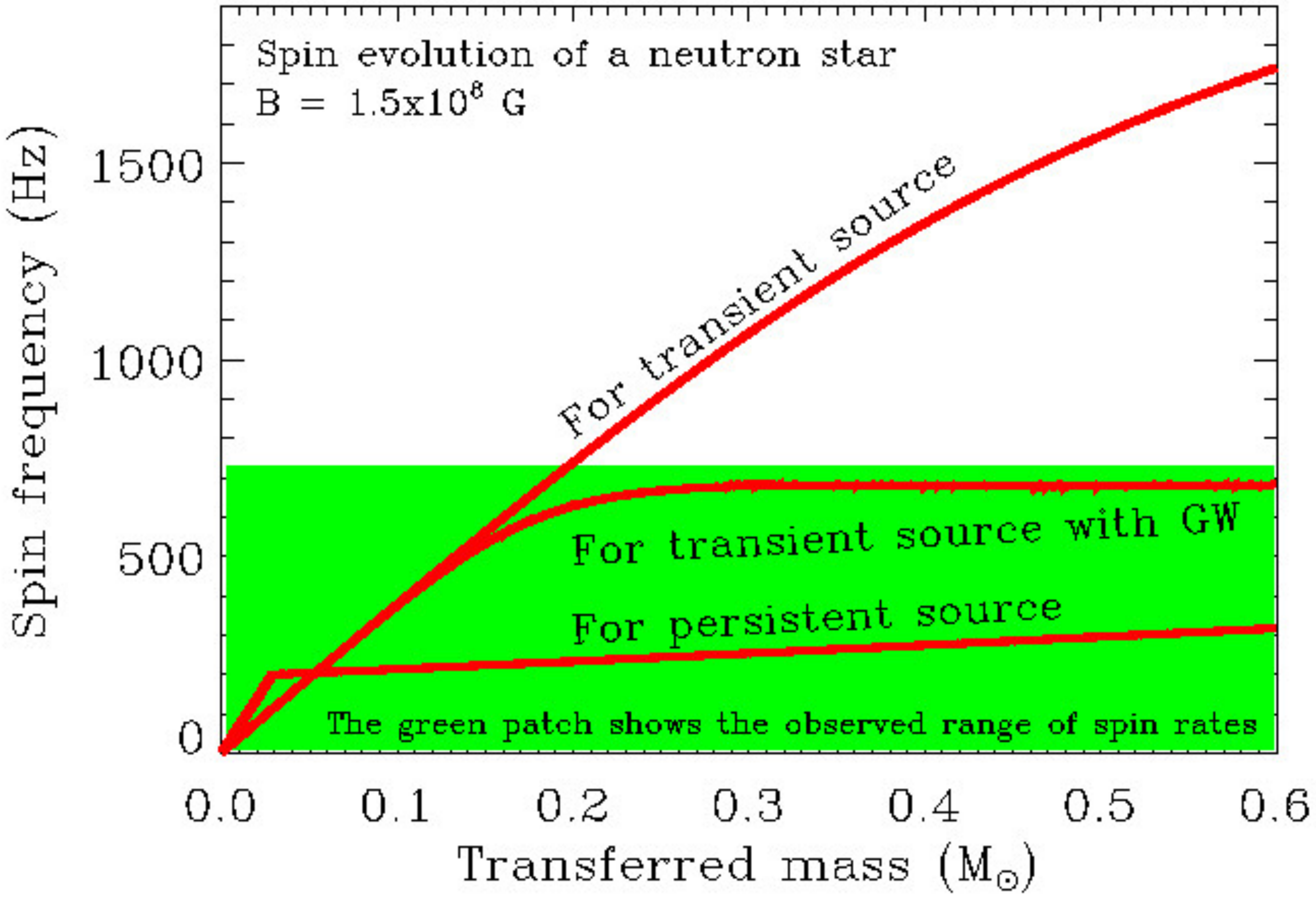
Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



So the spin equilibrium frequency for a transient source is higher than that for a persistent source, and the spin equilibrium frequency can be much larger than the observed upper limit of spin rates, indicating gravitational waves from some sources.

Neutron star's spin frequency ν evolves to become the spin equilibrium frequency

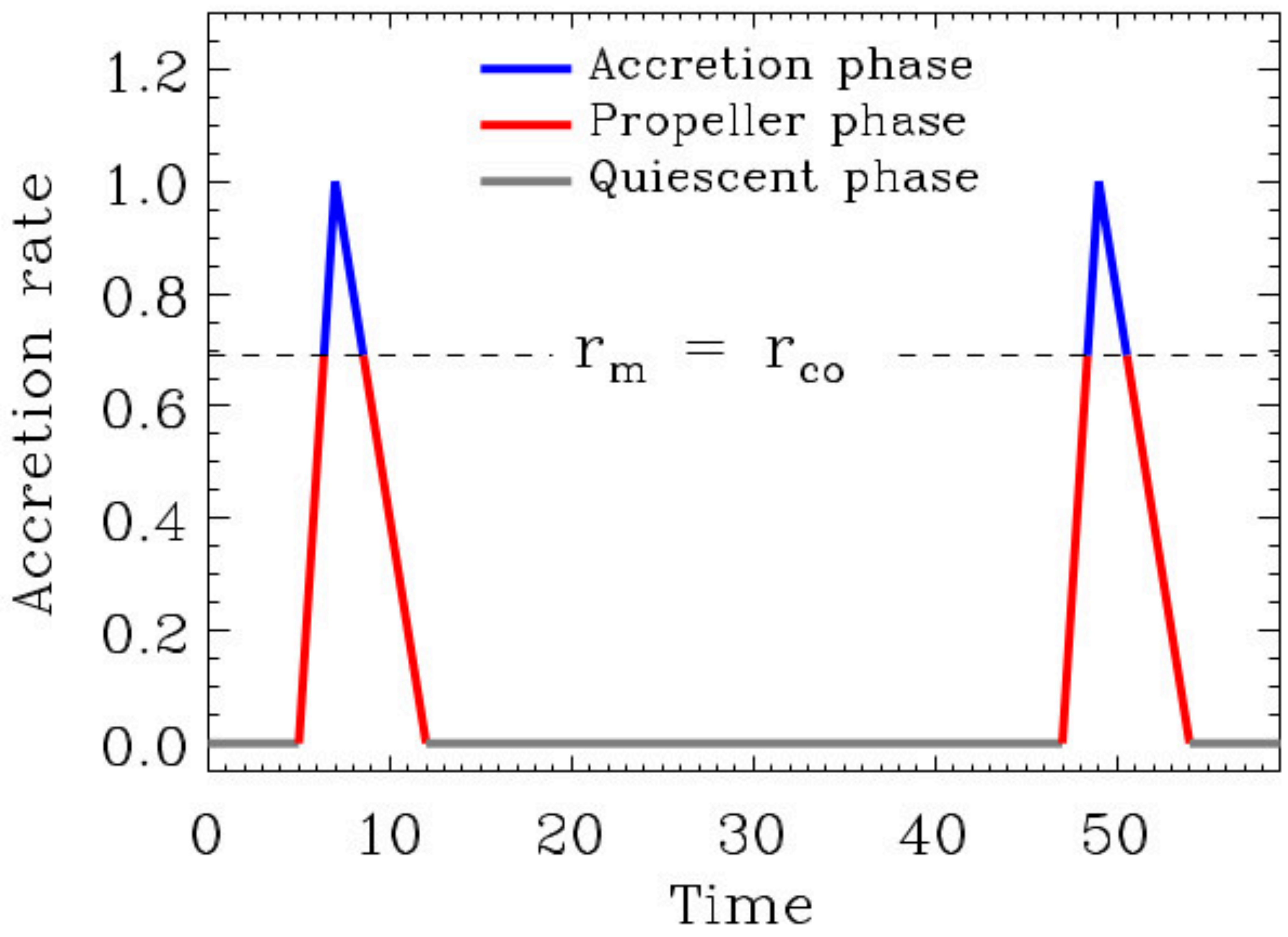
Why is the spin equilibrium frequency much larger for transient accretion?

What is the spin equilibrium condition for a transiently accreting pulsar?

First, we will discuss the concept of the spin equilibrium condition for transient accretion.

Then, we will derive a simple analytical expression of the spin equilibrium frequency and compare it to the numerically computed value to gain insight.

Transiently accreting neutron star: spin equilibrium



Magnetospheric radius

$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, where the accretion disk stops. ($\xi \sim 0.3$-1.4).

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$, where the Keplerian spin frequency is equal to the neutron star spin frequency.

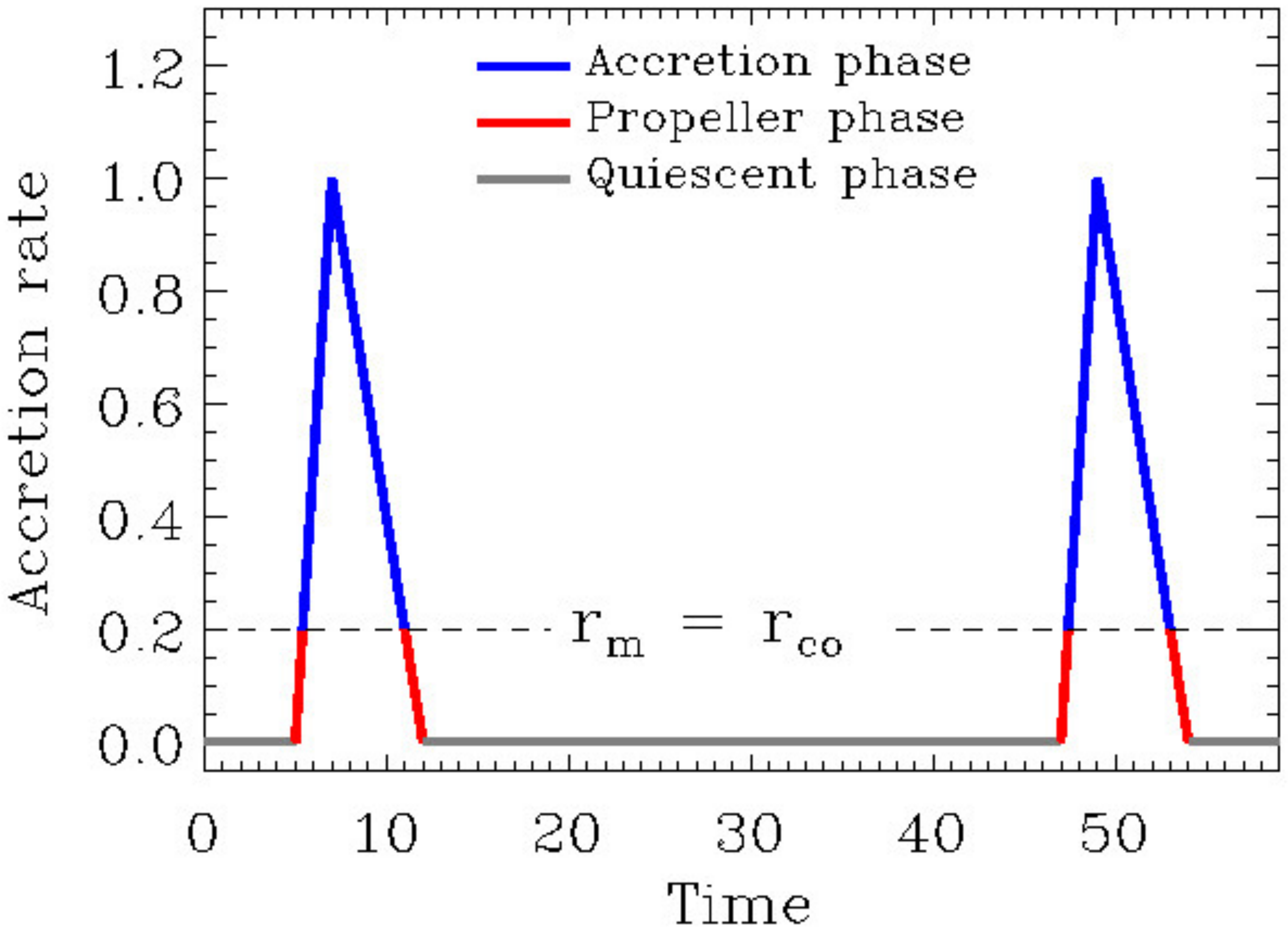
SB & D. Chakrabarty, 2017, ApJ, 835, 4

Recall:

(1) We need a different concept of spin equilibrium for transient accretion, because the standard condition ($r_m = r_{co}$) of spin equilibrium is satisfied for only one accretion rate during an outburst cycle.

(2) $r_m = r_{co}$ separates the spin-up phase and spin-down phase.

Transiently accreting neutron star: spin equilibrium



Magnetospheric radius

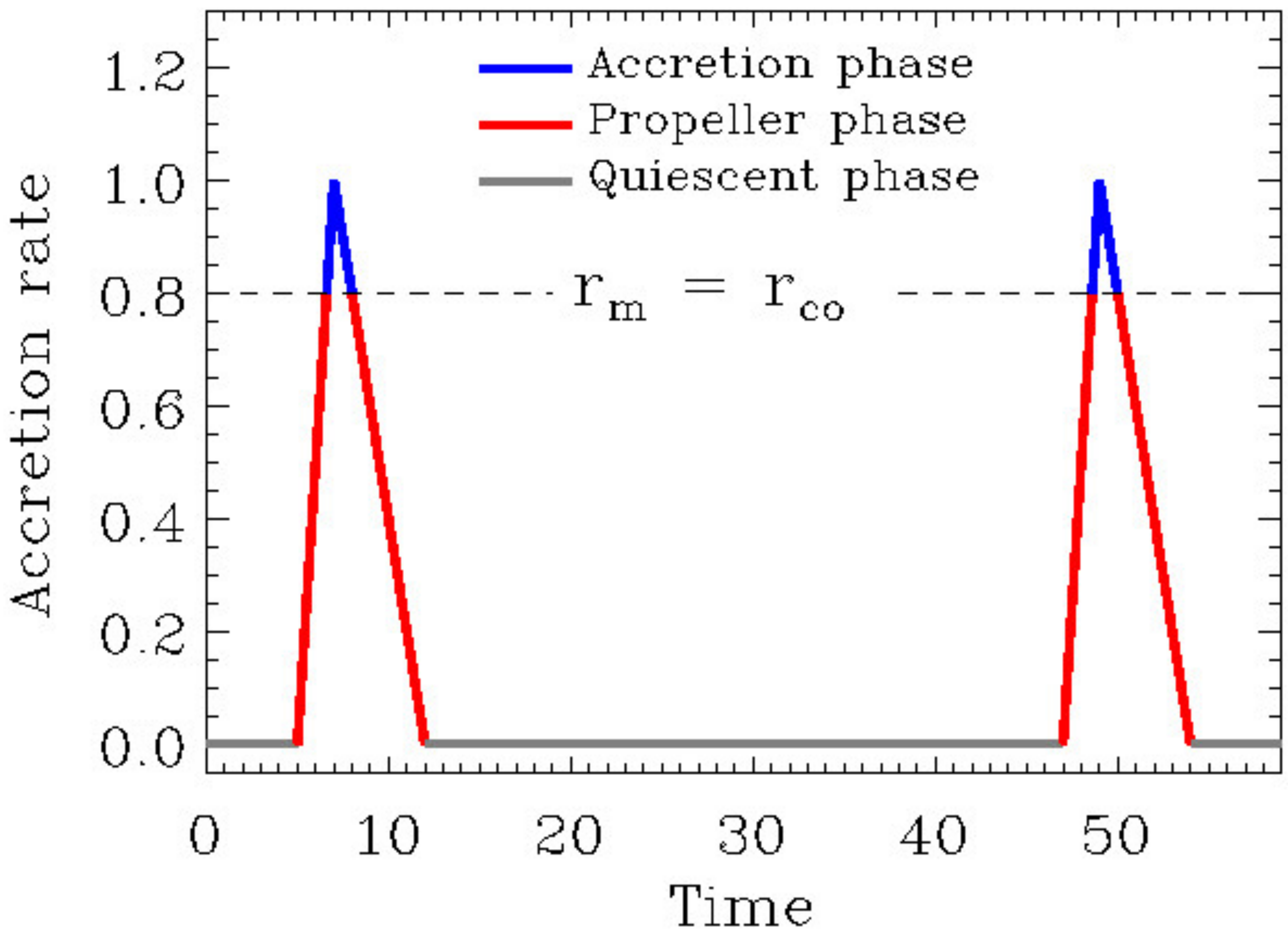
$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, where the accretion disk stops. ($\xi \sim$ 0.3-1.4).

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$, where the Keplerian spin frequency is equal to the neutron star spin frequency.

Low ν value → high r_{co} → $r_m = r_{co}$ condition at low accretion rate → net spin-up

Not in spin-equilibrium.

Transiently accreting neutron star: spin equilibrium



Magnetospheric radius

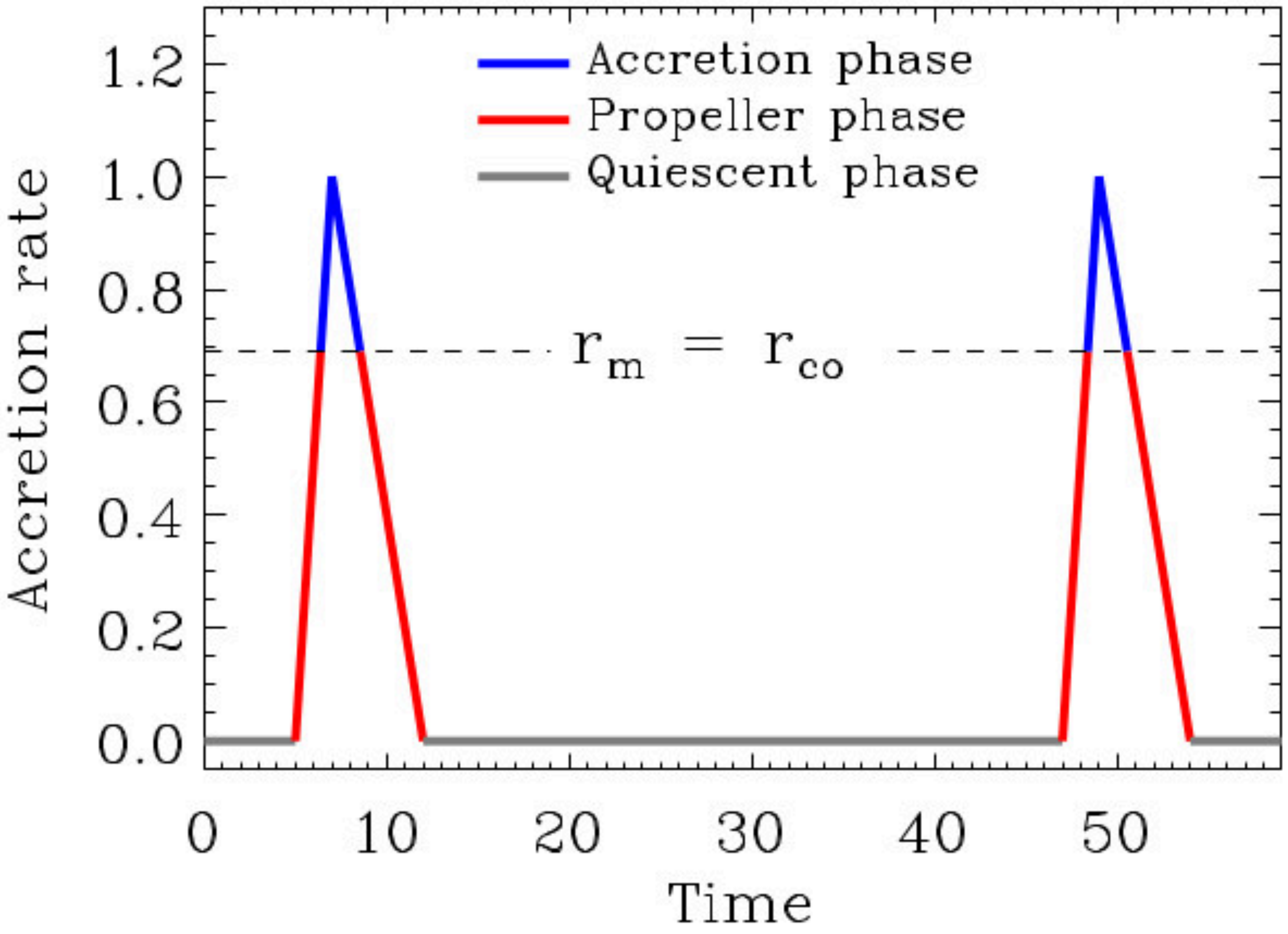
$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, where the accretion disk stops. ($\xi \sim 0.3$-1.4).

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$, where the Keplerian spin frequency is equal to the neutron star spin frequency.

High ν value → low r_{co} → $r_m = r_{co}$ condition at high accretion rate → net spin-down

Not in spin-equilibrium.

Transiently accreting neutron star: spin equilibrium



Magnetospheric radius

$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, where the accretion disk stops. ($\xi \sim$ 0.3-1.4).

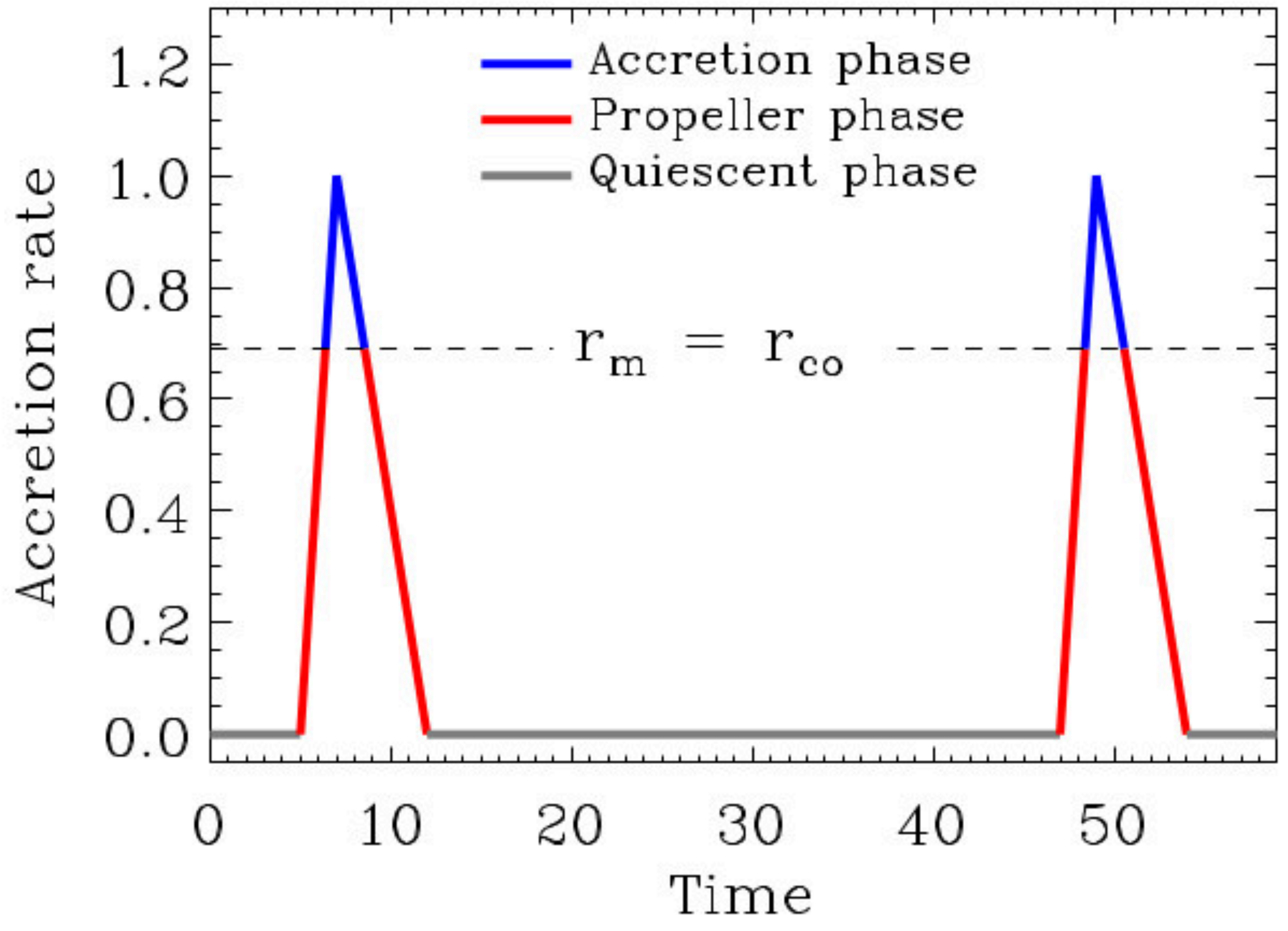
Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$, where the Keplerian spin frequency is equal to the neutron star spin frequency.

In spin-equilibrium. Here too a self-regulated mechanism operates.

For a transient source, the spin equilibrium is reached if total angular momentum transferred to the neutron star is zero during an outburst cycle.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Transiently accreting neutron star: spin equilibrium



Magnetospheric radius

$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$, **where the accretion disk stops.** ($\xi \sim$ **0.3-1.4).**

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$,

where the Keplerian spin frequency is equal to the neutron star spin frequency.

In spin-equilibrium. Here too a self-regulated mechanism operates.

For this spin equilibrium (i.e., zero angular momentum transfer in one accretion rate variation cycle), the $r_m = r_{co}$ condition gives the spin equilibrium frequency ($\nu_{eq,eff}$) for a transiently accreting neutron star.

$\nu_{eq,eff}$ **is the maximum frequency for a transiently accreting neutron star (because the star will spin down for a higher frequency).**

SB & D. Chakrabarty, 2017, ApJ, 835, 4

How do we analytically estimate the spin equilibrium frequency for a transiently accreting neutron star?

(A simple calculation to gain insight)

Analytical calculation of spin equilibrium frequency for transient accretion

Disk-magnetosphere interaction Torques

For the accretion phase:

$$\frac{\mathrm{d}J}{\mathrm{d}t} = \dot{M}\sqrt{GMr_{\mathrm{m}}} + \frac{\mu^2}{9r_{\mathrm{m}}^3}\left[2\left(\frac{r_{\mathrm{m}}}{r_{\mathrm{co}}}\right)^3 - 6\left(\frac{r_{\mathrm{m}}}{r_{\mathrm{co}}}\right)^{3/2} + 3\right]$$

For the propeller phase:

$$\frac{\mathrm{d}J}{\mathrm{d}t} = -\eta\dot{M}\sqrt{GMr_{\mathrm{m}}} - \frac{\mu^2}{9r_{\mathrm{m}}^3}\left[3 - 2\left(\frac{r_{\mathrm{co}}}{r_{\mathrm{m}}}\right)^{3/2}\right]$$

These expressions can be approximated (with a few percent error) to the following compact form: $\frac{\mathrm{d}J}{\mathrm{d}t} = \pm A\dot{M}^{6/7}$**, where *A* is a positive constant and $+$ and $-$ signs correspond to accretion (spin-up) and propeller (spin-down) phases respectively.**

SB & D. Chakrabarty, 2017, ApJ, 835, 4

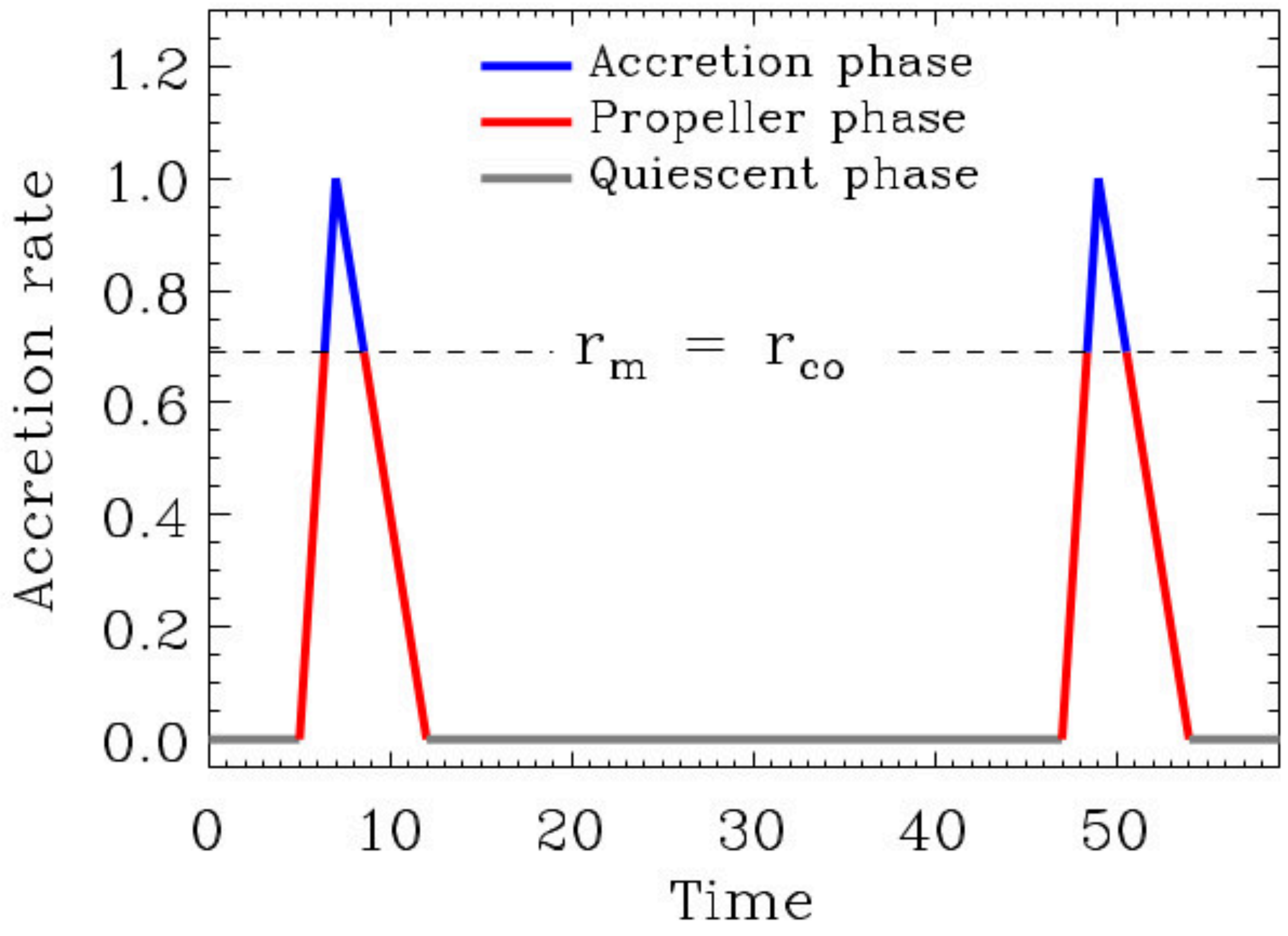
Analytical calculation of spin equilibrium frequency for transient accretion

Torque on the neutron star : $\frac{\mathrm{d}J}{\mathrm{d}t} = \pm A\dot{M}^{6/7}$

Total angular momentum transfer : $\Delta J = \int \mathrm{d}J$

$$= +[A\int \dot{M}^{6/7}dt]_{\mathrm{Acc}} - [A\int \dot{M}^{6/7}dt]_{\mathrm{Prop}}$$

$$= +[A_1\int \dot{M}^{6/7}d\dot{M}]_{\mathrm{Acc}} - [A_1\int \dot{M}^{6/7}d\dot{M}]_{\mathrm{Prop}} = 0$$



Here (for triangular outburst profile), $A_1 = A/(\mathrm{d}\dot{M}/\mathrm{d}t) = \mathrm{constant}$

This gives $\dot{M}_{\mathrm{max}}^{13/7} - \dot{M}_{\mathrm{eff}}^{13/7} = \dot{M}_{\mathrm{eff}}^{13/7}$

Here, $\dot{M}_{\mathrm{max}}$ is the accretion rate corresponding to the outburst peak, and $\dot{M}_{\mathrm{eff}}$ is the accretion rate corresponding to the transition between accretion and propeller phases.

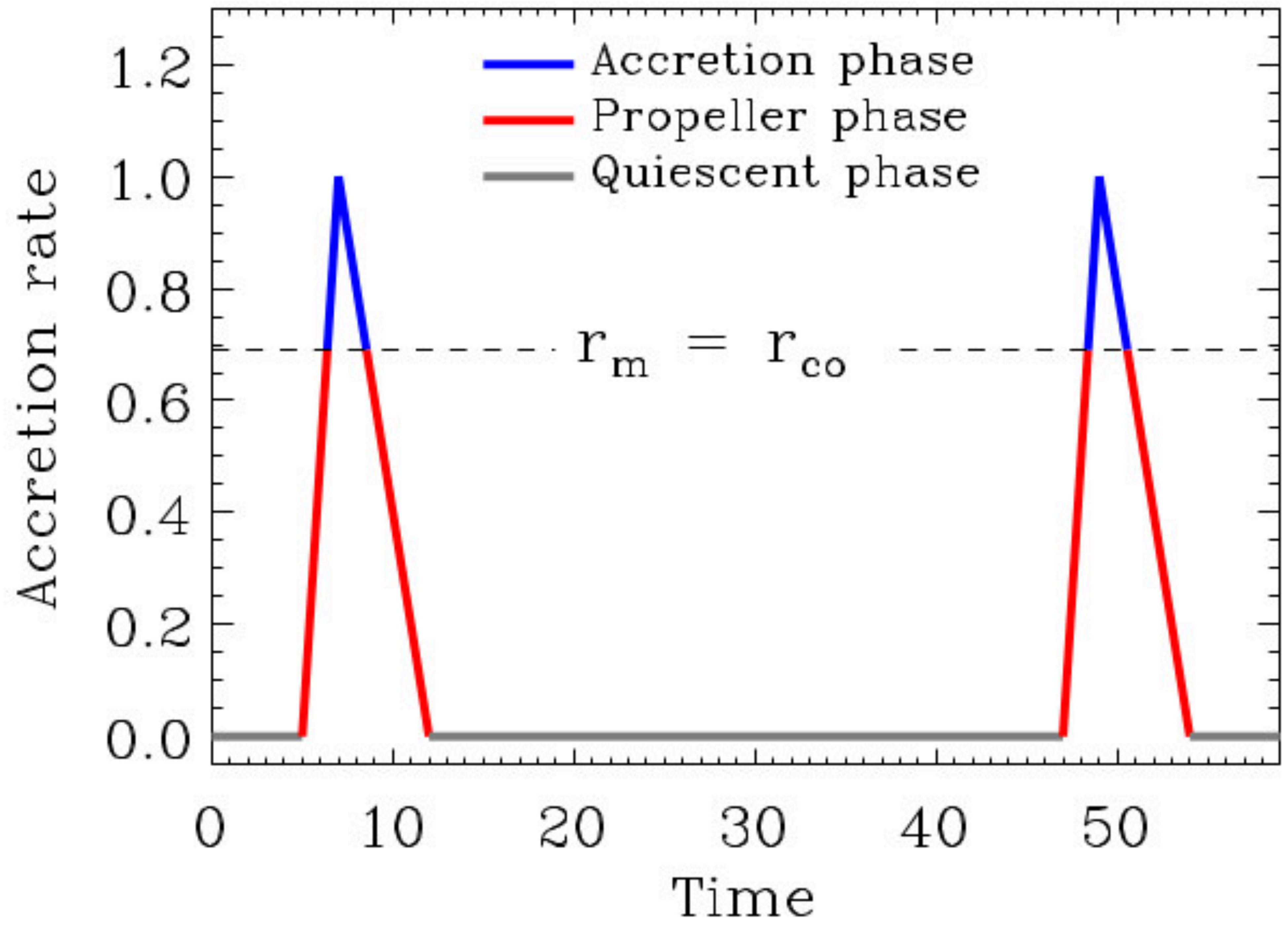
SB & D. Chakrabarty, 2017, ApJ, 835, 4

Analytical calculation of spin equilibrium frequency for transient accretion

Torque on the neutron star : $\frac{\mathrm{d}J}{\mathrm{d}t} = \pm A\dot{M}^{6/7}$

This gives $\dot{M}_{\mathrm{max}}^{13/7} - \dot{M}_{\mathrm{eff}}^{13/7} = \dot{M}_{\mathrm{eff}}^{13/7}$

Here, $\dot{M}_{\mathrm{max}}$ is the accretion rate corresponding to the outburst peak, and $\dot{M}_{\mathrm{eff}}$ is the accretion rate corresponding to the transition between accretion and propeller phases.



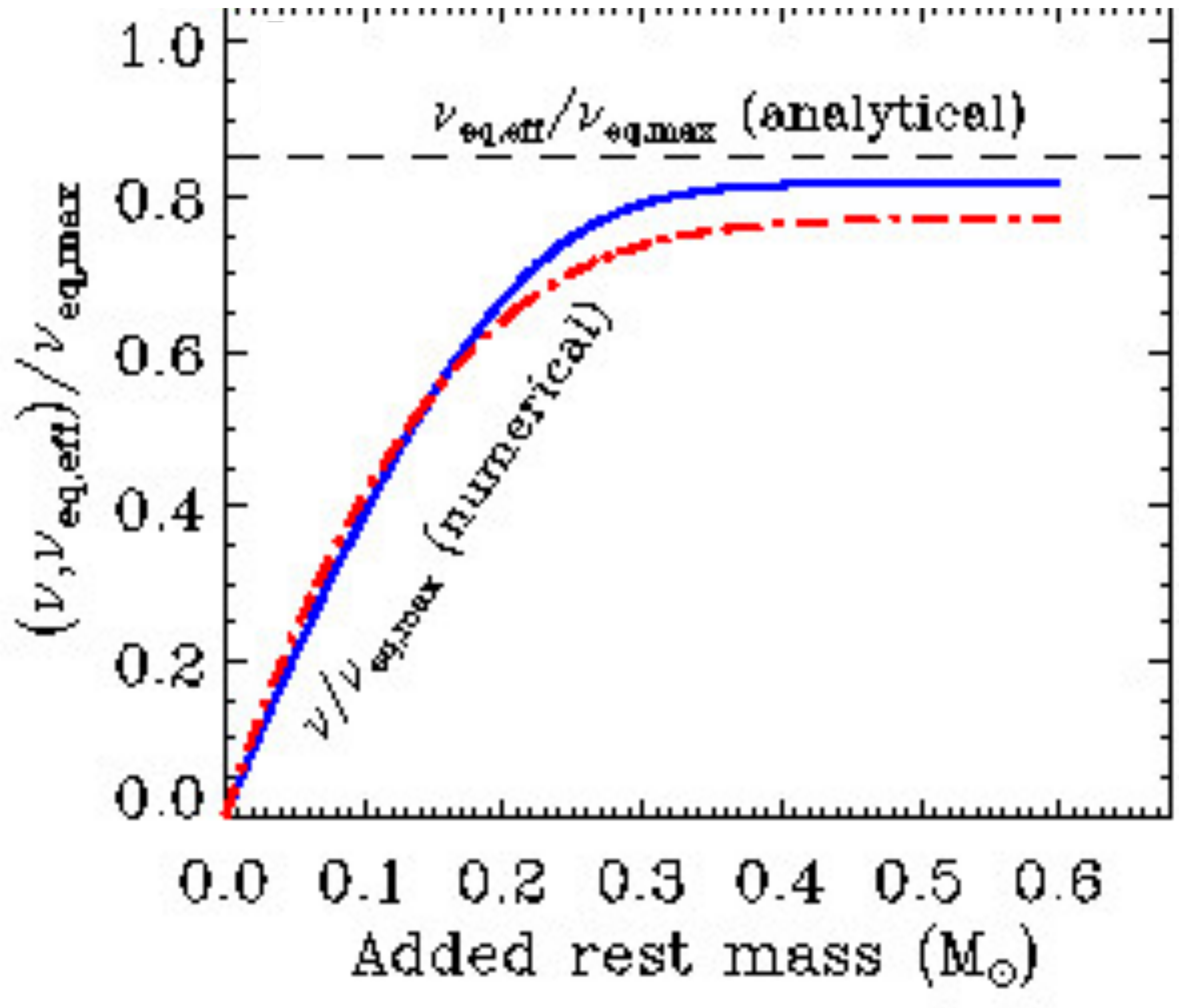
L.H.S. gives the positive angular momentum transfer in the accretion phase, while the R.H.S. gives the negative angular momentum transfer in the propeller phase.

This gives $\dot{M}_{\mathrm{eff}} = 0.69\dot{M}_{\mathrm{max}} \Rightarrow \nu_{\mathrm{eq,eff}} = 0.85\nu_{\mathrm{eq,max}}$ **[using** $r_{\mathrm{m}} = r_{\mathrm{co}} \Rightarrow \nu_{\mathrm{eq}} \propto \dot{M}^{3/7}$**]**

This gives a spin equilibrium frequency ($\nu_{\mathrm{eq,eff}}$) expression for transient accretion.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Transiently accreting neutron star: spin equilibrium
Comparison between numerical and simple analytical results



A simple analytical expression of spin equilibrium frequency is given by

$$\frac{\nu_{\mathrm{eq,eff}}}{\nu_{\mathrm{eq,max}}} = 0.85.$$

This matches the numerical results (blue [for simple torque] and red [for more realistic torque] curves) within a few percent.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Implication of the new picture: a counter-intuitive scenario

What have we found?

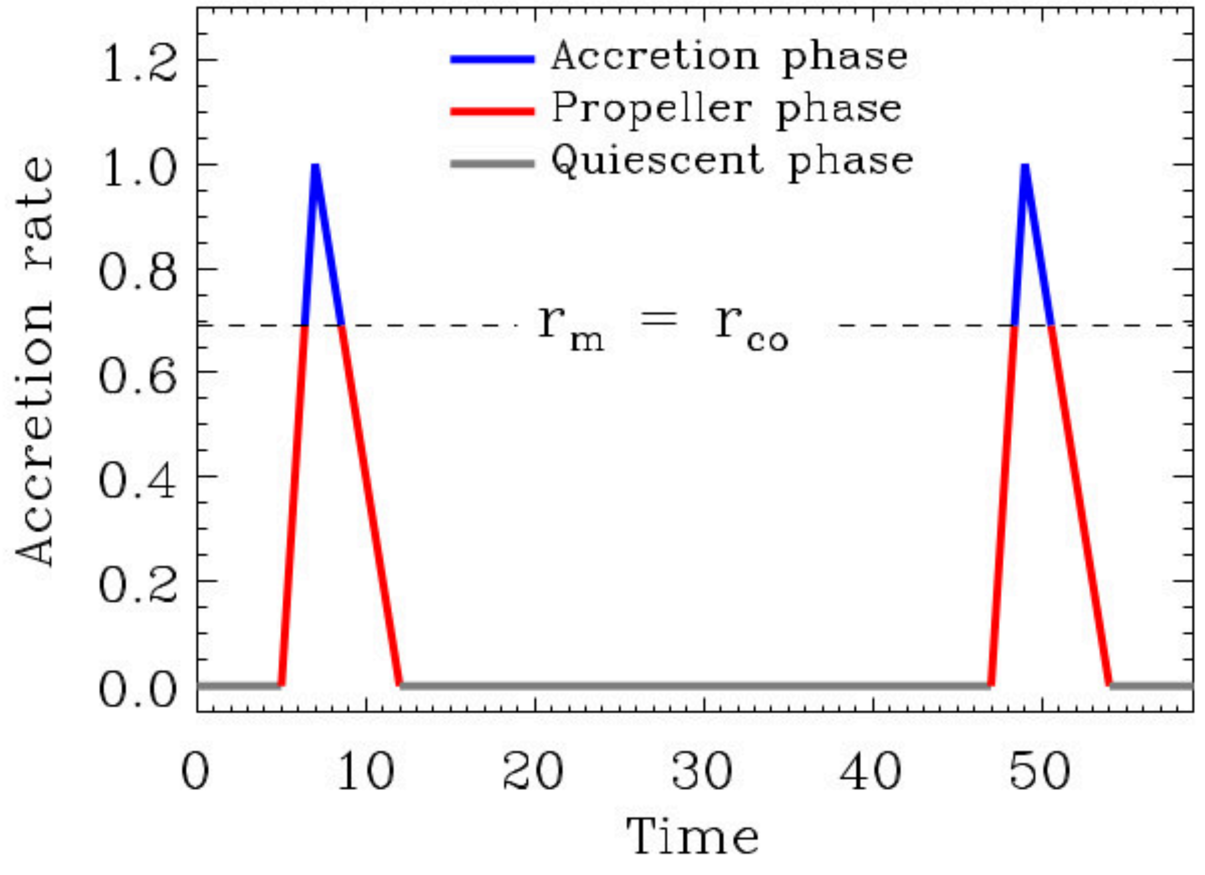
For persistent accretion, the neutron star spin frequency (ν) approaches $\nu_{\rm eq} \propto \dot{M}_{\rm av}^{3/7}$

For transient accretion, the neutron star spin frequency (ν) approaches $\nu_{\rm eq,eff} \propto \dot{M}_{\rm max}^{3/7}$

The peak accretion rate $\dot{M}_{\rm max}$ of an outburst is typically much higher than the long term average accretion rate $\dot{M}_{\rm av}$. Therefore, $\nu_{\rm eq,eff} \gg \nu_{\rm eq}$.

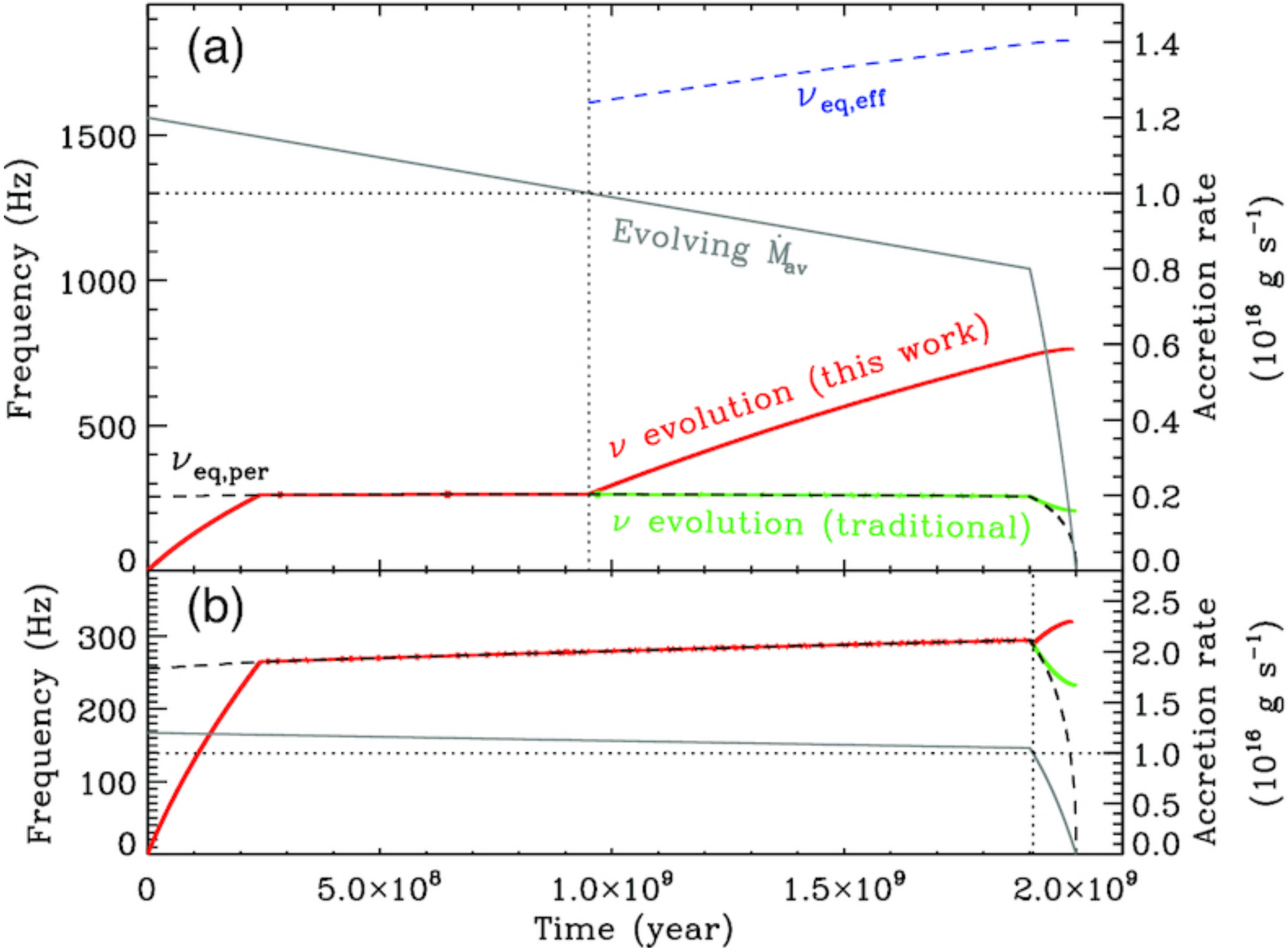
Therefore, the spin evolution of a neutron star in its LMXB phase can happen in two distinctly different modes — one when the accretion is persistent and another when the accretion is transient.

Transient accretion happens due to a thermo-viscous instability, when $\dot{M}_{\rm av}$ falls below a certain value. Thus ν can increase when accretion rate decreases, which is counter-intuitive.



SB, 2021, MNRAS, 502, L45

Implication of the new picture: a counter-intuitive scenario



The figure shows that the spin evolution of neutron stars can be much more complex than and the spin frequency can be drastically different from that predicted by the traditional/standard method.

SB, 2021, MNRAS, 502, L45

Takeaway message

1. Transient accretion has a crucial effect on the spin evolution of millisecond pulsars.

2. This effect indicates continuous gravitational wave emission from at least some millisecond pulsars.

Takeaway message

1. Transient accretion has a crucial effect on the spin evolution of millisecond pulsars.

2. This effect indicates continuous gravitational wave emission from at least some millisecond pulsars.

Thank you!